Robust Position Control of Induction Motor Using Discrete-Time Sliding Mode Control

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Abstract—**A new way of induction motor position control for high-performance applications is developed in this paper using discrete-time sliding mode control (DSMC). The proposed control structure includes an active disturbance estimator (ADE) in order to improve system robustness and accuracy. Experiments have verified high efficiency of the proposed servo-system under the influence of large parameter perturbations and external disturbances in the presence of un-modeled dynamics.**

Keywords—**Discrete-time sliding mode control, induction motors, position control, disturbance estimator, servo-systems**

I. Introduction

Exceptionally attractive features of the squirrel cage three-phase induction motor (IM) as reliability, high efficiency, ruggedness, low cost and no need for maintenance make the use of IM very important. However, due to their highly coupled nonlinear structure, IMs have been for years mainly used in unregulated drives. A real breakthrough in IM control was the design of field-oriented control (FOC) principle [1], which enables the decoupled control of rotor flux and electromagnetic torque. This new property has opened a wide door to IM applications such as velocity and positional systems, previously reserved only for DC motors.

Indirect FOC (IFOC) has become an industrial standard due to its implementation simplicity. In IFOC, the rotor flux vector position is evaluated using the slip estimate and the measured rotor position. IFOC incorporates two orthogonal current controllers and decoupling circuits. IFOC is very sensitive to the rotor resistance variation, which is the main drawback.

Sliding mode control (SMC) [2], the popular nonlinear robust control strategy, which is theoretically invariant to model uncertainties and external disturbances under matching conditions [3] in analog implementations, is very attractive for IM control [4]. Up to now, a lot of papers dealing with IM SMC have been reported in the literature. The main issue in servo applications is the position control, which is essential for any motion control. High-performance industrial applications require fast response, preferably without overshoot, high accuracy in steady state, good rejection of external disturbances and robustness to parameter perturbations. SMC methodology can in great deal meet those requirements. Unfortunately, the chattering, usually associated with classical SMC design, is a serious impediment for SMC application. Various SMC algorithms have been devised for IM position control, e.g. [5]-[7].

This paper proposes a new IM position control approach for high-performance applications. The proposed control system is based on DSMC that allows a simplified IFOC structure, where only rotor flux is indirectly regulated by *d*-axis stator current control. The torque current controller and decoupling circuits are not needed. The system robustness is improved by applying an active disturbance estimator (ADE) [8]. The designed servo-system ensures excellent dynamics and high accuracy in presence of internal and external disturbances.

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II. IM Mathematical Model

An IM model in the *d-q* synchronously rotating frame, under commonly used assumptions, can be expressed as

$$\dot{i}_{ds} = -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) i_{ds} + \omega_e i_{qs} + \frac{R_r L_m}{\sigma L_s L_r^2}\phi_{dr} + \frac{\omega_r L_m}{\sigma L_s L_r}\phi_{qr} + \frac{1}{\sigma L_s}u_{ds}, \quad (1)$$

$$\dot{i}_{qs} = -\left(\frac{R_s}{\sigma L_s} + \frac{R_r L_m^2}{\sigma L_s L_r^2}\right) i_{qs} - \omega_e i_{ds} + \frac{R_r L_m}{\sigma L_s L_r^2}\phi_{qr} - \frac{\omega_r L_m}{\sigma L_s L_r}\phi_{dr} + \frac{1}{\sigma L_s}u_{qs}, \quad (2)$$

$$\dot{\phi}_{dr} = \frac{R_r L_m}{L_r} i_{ds} - \frac{R_r}{L_r}\phi_{dr} + (\omega_e - \omega_r)\phi_{qr}, \quad (3)$$

$$\dot{\phi}_{qr} = \frac{R_r L_m}{L_r} i_{qs} - \frac{R_r}{L_r}\phi_{qr} - (\omega_e - \omega_r)\phi_{dr}, \quad (4)$$

$$J\ddot{\theta}_m + B\dot{\theta}_m + T_l = T_e, \quad (5)$$

$$T_e = \frac{3 p_p L_m}{2 L_r}(i_{qs}\phi_{dr} - i_{ds}\phi_{qr}), \quad (6)$$

where u_{ds}, u_{qs} are *d-q* components of stator voltage, i_{ds}, i_{qs} are stator current components and ϕ_{dr}, ϕ_{qr} are rotor flux components; R_s, R_r are stator and rotor resistances; L_s, L_r, L_m are stator, rotor and mutual inductances; $\sigma = 1 - L_m^2/(L_s L_r)$ is leakage factor; p_p is number of pole pairs; ω_e, $\omega_r = p_p\omega_m$, $\omega_m = \dot{\theta}_m$ are synchronous, rotor electrical and rotor mechanical angular velocities; θ_m is rotor shaft angular position; T_e, T_l are electromagnetic and load torques; J is rotor inertia and B is viscous friction.

