# Impact of Cross Phase Modulation on WDM Optical Systems

Georgi K. Georgiev<sup>1</sup>, Tchavdar Levtchev<sup>2</sup> and Nikolay M. Stoyanov<sup>3</sup>

*Abstract* – Cross phase induced nonlinear distortion and nonlinear time jitter are assessed preliminarily in this paper. A semi-analytical model is developed to evaluate the nonlinear time jitter introduced by cross phase modulation and information carrying capacity of the fibber in case of high optical power. The factors which cause these effects and the performance of the system with high spectrum efficiency are analyzed. The Obtained results with numerical simulations are revealed and discussed in this paper.

*Keywords* – Communication systems, Cross phase modulation (XPM), nonlinear optics, phase modulation, time jitter, nonlinear phase noise, WDM systems.

# I. INTRODUCTION

Multi-wavelength transmission systems with increasing number of channels and longer interamplifier spans require necessarily higher powers per channel to maintain acceptable signal-to-noise ratios (SNR's). Since the power in a channel directly affects the refractive index of the fiber, when there are power variations along the length of the fiber, they change the refractive index of the fiber. In these systems, the transmission distances will be limited mainly by fiber nonlinearities including cross-phase modulation (XPM) and self-phase modulation (SPM) [1].

Timing jitter is formed due to the phase distortions developed in the dispersive fiber. In the dispersion compensated standard single-mode fiber (SSMF, ps/nm/km), the high fiber the dispersion converts the phase modulation into intensity modulation, and results in signal distortion, which increases with bit rate [2].

To evaluate the impact of XPM in NRZ-OOK WDM optical system and the performance degradation caused by fiber nonlinearities, a semi-analytical model is constructed. Split step Fourier method is used to solve the pulse propagation problem in nonlinear dispersive media. The roots of this model lie in the derivation of the analytical time jitter formula related with XPM in WDM optical systems. In this model the dispersion and nonlinear effects are solved separately in time domain the first one and in frequency domain the last one.

The influence of linear and nonlinear transmission impairments are studied in single channel transmission, where an implementation of novel NRZ-advanced modulation format, provides a better dispersion tolerance, but suffers from strong nonlinear impairments.

Some of the previous analytical studies involved analyzing only the intensity modulation generated at the end of the fiber [3], does not involve the split-step Fourier transform [4]or Only intensity modulated systems are considered and the experiments [5]. Also they do not consider the delay between the probe and the pump channel which is implemented in this study. The XPM affects the signal quality when signal optical power is high enough and these effects become more and more important for high data rate long distance systems which require high signal optical power. Therefore the evaluation of the effect of cross phase modulation here would be helpful in order to fully investigate the performance of WDM systems and to overcome some of the limitations caused by this nonlinear effect.

# **II. INDUCED WAVEFORM DISTORTIONS**

The non-linear effects of the fibers play a detrimental role in the light propagation. Nonlinear Kerr effect is the dependence of refractive index of the fiber on the power that is propagating through it. This effect is responsible for SPM, XPM and for wave mixing (FWM).

# A. Calculation of phase shift and time jitter

In fibers, the refractive index always has some dependence on the optical intensity which is the optical power per effective area. In case of XPM, the phase shift depends on the power of other channel. The total phase shift can be represented as [1],

$$\theta_i^{NL} = \gamma L_{eff} \left( P_i + 2 \sum_{n \neq i} P_n \right)$$
(1)

where  $\theta_i^{NL}$  is the non-linear phase shift for the i<sup>th</sup> channel,  $L_{eff}$  is the effective length of the fiber,  $P_i$  and  $P_n$  are the power for the channels i and n. On the right-hand-side of equation (1), the first term represents effect of SPM and the second term represents that of XPM.

To analyze the effect of XPM and SPM, the nonlinear Schrödinger wave equation which is a mathematical representation for the nonlinear propagation of light signal through a fiber can be used [1].

The phase variation of the channel i induced by Kerr interaction with from channel k along the infinitesimal distance from z to z + dz is given by [3], Based on the pump-probe model [7, 8], the phase modulation of the first channel (probe) induced by the second channel (pump) is

<sup>&</sup>lt;sup>1</sup>Georgi K. Georgiev is with Ministry of the Interior of Bulgaria, E-mail: ggeorgiev@ncc.mvr.bg

<sup>&</sup>lt;sup>2</sup>Tchavdar Levchev is with Ministry of the Interior of Bulgaria, E-mail: nstoyanov@ncc.mvr.bg

<sup>&</sup>lt;sup>3</sup>Nikolay M. Stoyanov is with Ministry of the Interior of Bulgaria, E-mail: nstoyanov@ncc.mvr.bg

$$d\theta_i(z,\omega) = -2\gamma_i P_k(z,\omega) dz \tag{2}$$

Consider two channels, the pump channel k and the probe channel i. Due to the power in the pump channel, the phase of the probe channel varies along the length of the fiber. To evaluate the effect of this pump power, consider a standard SMF fiber of length L. Let dz be the small section of fiber for the analysis. The infinitesimal phase fluctuation created by optical power Pk of the channel k at fiber section dz will propagate to the end of the fiber of length L.

The power  $P_k(z,\omega)$  of the channel k with the assumption made by the small signal analysis [6] can be expressed as follows:

$$P_{k}(z,\omega) = P_{k}(0,\omega)\cos(q_{k}z)\exp(-az - j\omega/v_{g,k})$$
(3)  
$$q_{k} = -\beta_{2k}\omega^{2}/2$$
(4)

Here  $v_{g,k}$  denoting the group velocity,  $\alpha$  is the attenuation coefficient and  $\beta_{2,k}$  the dispersion. Due to chromatic dispersion of the fiber this small phase modulation will evolve into an intensity fluctuation through PM-IM (Phase modulation to Intensity modulation) conversion and also a phase delay will happen through a PM-PM (Phase modulation to Phase modulation) conversion [6]. As a result of cross-phase modulation, the phase variation of the signal channel (i) due to the presence of pump channel (k) in a short fiber section dz at z is [6]:

$$d\theta_i(z,\omega) = -2\gamma P_k \cos(q_k z) \exp(-\alpha z - j\omega z / v_k) dz \quad (5)$$

Along the fiber at the point z this phase modulation becomes:

$$\frac{d\theta_i(L, z, \omega) = \exp[-(j\omega/v_i)(L-z)]}{\cos[q_i(L-z)]d\theta_i}$$
(6)

Relative walk off length Lw between the two channels with wavelength separation  $\Delta\lambda$  and dispersion D, introduces the intensity changes due to the change in the relative alignments of the interacting channels. This relative change accounts for the small time difference between the pulses,  $\Delta t$  which characterizes this intensity changes as:

$$\theta_{xpm}(L,t) = 2\gamma \int_{0}^{L} P_{2}(0,t + L_{W1,2}z)e^{-\alpha z} dz$$
(7)

Assume the fiber chromatic dispersion parameter is constant along the fiber and neglecting the second order dispersion ( $\beta_{2,k}$  is wavelength independent) from (4),(5)and (6) can be obtained then at the receiver (z = L) the time jitter created by this short fiber section as:

$$dt(z,\omega) = -\frac{\lambda_i^2 \omega d\theta_i(z,\omega)}{c}$$

$$\int_0^L \exp\left[-(j\omega/\nu_i)(L-z)\right] \cos\left[q_i(L-z)\right] D(L-z) dz$$
(8)

#### B. The model

Pulse propagation in optical fibre is simulated by applying the split-step Fourier method to numerically solve the nonlinear Schrodinger equation (NSE) [1], which includes the effect of first order and second order GVD, self phase modulation (SPM) and cross phase modulation (XPM) due to fibre nonlinearity, and fibre attenuation,. This is a well established mathematical model and is extensively modelled fro all fibres in the transmission system, ie. for DSF and DCF. The instantaneous pulse envelope A(z,t) propagating inside the fibre is calculated by solving nonlinear Schrödinger equation numerically using the split-step Fourier method. Expressing equation (4.1) as

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A, \qquad (9)$$

where  $\hat{D}$  is the dispersion differential operator and  $\hat{N}$  is the nonlinearity operator:

$$\hat{D} = -\frac{\alpha}{2} - \frac{i}{2}\beta_2 \frac{\partial^2}{\partial T^2} + \frac{1}{6}\beta_3 \frac{\partial^3}{\partial T^3}$$
(10)

$$\hat{N} = j\gamma_{k} \left( \left| A_{k} \right|^{2} + \sum_{j=1, j \neq k}^{N} 2 \left| A_{j} \right|^{2} \right)$$
(11)

An algorithm is developed to synthesise the Split Step Model using the numerical methods. Various aspects of accuracy of the simulation of solitons evolution have been closely examined and considered. The algoritm is developed in the MATLAB platform. The exact solution describes the pulse envelope in segment  $z+\Delta z$  relative to its shape in the preceding segment z. Next consider dividing the fibre length into small steps of length  $\Delta z \approx dz$ . As these two operators appear to add by superposition, dispersive and nonlinear effects can be considered to act independently. That is, the two effects split.

In general, dispersion and nonlinearity act together along the length of the fibre. The Split Step Method obtains an approximate solution by assuming that in propagating the optical field over a small distance  $\Delta z$ , the dispersive and nonlinear effects can be pretended to act independently. More specifically, propagation from z to z+  $\Delta z$  is carried out in two steps as shown in Fig.1.

The drawback of using the SSF method is that it requires computations using two FFT per step size which are quite time consuming. Adding to that, it should be acknowledged that the step size used is usually held constant in system simulations.

In [9], several step-size selection schemes have been investigated which have found that in most cases, the constant step size method is the least efficient of all methods. In the abovementioned paper, a local error method for step-size selection is introduced.



Fig. 1. Split-Step Fourier Method

The local error is a measure of the error incurred in propagating through a single step  $\Delta z$ . This method is adopted to incorporate a variable step-size selection method in order to achieve more efficient propagation by decreasing the simulation time, while controlling the relative local error. Simulation semi-analytical model is constructed for simplicity to have two WDM channels, one operating at 1549.6nm and second at 1550.4nm. A block diagram of the model is shown on Fig.2.

Bernoulli binary generator is used to generate carrier, data random bit patter and modulation stream data. A discrete sampling method is employed with sampling time 10-18. Simple sum operator is to combine the two data streams and intensity fluctuations are added after that. The fiber



Fig. 2. Model block diagram

propagation model consists of 2 blocks (SMFiber1 and SMFiber2) and use described above split step Fourier method. The first one takes into account dispersion effects, second one represents a number of non-linear effects such as Brillouin, Rayleigh and Raman scattering by using nonlinear parameter  $\gamma$ , which affects optical propagation over the fiber. Also we have to take into account in our model, that the mean nonlinear phase is depending on the optical power and the walk-off length is the main constraint for nonlinear interaction between two or multiple pulses when the XPM effects occur [10,11]. More specifically, XPM can be considered to have no effects when the faster moving pulse completely walks through the slower moving pulses.

# III. DISCUSSION AND RESULTS

Modeling of the optical fiber system is quite different than the modeling of other types of communication systems, such as wireless system, due to its high bit rate, low bit error rate and its transmission media, which needs special consideration. The power in one channel affects the phase of the other channel and induces cross talk. If only one channel is transmitted, then it will not experience any cross talk since there is no neighboring channel to cause cross talk. This seldom happens in a WDM link since these links always carry multiple channels in the same link with narrow spacing between them. For simplicity and better understanding the worst effects of nonlinearities, first a single channel is considered. A launch power of 20 mW is used with 2.7 ps/nm-Km dispersion, with 100 Km fiber length. The figure 3 shows the eye diagram of the resulting channel after passing through the fiber and dispersion effects.



Intensity distortions and timing jitter are separate effects and they combine to produce more deleterious effects on the data transmission. The intensity distortion is due to the broadening of the upper rail corresponding to the transmission of "ones" and timing jitter is the broadening of edges of the eye corresponding to transition between "ones" and "Zeros".

The difference in the probe and the pump bit alignments plays a major role in the accumulation of the crosstalk.

If the edges are perfectly aligned, then it produces more time jitter and if they are not aligned exactly at the edges, then lesser time jitter is produced. The transfer function of the time jitter is shown on Fig.4



The probe waveform is multiplied with the time jitter created and the resulting eye diagram that is shown below (Fig.5).

Due to XPM considerable time jitter is accumulated along the length of the fiber. The time jitter is higher for higher pump channel powers and lower for lower pump channel powers. The difference in the probe and the pump bit alignments plays a major role in the accumulation of the crosstalk. If the edges are perfectly aligned, then it produces more time jitter and if they are not aligned exactly at the edges, then lesser time jitter is produced.



Fig.5. NRZ probe signal waveform with time jitter

The optical network with NRZ modulated waveforms in a four span system is analyzed. The power of the pump channel is varied between 5mW to 25 mW and the resulting time jitter and intensity modulation is analyzed. This system also has dispersion compensation which is 80% of the total length of the fiber.



The obtained results (dashed line) compared with model based on software package VPItranssmitionMarker4.5<sup>TM</sup> for launched power versus time jitter can be seen on Fig.6. The results are achieved for a 10-Gb/s systems using non-zero dispersion-shifted fiber, 50-GHz spacing (NZDSF) and D = 4ps/km/nm has a walk-off length Lw of 62.5 km. After the probe signal passes through the dispersion and SPM effects, an eye diagram is generated which will be undistorted and having a wide eye opening. This is taken as the reference and this waveform is multiplied with the XPM effects to analyze how much the probe signal will be affected by the XPM and other nonlinearities. The walk off is seen only at the end of the fiber while superimposing the time jitter on the probe wave, that's why it might have the higher jitter values than real systems and VPI-4.5 model. The model does not account for the dependence of the gain and noise spectra on the degree of amplifier saturation induced by the input signals. Since this paper considers the effect of SPM to be less, the power for the probe signal is considered to be very less in the order of 1 to 4 mW.

# IV. CONCLUSION

In this paper through the represented theoretical model we have investigate the performance degradation of NRZ modulation format for the influence of XPM induced nonlinear. The performance of the system was measured with different nonlinear phase shift. When there is high separation, the signals are well separated in the frequency domain which means that the effect of the walk off length is insignificant.

Although we have to confess with the increasing of the launch power also increases absolute SNR, the represented model has demonstrated that for low penalties, it is not beneficial to work high optical power, since it will generate nonlinear interference between channels and a settlement by compromise must be taken.

The optimal operation optical power range for each system configuration is the mid point where linear and nonlinear impairments balance each other.

### REFERENCES

- G.P.Agarwal, "Nonlinear Fiber optics", 3<sup>rd</sup> ed., Academic Press, San Diego, CA, 2001
- [2] M. Shtaif, "Impact of cross phase modulation in WDM systems," in Proceedings of the Optical Fiber Communication Conference, Baltimore, 2000, Paper ThM1 (invited)
- [3] G. Goeger, M. Wrage, and W. Fischler, "Cross-Phase Modulation in Multispan WDM systems with Arbitrary Modulation Fromats", IEEE photonics technology letters, Vol 16, No 8, pp.858 - 1860, August 2004.
- [4] F. Forghieri, R.W. Tkach, and A.R. Chraplyvy, "Fiber nonlinearities and their impact on transmission systems", Academic Press, San Diego, 1997.
- [5] Rongqing Hui, Kenneth R. Demarest, and Christopher Allen, "Cross phase modulation in multispan WDM optical fiber systems", J of Lightwave Tech., Vol 17, No. 6, pp.1018-1026, June 1999
- [6] J. Wang and K. Petermann, "Small signal analysis for dispersive optical communication systems," J.Lightwave Technol., vol 10, pp.99-100, Jan 1992
- [7] Chiang, T.-K., Kagi, N., Fong, T. K., Marhic, M. E., and Kazovsky, L. G. (1994). "Cross-phase modulation in dispersive fibers" IEEE Photon. Technol. Letters, vol. 6 no. 6, pp. 733-736.
- [8] Ho, K.-P. Performance degradation of phase-modulated systems with nonlinear phase noise, IEEE Photon. Technology Letters, vol. 15, no 9, pp. 1213-1215, 2003
- [9] O. V. Sinkin, Holzlohner, R., Zweck, J. & Menyuk, C. R., "Optimization of the Split-Step Fourier Method in Modeling Optical-Fiber Communications Systems", IEEE J. Lightwave Technology, vol. 21, no.1, pp. 61-68, Jan 2003
- [10] Hoon Kim, "Cross-phase-Modulation-Induced Nonlinear Phase Noise in WDM Direct-Detection DPSK Systems" Journal of Lightwave Technology, vol. 21, no. 8, pp. 1770-1774, August 2003.
- [11] Chiang, T.-K., Kagi, N., Marhic, M. E., and Kazovsky, L. G. (1996). Cross-phase modulation in fiber links with multiple optical amplifiers and dispersion compensators. J. Lightwave Technol., vol.14, no. 3, pp.249 -260