Analysis and Obtaining the Bit Error Rates (BER) in Turbo Code Decoder Algorithm

Bozhidar S. Stefanov[1]

Abstract – **Turbo code decoder algorithm is analysed in this paper. The performance of turbo code used in Code Division Multiple Access (CDMA) reverse or forward link under Additive White Gaussian Noise (AWGN) and slow fading channels is evaluated. The bit error rates (BER) of turbo code at low signal-to-noise ratio (SNR) are obtained by simulations.**

Keywords – **Turbo code, CDMA, AWGN,BER.**

I. Introduction

Error corrective coding is used to enhance the efficiency and accuracy of information transmitted. In a communication transmission system, data is transferred from a transmitter to a receiver across a physical medium of transmission or channel. The channel is generally affected by noise or fading which introduces errors in the data being transferred. Error-correcting code is a signal processing technique used for correcting errors introduced in the channel. It is done by encoding the data to be transmitted and introducing redundancy in it such that the decoder can later reconstruct the data transmitted using the redundant information [1].

A major concern in coding technique is the control of errors so that reliable communications can be obtained, i.e., original information is as close to information source as possible. There are many coding schemes available. Turbo code is the most exciting and potentially important development in the coding theory in recent years. This powerful code is capable of achieving near Shannon capacity performance [1, 2].

II. Explanation

A. Turbo codes system model

On Fig.1 we show the basic elements of a communication system with turbo code [3]. The source generates an information sequence of N symbols with a constant a priori probability distribution $P(u_k=u)$. The u_k denotes the transmitted symbol at time k with value (0, 1), i.e., $u_k \in \{0,1\}$. The u_k is encoded by two recursive systematic convolutional (RSC) encoders whose trellis states start at state $s_0(i)$, e.g. $s_0(1)$, and terminal at the final state $s_k(i) = s_N(1)$, which the final state returns to the starting state for encoding the next information block. k and i denote time index and state index, respectively. The encoder generates a sequence of N output coded symbols c_N.

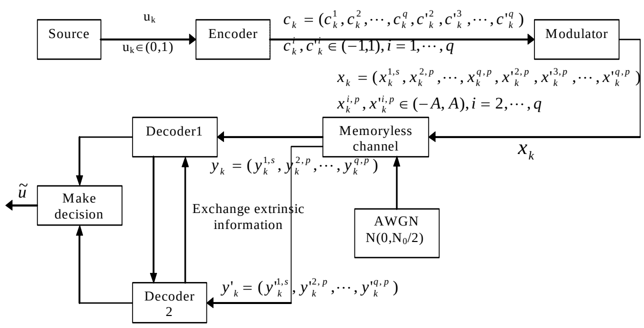

Fig.1 Block diagram of turbo code system model

The trellis state structure is supposed to be known at the receiver side. $c_k = \{c_k^1, c_k^2, \ldots, c_k^q, c'^2_k, c'^3_k, \ldots, c'^q_k\}$ represents one coded symbol at time k with a length of $2q$-1, where $c_k^1, c_k^2, \ldots, c_k^q$ are generated by the first RSC encoder, and $c'^2_k, c'^3_k, \ldots, c'^q_k$ are generated by the second RSC encoder. Each element c_k^l is binary signal, i.e., $c_k^l \in \{-1,1\}, l = 1, \cdots, q$. After modulation, the coded symbols are mapped one by one into transmitted signals x_k. $x_k = \{x_k^{1,s}, \mathrm{x}_k^{2,p}, \ldots, x_k^{q,p}, \mathrm{x}'^{2,p}_k, \mathrm{x}'^{3,p}_k, \ldots, x'^{q,p}_k\}$ represents the transmitted codeword at time k. $x_k^{1,s}$ and $\mathrm{x}_k^{2,p}, \mathrm{x}_k^{3,p}, \ldots, x_k^{q,p}, \mathrm{x}'^{2,p}_k, \mathrm{x}'^{3,p}_k, \ldots, x'^{q,p}_k$ are the systematic bit and the parity check bits for the kth symbl, respectively. The signal x_k is transmitted over the stationary memoryless channel [1,3]. At the destination, the decoder will evaluate the demodulator output y based on the statistic characteristic of the channel, i.e., the conditional probability density function of y_k, $p(y/c) \triangleq p(Y_k = y/C_k = c]$, $y_k = \{y_k^{1,s},\ \mathrm{y}_k^{2,p}, \cdots, y_k^{q,p}, \mathrm{y}'^{2,p}_k, \cdots, y'^{q,p}_k\}$ represents the received symbol at time k, and then make a decision.