Vector control principle, usually implemented by rotor flux oriented control, ensures decoupling of torque control and rotor flux control. Rotor flux is oriented towards the *d*-axis,

$$\phi_{dr} = \phi_r, \ \phi_{qr} = \dot{\phi}_{qr} = 0. \quad (7)$$

Using (7), (3) and (4) are reduced to

$$T_r\dot{\phi}_r + \phi_r = L_m i_{ds}, \quad (8)$$

$$\omega_s = \omega_e - \omega_r = \omega_e - p_p\omega_m = L_m i_{qs}/(T_r\phi_r), \quad (9)$$

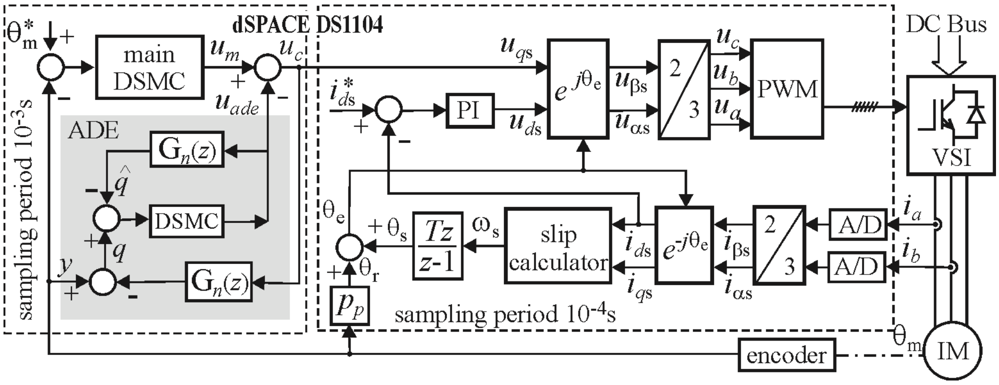

Fig. 1. Block diagram of the proposed robust position control system using simplified IFOC of IM.

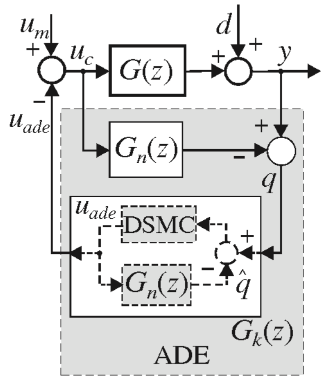

Fig. 2. Structure of ADE.

where $T_r = L_r / R_r$ is rotor time constant. The equations define the rotor flux dynamics and the slip frequency. As expected, the rotor flux is generated only by i_{ds}, and should be constant by controlling i_{ds} to keep a desired constant value i_{ds}^*. In the steady state the rotor flux is given by

$$\phi_r = L_m i_{ds}^* . \tag{10}$$

Substituting (7) and (10) into (6), the expression for the electromagnetic torque becomes

$$T_e = k_t i_{qs} ,\ k_t = (3p_p/2)(L_m^2/L_r) i_{ds}^* , \tag{11}$$

which is linearly dependent on i_{qs}, showing that both rotor flux and electromagnetic torque can be controlled separately.

By virtue of (7), (9), (10) and (11), under assumption that flux current controller ensures $i_{ds} = i_{ds}^*$, and by neglecting electrical time constant $T_{el} = \sigma L_s / R_s$, IM model (1)-(6) is reduced to a second order perturbed system, given by

$$\begin{gathered}\dot{\mathbf{x}} = (\mathbf{A} + \Delta\mathbf{A})\mathbf{x} + (\mathbf{b} + \Delta\mathbf{b})u + \mathbf{t}T_l, \quad y = \mathbf{g}\mathbf{x}, \\ \mathbf{x} = [x_1 \quad x_2]^{\mathrm{T}} = [\theta_m \quad \omega_m]^{\mathrm{T}}, \quad u = u_{qs}, \\ \mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-B}{J} \end{bmatrix}, \Delta\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-2k_t^2 L_s L_r^2}{3JL_m^2(R_s L_r + R_r L_s)} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ \frac{k_t}{JR_s} \end{bmatrix}, \\ \Delta\mathbf{b} = \begin{bmatrix} 0 & \frac{-k_t R_r L_s}{JR_s(R_s L_r + R_r L_s)} \end{bmatrix}^{\mathrm{T}}, \mathbf{t} = \begin{bmatrix} 0 & \frac{-1}{J} \end{bmatrix}^{\mathrm{T}}, \mathbf{g} = \begin{bmatrix} 1 & 0 \end{bmatrix}. \end{gathered} \tag{12}$$

