Outage Probability of Dual-Hop Non-regenerative Relaying System over Weibull Fading Channels

Aleksandra M. Cvetkovic[1], Dejan N. Milic [2], Jelena A. Anastasov [3]

Abstract - **In this paper analytical approach for evaluating performance of dual-hop cooperative link over Weibull fading channel is presented. The expressions for the outage probability for both, channel state information and fixed gain relay are derived. Numerical results are also presented. Performances are compared for both types of non-regenerative relays.**

Keywords – **dual hop cooperative system, non-regenerative relay, outage probability, Weibull fading.**

I. Introduction

Recently, non-regenerative dual-hop cooperative links have gained great interest in wireless communication systems [1]. When the annoyances over direct transmission path are evident, the third terminal between the source and destination one, may be used. The scheme is known as a cooperative diversity scheme with dual-hop transmission link which enables better communication between nodes [2].

The concept of cooperative diversity is often used in diversity systems. Mobile users relay signals for each other and exploit all the benefits of spatial diversity. There is a number of research papers on multiuser spatial diversity systems with channel-state-information-(CSI) based relays over Rayleigh [3] and Nakagami-*m* [4] fading channels. Certain low complexity cooperative protocols using the three-terminal cases have been proposed [4, 5]. These protocols have been applied with different relaying models in regenerative and non-regenerative relays. Amplify-and-forward is a non-regenerative relaying model, while decode-and-forward is a regenerative one. CSI-based relays use instantaneous CSI of incoming signals to control the output gain and thus limit the power of transmitted signal. Non-regenerative relay is an alternate configuration employing the fixed gain relay. The case of fixed gain relay has a simpler practical realization [4, 5].

In wireless communications, the presence of reflectors in the environment surrounding a transmitter and receiver create multiple paths that a transmitted signal can traverse. As a result, the receiver sees the superposition of multiple copies of the transmitted signal, each traversing a different path [6, 7]. Recently, Weibull fading channel model has been used to analyze performance in mobile communication environment. Specifically, it has been used for evaluation of statistical parameters of different space-diversity techniques, such as outage probability and average output SNR [8, 9]. Weibull distribution is often used to describe environments in which Rayleigh distribution is not an adequate choise.

In this paper, an approach to the performance analysis of non-regenerative dual-hop cooperative system over Weibull fading channel is presented. Useful analytical and numerical results are derived. The outage probability function is evaluated for both channel state information and fixed gain relays, using selection combining receiver. Numerical results are presented for variety of average signal-to-noise ratio threshold parameters on direct path.

II. Model of Non-Regenerative Cooperative System

Model of a wireless communication system using the cooperative diversity link considered in this paper, is shown in Fig. 1.

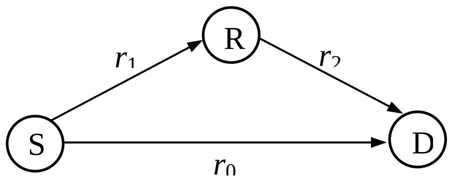

Fig. 1. Model of system

The source terminal *S* communicates with the destination terminal *D* over direct path as well as over the cooperative one. Period of transmission is divided into two signaling intervals. During the first interval, terminal *S* communicates with the relay terminal *R* and the destination terminal *D*. During the second interval, only the relay terminal *R* communicates with the terminal *D* [2]. The destination terminal combines the received signals using a SC. Assuming that *S* is transmitting a signal with an average power normalized to unity, the instantaneous equivalent SNR of the dual-hop path can be expressed as [2]:

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$$\gamma_{end} = \frac{\left(\frac{r_1^2}{N_{0,1}}\right)\left(\frac{r_2^2}{N_{0,2}}\right)}{\left(\frac{r_2^2}{N_{0,2}}\right)+\left(\frac{1}{g^2 N_{0,1}}\right)} \quad (1)$$

In Eq. (1), r_i represents fading amplitude on i-th path, where i=0, 1, 2 (with r_0 representing the fading amplitude on direct path), $N_{0,i}$ represents one-sided power spectrum density of additive white Gaussian noise on i-th path, and g is the relay gain. Since r_i is modeled as Weibull random variable (RV), the instantaneous SNR, $\gamma_i = r_i^2 / N_{0,i}$, is a distributed RV with probability density function (PDF) given by:

$$f_{\gamma_i}(\gamma) = \frac{\alpha_i}{2\bar{\gamma}_i}\gamma^{\alpha_i/2-1}\exp\left\{-\frac{\gamma^{\alpha_i/2}}{\bar{\gamma}_i}\right\} \quad (2)$$

where α_i represents the Weibull fading parameter. With increasing α_i, fading severity becomes less significant, and vice-versa. For $\alpha_i = 2$ Weibull distribution becomes a Rayleigh distribution. We consider a case where fading parameter over direct link is α_0, and fading parameters over the two indirect relaying links are equal, $\alpha_1=\alpha_2=\alpha$. Cumulative distribution function (CDF) of a Weibull RV can be written as:

$$F_{\gamma_i}(\gamma) = 1-\exp\left\{-\frac{\gamma^{\alpha_i/2}}{\bar{\gamma}_i}\right\} \quad (3)$$

When considering a CSI relay, the gain is used to adaptively limit the output power, and is given by [10]:

$$g_1^2 = \frac{1}{r_1^2 + N_{0,1}} \quad (4)$$

Therefore, the instantaneous equivalent SNR of the dual-hop path can be expressed as [3]:

$$\gamma_{eq1} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} \quad (5)$$

By contrast, when R has a fixed gain given by [11]:

$$g_2^2 = \frac{1}{CN_{0,1}} \quad (6)$$

instantaneous SNR of dual-hop path can be expressed as in [11]:

$$\gamma_{eq2} = \frac{\gamma_1\gamma_2}{C+\gamma_2} \quad (7)$$

III. OUTAGE PROBABILITY

If there is selection combining in terminal D, equivalent output SNR is evaluated as $\gamma_{eq,sc}=\max(\gamma_{eqi}, \gamma_0)$. The outage probability, for independent signal, can be defined as probability which falls below a given threshold and expressed as [12]:

$$P_{out} = P_r\left[\gamma_{eq,sc} \le \gamma_{th}\right] = F_{\gamma_0}(\gamma_{th})F_{eqi}(\gamma_{th}) \quad (8)$$

In the above equation, $F_{\gamma 0}$ is CDF on the direct path and can be evaluated based on Eq. (3). CDF of equivalent SNR of the relayed fading channel is evaluated next. Moreover, Weibull fading parameter α_0 over direct path is less than the two fading parameters α over cooperative link, corresponding to a situation where a direct path is affected by a more severe fading. It follows that fading severity over cooperative path is less than that one over direct path. For the case of non-regenerative link with fixed gain relay, the cumulative density function is given by:

$$\begin{aligned}
F_{eq1}(\gamma) &= \int_0^\infty P_r\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} \le \gamma \middle| \gamma_2\right) f_{\gamma_2}(\gamma_2)d\gamma_2 = \\
&\int_0^\gamma P_r\left(\gamma_1 \ge \frac{\gamma(\gamma_2+1)}{\gamma_2-\gamma} \middle| \gamma_2\right) f_{\gamma_2}(\gamma_2)d\gamma_2 + \\
&\int_\gamma^\infty P_r\left(\gamma_1 \le \frac{\gamma(\gamma_2+1)}{\gamma_2-\gamma} \middle| \gamma_2\right) f_{\gamma_2}(\gamma_2)d\gamma_2 = \\
&1-\int_\gamma^\infty \frac{\alpha}{2\bar{\gamma}_2}\gamma_2^{\alpha/2-1}\exp\left\{-\frac{\gamma^{\alpha/2}}{\bar{\gamma}_1}\left(\frac{\gamma_2+1}{\gamma_2-\gamma}\right)^{\alpha/2}\right\} \\
&\exp\left\{-\frac{\gamma_2^{\alpha/2}}{\bar{\gamma}_2}\right\}d\gamma_2
\end{aligned} \quad (9)$$

By substituting Eqs. (9) and (3) into Eq. (8), the outage probability and cumulative density function for dual-hop link, when gain relay is variable and based on channel state information, can be evaluated.

For the case of fixed gain relay CDF can be written as:

$$\begin{aligned}
F_{eq2}(\gamma) &= \int_0^\infty P_r\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1} \le \gamma \middle| \gamma_2\right) f_{\gamma_2}(\gamma_2)d\gamma_2 = \\
&\int_0^\infty P_r\left(\gamma_1 \le \frac{\gamma(\gamma_2+C)}{\gamma_2} \middle| \gamma_2\right) f_{\gamma_2}(\gamma_2)d\gamma_2 = \\
&1-\int_0^\infty \frac{\alpha}{2\bar{\gamma}_2}\gamma_2^{\alpha/2-1}\exp\left\{-\frac{\gamma^{\alpha/2}}{\bar{\gamma}_1}\left(\frac{\gamma_2+C}{\gamma_2}\right)^{\alpha/2}\right\} \\
&\exp\left\{-\frac{\gamma_2^{\alpha/2}}{\bar{\gamma}_2}\right\}d\gamma_2
\end{aligned} \quad (10)$$

By substituting Eqs. (10) and (3) into Eq. (8), the outage probability for the system shown at Fig. 1, using the fixed gain relay, can also be evaluated.

IV. NUMERICAL RESULTS

In this section, according to analytical expressions for outage probability of cooperative dual-hop non-regenerative link with fixed gain relay, numerical results are presented in the figures below. Fig. 2 depicts the outage probability as a function of the average SNR of the direct link, $\overline{\gamma}_0$, for both types of relays. Thereat, CSI-based relay and fixed gain relay are considered. Comparing the performance between relay over cooperative link and transmission over direct path, it is evident that the outage probability of non-regenerative relay is less than that of direct link. It can also be seen that for large SNR values performance gain of both non-regenerative relays increases.

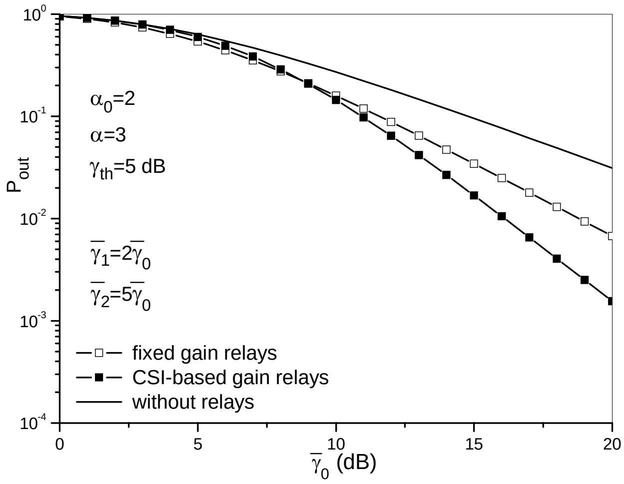

Fig. 2. Outage probability of both non-regenerative relays and direct path relay versus average SNR $\overline{\gamma}_0$

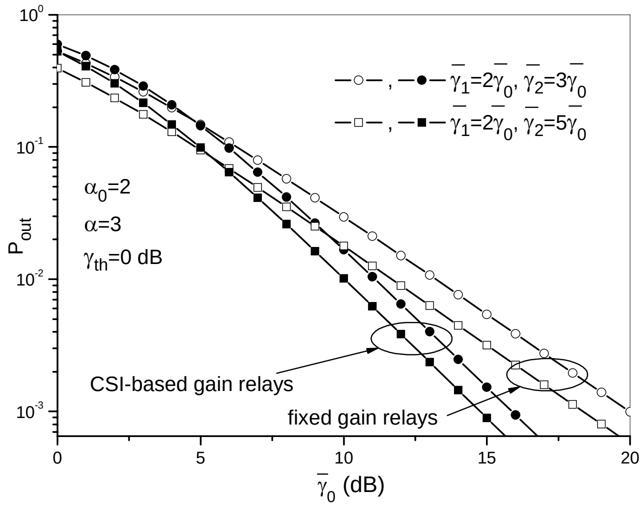

Fig. 3. Outage probability of dual-hop cooperative links with CSI-based relay and fixed gain relay versus SNR $\overline{\gamma}_0$

In Fig. 3 and Fig. 4 comparation between CSI-based and fixed gain relay performance is depicted graphicaly. In Fig. 3, outage probability for different SNR values of transmission from terminal R to destination terminal D, is presented. If average SNR values of cooperative transmission (from terminal R to destination terminal D) increase, then the outage probability decreases. For larger values of $\overline{\gamma}_2$, dual-hop systems employed with fixed gain relays outperform those with variable gain. In Fig. 4, we consider influence of fading severity on outage probabilities. Furthermore, it is observed that as α increases, system performance becomes better. Considering the higher complexity nature of CSI-based relays, our results show that fixed gain relays may serve as an efficient replacement in relayed transmission. This is evident for lower SNR values where performance improvement of CSI-based relays is insignificant. When the fading severity is relatively low, fixed-gain relays can be effective up to a higher range of SNR values.

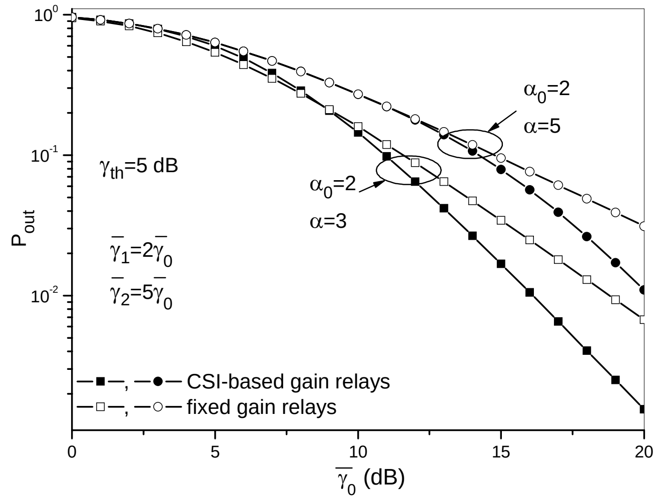

Fig. 4. Outage probability of non-regenerative cooperative link for different transmission conditions

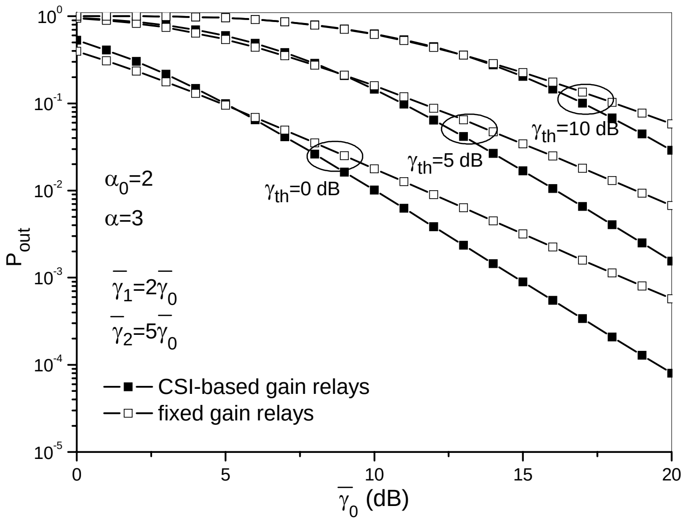

Fig. 5. Outage probability of dual-hop non-regenerative relaying system versus threshold γ_{th}

Also, it is interesting to note that for medium and large SNR values CSI-based relays outperforms those with fixed gain (Fig. 5). At the other side, for low SNR values fixed gain relay outperforms CSI-based relay. This happens due to the fact that the maximum value of the CSI-based gain relays, $g_1=1/N_{0,}$ when α_1->0, which is very possible in the low average SNR regime [2].

V. CONCLUSION

In this paper, the performance analysis of dual-hop cooperative diversity system using non-regenerative relays and selection combining at destination terminal has been considered. Analytical and numerical results of outage probability over Weibull fading channels has been presented. It can be seen that dual-hop links outperform the systems with direct links. Also, our results show that fixed gain relays can in some cases be used in place of CSI-based relays of higher comlexity. If the terminal *R* is optimaly located, fading severity will decrease, thus providing better system performance.

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