B. Turbo decoder in AWGN channel

AWGN channel model is a simple and common channel model in a communication system. It is easier to be studied. In this section, a turbo code decoding algorithm under AWGN channel will be discussed. Fig.2 shows a block diagram of a turbo decoder where: π is the notation for interleaver, π^{-1} is the notation for de-interleaver [4].

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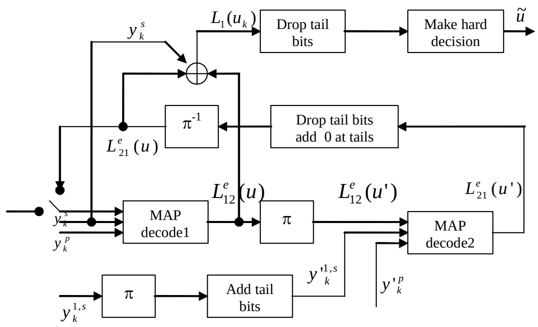

Fig.2 Block diagram of turbo decoder

A log ratio of the posteriori probability of u_k conditioned on the received signal y is defined as:

$$L(u_k) \triangleq \log\left[\frac{P(u_k = 1/y_1^N)}{P(u_k = 0/y_1^N)}\right] \quad (1)$$

The decoding decision of $\tilde{u}_k$ is made based on the sign of $L(u_k)$, i.e.:

$$\tilde{u}_k = sign[L(u_k)] \quad (2)$$

$L(u_k)$ is computed by three terms which are *L_apriori*, *L_channel* , and $L^e(u_k)$. *L_apriori* is a priori information based on the input bit u_k at time k. It is provided by the previous decoder. *L_channel* is the received systematic bit at time k.

$$L(u_k) = L\_apriori + L\_channel + L^e(u_k), \quad (3)$$

where *L_apriori* and *L_channel* denote $L^e(u_k)$ and $Lc \cdot y_k^{1,s}$ respectively. The summation over all the possible transition branch pair (s_{k-1}, s_k) at time k given input u_k=0. Lc is the channel reliable factor, its computation is given as the following:

$$Lc = \frac{4 \cdot A \cdot SNR\_b}{p}, \quad (4)$$

where A=1 for AWGN channel, SNR_b is the uncoded bit-energy-to-noise-ratio E_b/N_0, p denotes $1/r_c$, r_c is code rate of the turbo encoder.

$L^e(u_k)$ is an extrinsic information based on all parity and systematic information except the systematic value at time k:

$$L^e(u_k) \triangleq \log\frac{\sum_{u^+}\tilde{\alpha}_{k-1}(s')\cdot\gamma_k^e(s',s)\cdot\tilde{\beta}_k(s)}{\sum_{u^-}\tilde{\alpha}_{k-1}(s')\cdot\gamma_k^e(s',s)\cdot\tilde{\beta}_k(s)}, \quad (5)$$

At any given iteration, decoder 1 $L_1(u_k)$ is computed as:

$$L_1(u_k) = Lc \cdot y_k^{1,s} + L_{21}^e(u_k) + L_{12}^e(u_k), \quad (6)$$

$$\tilde{u}_k = \mathrm{sign}[\mathrm{L}_1(u_k)],$$

where $L_1(u_k)$ is given in equation (3). $L_{21}^e(u_k)$ is extrinsic information for decoder 1 derived from decoder 2, and $L_{12}^e(u_k)$ is the third term in equation (3) which is used as the extrinsic information for decoder 2 derived from decoder 1. The decoders are sharing the information with each other [5]. The value of $L_1(u_k)$ decides the degree of the reliability of $\tilde{u}_k$.

III. Implementation of Rayleigh Fading Generators

The characteristics of the Rayleigh fading process will affect the Turbo decoder performance. Two different structures are used to generate Rayleigh fading processes in the simulation. One is to generate an uncorrelated Rayleigh fading process, the other is to generate a correlated Rayleigh fading process.

A. Uncorrelated Rayleigh fading generator

This generator generates a_s and a_c which are i.i.d. Gaussian distributed variables with zero mean and unit variance, and $\alpha = \sqrt{\frac{(a_s)^2 + (a_c)^2}{2}}$. The auto-correlation of the random variable α is time independent (uncorrelated) [6]

B. Correlated Rayleigh fading generator

Another simulator to generate Rayleigh fading channel gain is by using mathematical functions, which is called Jake fading generator a_c and a_s are given as the following [6,7]:

$$a_c = \frac{2}{M_0}\left(\sum_{n=1}^{M_0}\cos\beta_n\cos\omega_n t + \sqrt{2}\cos\zeta\cos\omega_n t\right), \quad (7)$$

$$a_s = \frac{2}{M_0 + 1}\left(\sum_{n=1}^{M_0}\sin\beta_n\cos\omega_n t + \sqrt{2}\sin\zeta\cos\omega_n t\right), \quad (8)$$

$$A_1 = \frac{\left[(a_c)^2 + (a_s)^2\right]^{\frac{1}{2}}}{\sqrt{2}}, \quad (9)$$

and $M_0 = \frac{1}{2}(\frac{M_1}{2} - 1)$, $\beta_n = \frac{\pi \cdot n}{M_0}$, ζ= $\pi/4$, and $\omega_n = \omega_m\cos(\frac{2\cdot\pi\cdot n}{M_1})$, where M_0 is the number of low frequency oscillators with frequencies equal to ω_n. a_s and a_c are approximately Gaussian random processes with zero means and unit variances [8]. Then $A_1 = \sqrt{\frac{(a_s)^2 + (a_c)^2}{2}}$ is Rayleigh distributed. The autocorrelation of A_1 is given as $J_0(\omega_m\tau)$, where ω_m is the Doppler frequency and $J_0(x) = \frac{2}{\pi}\cdot\int_0^{\frac{\pi}{2}}\cos(x\cdot\cos\varphi)d\varphi$. In the simulations, the variance of α is selected to make the average power of the received signal equal to 1 and M_0 is 8, n∈1,2,…,M_0. Fig.3 and Fig.4 are two samples of the random process generated by the two generators discussed above. The curves represented by {A_i} in solid line and {nr_i} in dashed line are corresponding to the correlated and uncorrelated Rayleigh fading processes with $\sigma^2=1$ respectively. From the figures, it is observed that the correlation of the Rayleigh fading process decreases when the Doppler frequency increases.

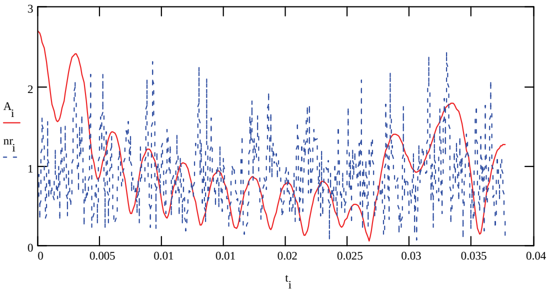

Fig.3 Realizations of correlated and uncorrelated Rayleigh fading processes with Doppler frequency f_d =178 Hz

In Fig.4, the moving speed v_c is 60 miles per hour (mph), the carrier frequency f_c=2 GHz, and the Doppler frequency f_d=v_c*f_c/c=178 Hz, where c is the light speed 3×10[8] m/sec. In Fig. 4, the moving speed v_c is 10 mph, the carrier frequency f_c=2GHz, and the Doppler frequency f_d =29.8 Hz.

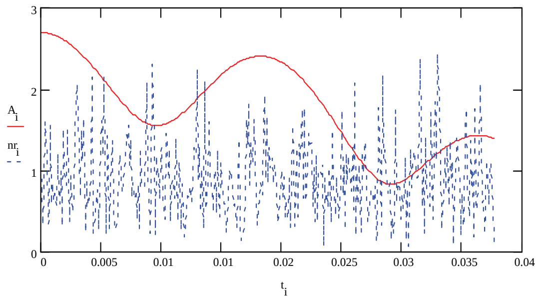

Fig.4 Realizations of correlated and uncorrelated Rayleigh fading processes with Doppler frequency f_d=29.8 Hz

IV. Simulation and Results

We presents the simulation results of turbo code performance under AWGN and fading channel environments. The simulation uses the turbo recursive systematic convolutional encoder with generator matrices (1,15/13,17/13) and a shift register memory 4. The channel gain is generated by using either the Jack fading or Rayleigh fading generator. The frame size either is 384 or 20736 bits/frame.

A. Effects of the number of iterations on BER

Increasing the number of iterations is not much help in the low regions of SNR. In the middle to high regions of SNR, when the number of iterations increases from 1 to 3, the performance of the turbo decoder improves dramatically. In other words, BER decreases dramatically. This is due to the decoder 1 and decoder 2 share the information and make more accurate decisions. As the number of iterations increases, the performance of the turbo decoder improves. However, after the number of iterations reaches a certain value, the improvement is not significant. It can be explained that decoder 1 and 2 already have enough information, further iterations do not give them more information. Fig.6 shows the convergence of the decoding iterations. The correlated Rayleigh fading channel corresponds to the Doppler frequency f_d=268.2Hz, with a SNR_b of 4 dB, and a code rate of 1/3. The uncorrelated Rayleigh fading corresponds to a SNR_b of 4 dB and a code rate of 1/3. The AWGN channel corresponds to a SNR_b of 1.2 dB and code rate of 1/3. The conclusion is that 3 iterations are good enough to get reasonable results in the middle region of SNR_b, and 2 iterations, for high region of SNR_b. High region is defined as SNR_b>1.4 dB, middle range 1.4 dB >SNR_b>0.3 dB in AWGN channel. In uncorrelated Fading channel, high region is defined as SNR_b>4 dB, middle range as 4 dB>SNR_b>2.5 dB.

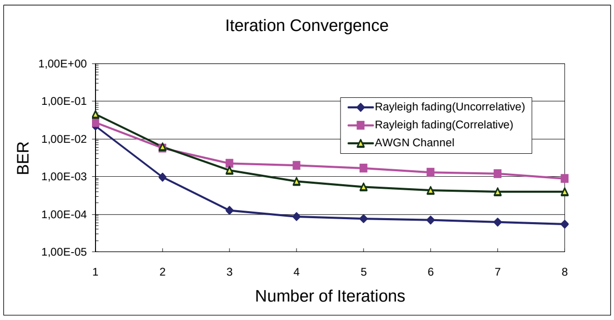

Fig.5 Iteration convergence and BER

In the simulations, a criterion is applied to stop the iterations. The simulation result shows that at SNR>3.2 dB under uncorrelated Rayleigh fading channel, the decoding errors can be reduced greatly after two to three iterations. Therefore, when five iterations are used in this region, more than half of the time the decoder is dealing errorless codes, which is a waste of time. By dynamically applying the criterion to stop the iterations greatly saves the decoding time in region of high SNR, hence, improve the decoding efficiency. The smaller the frame size, the more time it saves. From which obviously it can be seen that some penalty is added if the iteration stops earlier. The simulation is given at the data frame size=378 bits, code rate=1/3, iterion=5. For correlated Rayleigh fading channel, Doppler frequency f_d=178Hz was used.

B. Larger frame size gets better performance

The larger the frame size, the bigger the S-window. Therefore, it will produce larger distance by using an interleaver. The correlation between the two adjacent bits will become smaller. Hence the decoder gives better performance. The simulation results verified this conclusion. However, since turbo code is a block code, it causes a time delay before getting the complete decoding output. Increasing the frame size also increases the delay time. Fig.6 shows the BERs of turbo code under uncorrelated Rayleigh fading channel with the code rate=1/3, iteration=3, frame size L=384 bits (line with triangle) and L=20730 bits (line with diamonds). From the figure we can see that the turbo code with lager frame size has better performance.

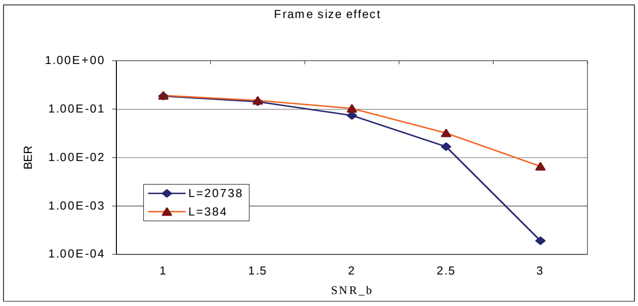

Fig.6 Effects of frame size on BER

C. Effects of Puncturing on BER

When the code rate is decreased, more bits have to be punctured. The bandwidth requirement is also decreased. This means that the performance of the turbo code will also degrade in general. Fig.7 shows the effects of the punctuation on BER. The higher the code rate, the lower the BER. In the simulation, decode iteration=3, frame size=384, uncorrelated Rayleigh fading environment applied. The three curves are corresponding to code rate=1/2, 1/3, and 1/4, respectively.