In this approximated model the matching conditions are fulfilled, i.e. $\mathrm{rank}[\mathbf{b} \mid \Delta\mathbf{A} \mid \Delta\mathbf{b} \mid \mathbf{t}] = \mathrm{rank}[\mathbf{b}]$. Since matching conditions hold, (12) can be represented as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{j}v ,\ y = \mathbf{g}\mathbf{x} , \tag{13}$$

without loss of generality, where $\mathbf{j}v(t)$ is matched equivalent disturbance, i.e. $\mathrm{rank}[\mathbf{b} \mid \mathbf{j}] = \mathrm{rank}[\mathbf{b}]$.

The discrete-time representation of the control system (13), assuming zero-order-hold applied to the control signal $u(t) = u(kT)$, $kT \le t < (k+1)T$, $k \in N_0$, is obtained as

$$\begin{gathered}\mathbf{x}(k+1) = \mathbf{A}_d \mathbf{x}(k) + \mathbf{b}_d u(k) + \mathbf{w}(k),\ y(k) = \mathbf{g}\mathbf{x}(k), \\ \mathbf{A}_d = e^{\mathbf{A}T},\ \mathbf{b}_d = \int_0^T e^{\mathbf{A}t} \mathrm{d}t \mathbf{b},\ \mathbf{w}(k) = \int_0^T e^{\mathbf{A}t} \mathbf{j} v(kT + T - t) \mathrm{d}t ,\end{gathered} \tag{14}$$

where k stands for kT; T is a sampling period.

III. IM Position Control scheme

The proposed IM position control scheme, relying on simplified IFOC, is given in Fig. 1. Unlike the standard IFOC, in the suggested simplified IFOC there is only flux current PI controller. The torque current controller and decoupling circuits are excluded. The rotor flux vector angular position θ_e is obtained using slip estimate and measured rotor position. The simplest slip estimation is applied, $\omega_s = i_{qs} / (T_r i_{ds})$. The rotor resistance variation due to machine thermal changes results in inaccurate slip estimation, and consequently in incorrect rotor flux position. This leads to a violation of the ideal decoupling between torque and rotor flux, which deteriorates dynamic behavior. This phenomenon has not been isolatedly handled in this paper, although there exists a variety online rotor resistance identification techniques, which overcome this problem. Impact of the rotor resistance variation is here treated as a system perturbation, which is submitted to the robustness of the proposed scheme.

In addition to the main position DSM controller, the proposed control structure incorporates ADE, Fig. 1, in order to improve servo-system robustness and accuracy [9]. In ADE, a conventionally used passive digital filter $G_k(z)$ is replaced by another DSM controlled subsystem [8], Fig. 2 (dashed line). This increases efficiency of the compensation of external disturbances and parameter perturbations, which are jointly regarded as an equivalent disturbance. The control structure in Fig. 2 consists of a real plant $G(z)$ and ADE in the local loop. Equivalent disturbance q is evaluated inside the ADE employing discrete transfer function of the plant nominal model $G_n(z)$, which can be easily obtain by applying the z-transform on (14). The nominal model inevitably differs from the real plant. Signal $\hat{q}$ is an estimate of the compensated part of the equivalent disturbance.

It is shown in [8] that if DSM controller within ADE ensures $\hat{q} = q$, the equivalent disturbance is completely compensated and the plant has nominal behavior. In general, DSMC systems ensure only quasi-sliding mode [10]. A small but bounded difference between q and $\hat{q}$ will exist, implying that total disturbance rejection cannot occur, and in reality the obtained plant behavior is almost nominal. From the control design aspect, equivalent disturbance compensation is in this obtained by discrete-time tracking control within ADE with measurable but not known in advance referent signal $q(k)$.

Both controllers, in the main loop and within ADE, handle tracking tasks, with reference inputs $r(k) = \theta_m^*(k)$ and $q(k)$, respectively. The controller in ADE steers the nominal model not the real plant, thus all of the state variables are available and the control subsystem in ADE is free of disturbances. Since ADE forces the real plant to behave as the nominal system, the main controller also governs nominal plant. Hence, the tracking DSM controller, designed for the nominal system, may be used in the main loop as well as in ADE. Furthermore, the required state coordinates, feeding the main controller, are reliably obtained by an observer, since observation error depends only on the uncompensated part of the equivalent disturbance. Thus, the proposed servo-system requires only system output measurement.

IV. DSM Tracking Controller Design

As previously stated, both DSM controllers should ensure satisfactory output tracking of an unknown but measurable input signal $r(t)$ in the nominal system (main controller tracks θ_m^* and ADE controller tracks q). Since control objective is to force the output $y(t)$ to track the reference $r(t)$, it is desirable to transform the system model (13) into canonical tracking error space, $e_1 = r - x_1$, $e_2 = \dot{e}_1 = \dot{r} - x_2$. The required model is obtained as

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} - \mathbf{b}(u + b^{-1}v_r), \qquad (15)$$

where $v_r(t) = \ddot{r}(t) + a\dot{r}(t)$, $a = B/J$, $b = k_t/(JR_s)$. Additional matched disturbance term $\mathbf{b}b^{-1}v_r(t)$ occurs due to the transformation, and appears because the reference signal varies in time. The referent signal is not analytically known, hence the total elimination of its derivatives through the control part generally is not possible. Note also that in the case of a step input $v_r(t) = 0$.

Under the assumption that the referent signal is sampled, i.e. $r(t) = r(kT)$, $kT \le t < (k+1)T$, implying that the disturbance caused by the reference is also discrete, $v_r(t) = v_r(kT)$, the discrete-time model of the system (15) is given by

$$\mathbf{e}(k+1) = \mathbf{A}_d\mathbf{e}(k) - \mathbf{b}_d(u(k) - b^{-1}v_r(k)), \qquad (16)$$

with $\mathbf{A}_d$ and $\mathbf{b}_d$ defined in (14).

To attain zero tracking error employing DSM strategy, it is necessary to establish a discrete-time sliding mode along the sliding surface $s(k) = 0$, defined by the switching function

$$s(k) = \mathbf{c}\mathbf{e}(k), \quad \mathbf{c} \in \Re^{1\times 2}. \qquad (17)$$

If stable sliding dynamics is secured by appropriate selection of the vector $\mathbf{c}$, the state space origin will be reached, and there will be an ideal tracking. Since most of servo-systems do not allow chattering, only the “chattering free” DSMC algorithms may be considered to provide the desired motion.

Control algorithm [11], which represents a combination of two DSMC principles, reaching law and boundary layer, is adopted in this paper. A system trajectory approaches boundary layer according to some prescribed discrete-time reaching law [12]. Once the trajectory enters the predefined boundary layer, linear equivalent control is applied and system reaches the sliding surface in one step, [13]. The designed control law that ensures a desired reaching dynamics, defined by $\Phi(s)$, and partially compensates $v_r(k)$ is given as

$$u_{sm}(k) = (\mathbf{c}\mathbf{b}_d)^{-1}\mathbf{c}(\mathbf{A}_d - \mathbf{I})\mathbf{e}(k) + (\mathbf{c}\mathbf{b}_d)^{-1}\Phi(s(k)) + b^{-1}\tilde{v}_r(k), \qquad (18)$$

where $\tilde{v}_r(k) = a\dot{r}(k)$. If $\Phi(s(k)) = \min(|s(k)|, \sigma T)\operatorname{sgn}(s(k))$, the resulting reaching law outside the boundary layer $|s(k)| \ge \sigma T$ is, using (16)-(18), given by

$$s(k+1) = s(k) - \sigma T \operatorname{sgn}(s(k)) + \mathbf{c}\mathbf{b}_d b^{-1}\ddot{r}(k). \qquad (19)$$

It can be easily proven that if $|\ddot{r}(k)| \le M, \forall k$, finite time boundary layer convergence is obtained if the switching gain σ satisfies $\sigma > |\mathbf{c}\mathbf{b}_d| b^{-1}M/T$. Due to the uncompensated part of $v_r(k)$, linear control inside boundary layer $|s(k)| < \sigma T$ ensures only quasi-sliding mode in the domain $S_{qs} = \{\mathbf{e} \mid |s(\mathbf{e})| \le |\mathbf{c}\mathbf{b}_d| b^{-1}M\}$. In the case of step and ramp inputs, $M = 0$ and ideal discrete-time sliding mode occurs ($s(k+1) = 0$). Consequently, the designed DSM tracking controller provides ideal tracking of step and ramp references. The tracking of parabolic inputs has a constant error.

To improve system performances a specific integral action, described in [14], is introduced, which is active only inside the boundary layer, i.e. during the linear control mode. The control law is now formed as

$$u(k) = u_{sm}(k) - u_I(k), \qquad (20)$$

where the integral action is given by

$$u_I(k) = \begin{cases} 0, & |s(k)| \ge \sigma T, \\ hs(k) + u_I(k-1), & |s(k)| < \sigma T, \end{cases} \quad 0 < h < 1/T. \qquad (21)$$

Such extension improves controller tracking capabilities to ideal tracking of references (θ_m^*, q) up to parabolic forms, without any degradation of the system dynamics.

In the DSM based servo-system with ADE, Fig. 1, the overall control effort $u_c(k)$ is formed as

$$u_c(k) = u_m(k) - u_{ade}(k), \qquad (22)$$

where $u_m(k)$ and $u_{ade}(k)$ are the outputs of the main and ADE controllers, respectively. Each controller is realized using the same control law, described by (20), (18) and (21).

V. Experimental Results

The effectiveness of the proposed control structure has been investigated by experiments conducted on a servo-system with a three-phase IM, whose nominal parameters are given in Table I. Tests have been run in an experimental platform presented in Fig. 3. The applied incremental encoder gives an angle resolution of $3.8 \cdot 10^{-4}$ rad. The control part of the positional servo-system is implemented by dSPACE DS1104 R&D controller board, installed on a host computer. Parameters of the both controllers are summarized in Table II. Besides measured position, the main controller consumes velocity estimate from the conventional Luenberger observer.

The designed IM positional servo-system is tested under action of external disturbances, unmodeled dynamics and

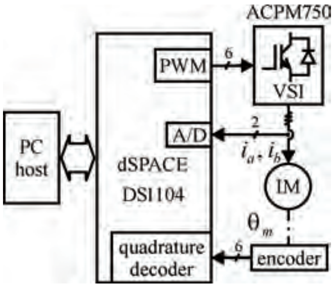

Fig. 3. Experimental setup.

TABLE I
IM NOMINAL PARAMETERS

Δ-connection	R_s=24.6 Ω
P_p=1	R_r=16.1 Ω
f_n=50 Hz	L_s=1.48 H
P_n=0.37 kW	L_r=1.48 H
I_n=1.7 A	L_m=1.46 H
T_n=1.3 Nm	J=3.5·10^{-4} kgm^2
n_n=2820 rpm	B=3·10^{-4} Nm/rad/s

TABLE II
DSM CONTROLLERS PARAMETERS

	Main Controller	ADE Controller
control law equations	(20), (18) and (21)	(20), (18) and (21)
switching gain	σ=50	σ=50
integral gain	h=10	h=10
SM dynamics eigen.	z_1=e^{-5T}, T=0.001 s	z_1=e^{-50T}, T=0.001 s
switching funct. vector **c**	[-0.0663 –0.0133]	[-0.6485 –0.013]

parameter perturbations. The system is subjected to a constant load torque $T_l(t) = 0.65$ Nm at the moment $t = 2.5$ s, which is 50% of the nominal torque. Rotor inertia is increased to $1.9J_n$. Also, an impact of the rotor resistance variation is emulated by employing in the slip calculator a 100% larger value ($32.2\,\Omega$) than the nominal one. Step response of such system is given in Fig. 4a. The effects of the counted disturbances are visible only in the enlarged scale, Fig. 4b. It is obvious that ADE considerably compensates the impact of the internal and external disturbances onto the system behavior, resulting in an excellent performance.

Apart from great robustness, ADE also increases steady-state accuracy. In fact, according to the Fig. 4c, it is apparent that the system provides maximal possible accuracy even in the presence of system perturbations, since the obtained error is within the measurement resolution. Finally, the overall control signal of the proposed system is displayed in Fig. 4d.

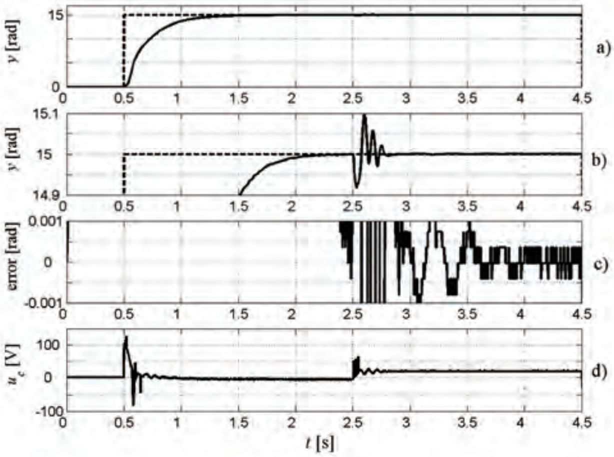

Fig. 4. a) Position response; b) Position response (enlarged scale); c) Steady-state positioning error; d) