An Experimental Setup for Studying Sampling and Signal Reconstruction Process

Milica B. Naumović[1] and Branislav Petrović[2]

Abstract – **This paper is devoted to the signal sampling and reconstruction processes in the sampled-data control systems. Some phenomena that have been discussed in the literature only by several authors are experimentally verified.**

Keywords – **Sampling process, signal reconstruction process, zero-order hold device, frequency response, sampled-data control systems.**

I. Introduction

The purpose of the present paper is primarily pedagogical. Exercises with laboratory experiments in teaching of digital control require a model of sampling and signal reconstruction process. The mathematical models that give the relationships between the continuous-time signals in a sampled-data control system are well-known in the literature. Namely, in sampled-data control, hold circuits are used to convert the discrete-time signals from digital compensators into the continuous-time signals to be applied to the continuous-time objects. Hold circuits can be viewed also as filters which attenuate the high frequency alias spectra generated by sampling continuous-time signals. Recall, that both zero-order hold (ZOH) and first-order hold (FOH) are typical hold circuits. In industrial applications, however, the zero-order hold seems to be particularly popular although the use of the first-order hold leads into reduction of the response intersample ripple. The primary reason for this is that a zero-order hold can be implemented quite easily by using the function of D/A converters, while a first-order hold can be implemented only with the aid of some additional analog circuits. Another reason might be that, when viewed as continuous-time filters, the phase lag of a first-order hold for high-frequency ranges is greater than that of a zero-order hold, which seems to be a disadvantage from the point of view of closed-loop stability.

The aim of this paper is to give more insight into the sampling and reconstruction processes. For this purpose, a sample-and-zero-order hold experiment model is implemented using the standard analogue and digital integrated circuits. Some results based on the frequency responses are discussed and serve for verification of the theoretical consideration that can be found in the literature [1]-[4].

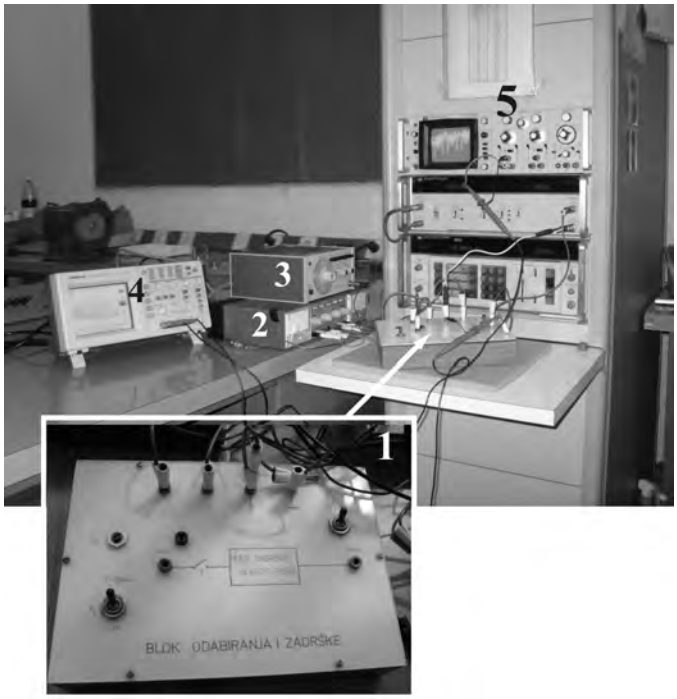
[1]Milica B. Naumović is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: milica.naumovic@elfak.ni.ac.yu

[2]Branislav Petrović is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: branislav.petrovic@elfak.ni.ac.yu

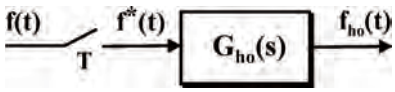
II. Laboratory Setup

This paper deals with an experiment for testing and describing some phenomena occur during the signal sampling and reconstruction processes. As it is shown in Fig. 1a, the experimental setup consists of several functional elements as follows:

1. Model of sample-and-zero-order hold process, **2.** Power supply Iskra MA 4170, **3.** Function generator Iskra MA 3732, **4.** Tektronix TDS 1002 Digital Storage Oscilloscope, and **5.** HP 3042A Automatic Network Analyzer.



a.



b.