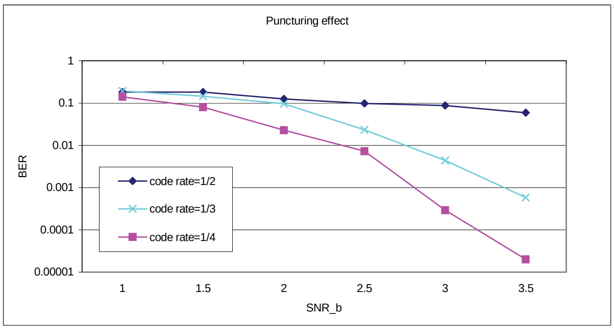

Fig.7 Effects of puncturing on BER

D. Effects of Doppler frequency on BER

Fig.8 shows the effects of Doppler frequency on BER of the turbo code. The Jake fading generator is used in the simulations.

The three curves are corresponding to v_c=60 mph (f_d=178 Hz), 24 mph (f_d=71.5 Hz), and 2.5 mph (f_d=7.5 Hz), respectively, when the carrier frequency is f_c=2 GHz. The code rate ¼, frame size 384 bits, iteration 3 are used for the three cases in the simulation. The higher the Doppler frequency, the less the correlation of the fading process, hence the better the performance.

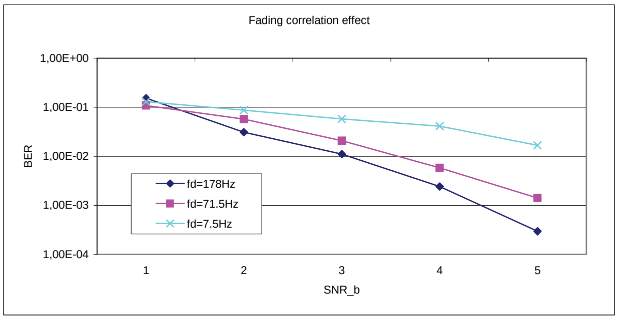

Fig.8 Effects of Doppler frequency on BER

V. CONCLUSION

The simulation results show that turbo code is a powerful error correcting coding technique under SNR environments. It has achieved near Shannon capacity. However, there are many factors need to be considered in the turbo code design. First, a trade-off between the BER and the number of iterations need to be made, e.g., more iterations will get lower BER, but the decoding delay is also longer. Secondly, the effect of the frame size on the BER also needs to be considered. Although the Turbo code with larger frame size has better performance, the output delay is also longer. Thirdly, the code rate is another factor that needs to be considered. The higher coding rate needs more bandwidth.

From the simulation results, it is observed that the behavior of the turbo decoder is quite different under different channel environments. The performance of the turbo code is much worse under correlated Rayleigh fading channel than that of AWGN or uncorrelated Rayleigh fading channels. Another drawback of the Turbo code is its complexity and also the decoding time.

REFERENCES

[1] J. Proakis, *Digital communications*, 4th edition, McGraw-Hill, New York, 2000

[2] C. Berrou, “The ten–year old turbo codes are entering into service”, IEEE Communications Magazine, vol.41, no.8, pp. 110–116, 2003

[3] T. Iliev, G. Hristov, “Simulation estimation of the forward error correction of turbo codes”, TELSIKS’2007, Conference Proceedings, pp. 521 – 524, Nis, Serbia, 2007

[4] C. Berrou, R. Pyndiah, P. Adde, C. Douillard and R. Le Bidan, “An overview of turbo codes and their applications”, The European Conference on Wireless Technology, pp. 1–9, 2005

[5] Interim V&V Text for CDMA-2000 Physical Layer (Revision 8.3), TIA, 1999.

[6] E. Hall, “Performance and design of Turbo codes on Rayleigh fading channels” Master's Thesis, University of Virginia, May 1996.

[7] F. Dowla, *Handbook of RF and Wireless Technologies*, Elsevier, 2004

[8] W. Jakes, Jr., *Microwave Mobile Communication*, New York: Wiley, 1974.