Fig. 1. Laboratory setup:
a. The photograph; b. The block diagram of the idealized model of the sample-and-zero-order hold circuit

The boxed photograph in Fig. 1a is a close-up of the top of the evaluation board, based on a low-cost single monolithic chip AD582. This sample-and-hold circuit consists of a high performance operational amplifier, a low leakage analog switch and a JFET integrating amplifier. An external holding

capacitor, connected to the circuit, completes the sample-and-zero-order hold function [5]. Notice that convenient sockets on the front panel of the model, made at the Department of Automatic Control [6], [7], allow us to quickly bring all input and output signals.

HP 3042A Network Analyzer is designed in the mid 1980s to meet the demand for precise and fast characterization of both active and passive linear two-port devices. It is a powerful bench system that makes digital amplitude, phase and group delay response measurements over a $50\ \mathrm{Hz}$ to $13\ \mathrm{MHz}$ frequency range.

III. Time-domain Analysis

Recall that the linear behavior of the hold circuit may be modeled as it is shown in Fig. 1b. We must emphasize that the signal $f^*(t)$ in Fig. 1b is not expected to represent a physical signal in the hold circuit, but is introduced to allow us to obtain a transfer function model of the zero-order hold (ZOH) operation and to make an appropriate input-output model of the hold action. Namely, from the impulse-modulation model the sampled representation $f^*(t)$ of the input continuous-time signal $f(t)$ follows

$$f^*(t) = f(t)i(t)\,. \tag{1}$$

The carrier signal is given in the form of the unit impulse train

$$i(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)\,, \tag{2}$$

where $\delta(t)$ is a delta function.

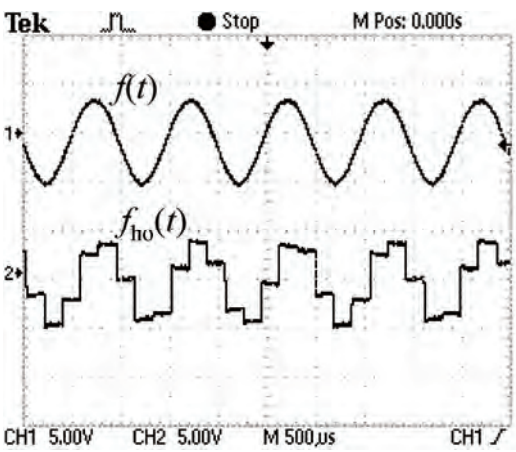


Fig. 2. Waveforms $f(t)$ and $f_{\mathrm{ho}}(t)$; horizontal scale is $0.5\ \mathrm{ms/div}$, vertical scale is $5\,\mathrm{V/div}$

The idealized model of a sample-and-hold circuit is obtained by combining a sampler, described by using impulse modulation (1)-(2), with a zero-order-hold circuit given by transfer function

$$G_{\mathrm{ho}}(s) = \frac{1-\mathrm{e}^{-sT}}{s}\,. \tag{3}$$

Fig. 2 visualizes output of the sampler/zero-order hold $f_{\mathrm{ho}}(t)$ for a sinusoidal input $f(t)$. Thus, in the case of a (relative low) sampling frequency of 5 samples per cycle of the input sine wave, the ZOH output is a piecewise constant waveform is shown in Fig. 2.

IV. Frequency-domain Analysis

Sampling and signal reconstruction processes may also be considered in the frequency domain as follows.

Assume that a sinusoidal signal

$$f(t) = \sin(\omega t + \varphi) = \mathrm{Im}\left[\mathrm{e}^{j(\omega t+\varphi)}\right] \tag{4}$$

is applied to the input of sample-and-hold circuit, shown in Fig. 1b. Notice that the sine wave used during the experimentation is imperfect with frequency content as shown in Fig. 3. The spectrum of the sinusoid has components not only at the fundamental frequency but also at other frequencies.

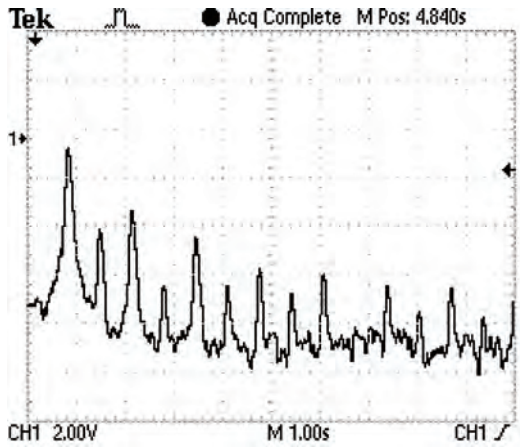


Fig. 3. Frequency spectrum of the periodic input signal $f(t)$ $[5\,\mathrm{V/div}]$ with sweep rate of $1.5\ \mathrm{kHz}$ per division (second)

The Fourier series representation of the periodic unit impulse train (2) is

$$i(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \mathrm{e}^{jk\omega_s t} = \frac{1}{T}\left\{1 + 2\sum_{k=1}^{\infty}\cos(k\omega_s t)\right\}, \tag{5}$$

where T is the sampling period and $\omega_s = 2\pi/T$ is the corresponding sampling frequency in radians per second. Series expansion of the output of the sampler $f^*(t)$ then becomes

$$f^*(t) = \frac{1}{T}\left\{\sin(\omega t+\varphi) + 2\sum_{k=1}^{\infty}\cos\left(k\omega_s t\right)\sin(\omega t+\varphi)\right\}$$

$$= \frac{1}{T}\left\{\sin(\omega t+\varphi) + \sum_{k=1}^{\infty}\left[\sin(k\omega_s t+\omega t+\varphi) - \sin(k\omega_s t-\omega t-\varphi)\right]\right\} \tag{6}$$

The signal $f^*(t)$ has a component with the frequency ω of the input signal multiplied by $1/T$. This signal also has components corresponding to the sidebands $k\omega_s \pm \omega$. It is customary to consider only positive frequencies. The frequency content of the sample-and-zero-order hold device output $f_{\mathrm{ho}}(t)$ with the phenomenon of aliasing is given in Fig. 4. Thus, the output signal $f_{\mathrm{ho}}(t)$ has components with the fundamental frequency ω and the sidebands $k\omega_s \pm \omega$. The sampling is a linear operation, but it is not a time invariant process and some new frequencies are created. The fundamental alias for a frequency $\omega_1 > \frac{\omega_s}{2}$, where $\frac{\omega_s}{2}$ is the Nyquist frequency, is given by [1], [7]

$$\omega_{\mathrm{a}} = \left|\left(\omega_1 + \frac{\omega_s}{2}\right)\mathrm{mod}\left(\omega_s\right) - \frac{\omega_s}{2}\right| . \tag{7}$$

It is well-known that to avoid aliasing, we must either choose the sampling frequency high enough with regard to the sampling theorem, or use a prefilter ahead of the sampler to reshape the frequency spectrum of the signal before it is sampled.

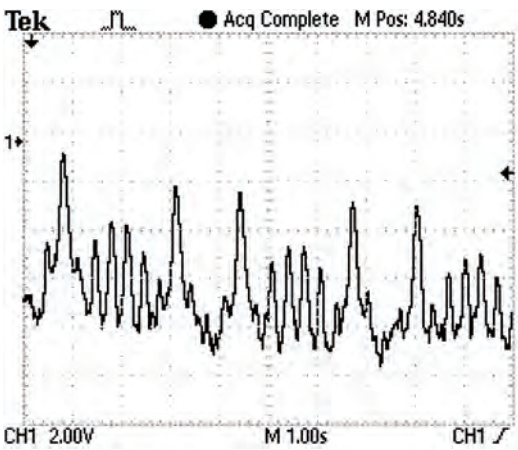


Fig. 4. Frequency content of the output signal $f_{\mathrm{ho}}(t)$ [5 V/div] with sweep rate of 1.5 kHz per division (second)

Thus, the output signal $f_{\mathrm{ho}}(t)$ of the sample-and-hold circuit, shown in Fig. 1b is obtained by linear filtering of the signal $f^*(t)$ with a system having the transfer function (3). To obtain the frequency responses (magnitude and phase) of the zero-order hold, consider the following expressions:

$$\left|G_{\mathrm{h0}}(j\omega)\right| = \frac{2\pi}{\omega_s}\left|\mathrm{sinc}\left(\pi\frac{\omega}{\omega_s}\right)\right| \tag{8}$$

and

$$\arg G_{\mathrm{h0}}(j\omega) = -\pi\frac{\omega}{\omega_s} + \theta \quad , \quad \theta = \begin{cases} 0 & \sin(\pi\omega/\omega_s) \geq 0 \\ \pi & \sin(\pi\omega/\omega_s) < 0 \end{cases} . \tag{9}$$

Plots of the magnitude and phase of the sample-and-zero-order hold circuit are shown in Fig. 5. Notice that the measured frequency responses, given in Fig. 5, fit the amplitude and phase of $G_{\mathrm{h0}}(j\omega)$ given by (8) and (9) well, but some additional explanations are necessary.

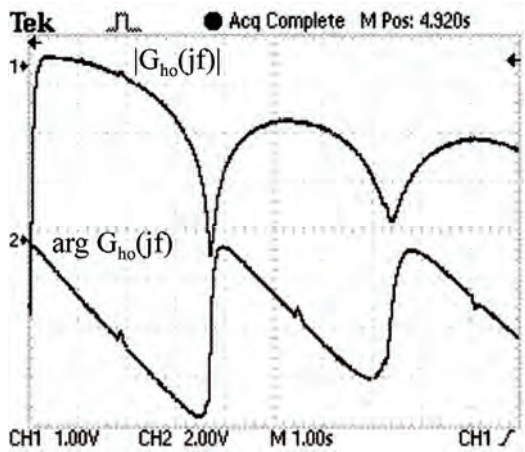


Fig. 5. Magnitude $\left|G_{\mathrm{h0}}(jf)\right|$ [20 dB/div] and phase $\arg G_{\mathrm{h0}}(jf)$ [180°/10 V] frequency responses of zero-order hold device in a single sweep of 1.5 kHz per division (second)

Recall that if the input signal is a sine wave with frequency ω given by (4), the output signal $f_{\mathrm{ho}}(t)$ has components with the fundamental frequency ω and the sidebands $k\omega_s \pm \omega$, $k = 1, 2, \ldots$.

Consider first the case where $\omega \neq k\frac{\omega_s}{2}$, $\frac{\omega_s}{2}$ is the Nyquist frequency. The fundamental component of the output is

$$f_{\mathrm{h0}}(t) = \frac{1}{T}\mathrm{Im}\left[G_{\mathrm{h0}}(j\omega)\,\mathrm{e}^{j(\omega t+\varphi)}\right]. \tag{10}$$

Now let frequency be $\omega = k\frac{\omega_s}{2}$. The frequency of one of the sidebands $k\omega_s - \omega$ coincides with the fundamental frequency ω. Thus, two terms contribute to the component with frequency ω and for $k = 1$, based on the principle of superposition, the component is

$$f_{\mathrm{h0}}^{0}(t) = \frac{1}{T}\mathrm{Im}\left\{G_{\mathrm{h0}}(j\omega)\,\mathrm{e}^{j(\omega t+\varphi)} - G_{\mathrm{h0}}(j\omega)\,\mathrm{e}^{j(\omega t-\varphi)}\right\} . \tag{11}$$

Applying some trigonometric rules, we find

$$f_{h0}^{0}(t)=\frac{1}{T}\mathrm{Im}\left\{\left(1-\mathrm{e}^{-j2\varphi}\right)G_{h0}(j\omega)\,\mathrm{e}^{j(\omega t+\varphi)}\right\}$$
$$=\frac{1}{T}\mathrm{Im}\left\{2\,\mathrm{e}^{\mathrm{j}\left(\frac{\pi}{2}-\varphi\right)}\sin\varphi G_{h0}(j\omega)\,\mathrm{e}^{j(\omega t+\varphi)}\right\}. \tag{12}$$

It is well-known that the steady-state response of a linear system to a sinusoid of unit amplitude and frequency ω is the sinusoid of the same frequency as the input signal, but of different amplitude and phase. We see immediately that the transmission of the fundamental frequency ω is characterized by [1]

$$G_{h0}^{0}(j\omega)=\begin{cases}\frac{1}{T}G_{h0}(j\omega) & \omega\neq k\frac{\omega_s}{2}\\ \frac{2}{T}G_{h0}(j\omega)\,\mathrm{e}^{j(\frac{\pi}{2}-\varphi)}\sin\varphi & \omega=k\frac{\omega_s}{2}\end{cases}\qquad k=1,2,\ldots \tag{13}$$

From the previous consideration one main conclusion can be drawn. It is the fact that the signal transmission at the Nyquist frequency critical depends on φ, i.e., how the sinusoidal input signal is synchronized with respect to the sampling instants. Note, that this phenomenon has been observed and discussed only by several authors of the textbooks in the context of digital control systems, such as [1] and [2]. The effect of the unsynchronization between input and clock signals during the experimentation is visualized in Fig. 5 by the appearance of impulses at $k\omega_s/2$, $k=1,2,\ldots$.

It is easy to extend the analysis to the case of closed-loop computer-controlled system shown in Fig. 6. In a similar way as in previous analysis it can be shown that the results in a computer-controlled system depend critically on how the input is synchronized with the clock of the microcomputer [1].

V. Conclusion

This paper is concerned with the signal sampling and reconstruction processes in the sampled-data control systems. An experimental platform is designed for studying some phenomena that have been discussed in the literature only by several authors. In order to support learning of automatic control at the Faculty of Electronic Engineering, University of Niš, a web-based laboratory was established [8]. The paper presents some initial outcomes in order to stimulate future research in connection with creating a laboratory environment for remote monitoring of sample-and-hold experiment model.

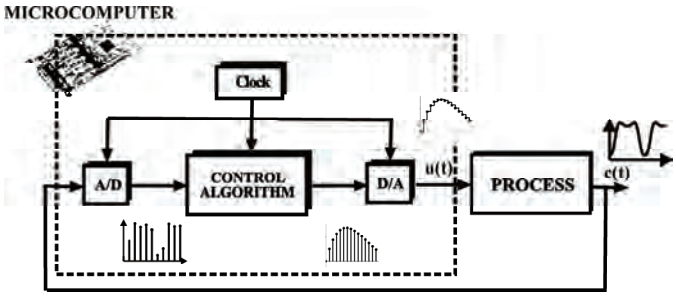


Fig. 6. Schematic diagram of a closed-loop computer-controlled system

References

[1] K. J. Åström, B. Wittenmark, *Computer-Controlled Systems - Theory and Design*, Englewood Cliffs, N.J.: Prentice-Hall, 1984.

[2] C. L. Phillips, H. T. Nagle, *Digital Control System Analysis and Design*, Englewood Cliffs, N.J.: Prentice-Hall, 1984.

[3] G.F. Franklin, J.D. Powell, *Digital Control of Dynamic Systems*, Reading, MA: Addison-Wesley, 1980.

[4] K. Ogata, *Discrete-Time Control Systems*, Englewood Cliffs, N.J.: Prentice-Hall, 1995.

[5] Analog Devices, Inc., http://www.analog.com (accessed on May 11th 2008).

[6] M. B. Naumović, "Laboratory model of the sampling and signal reconstruction process", *Proc. JUREMA* 31, Zagreb, 1986, pp. 133-136 (in Serbian).

[7] M. B. Naumović, *Problem Solver in Digital Control Systems, Part I: Discrete Signals*, Faculty of Electronic Engineering, Niš, 1997 (in Serbian).

[8] M.B. Naumović, and D. Živanović, "Remote Experiments in Control Engineering Education Laboratory", *International Journal of Online Engineering*, (ISSN 1861-2121), vol. 4, No. 2, 2008, pp. 48-53.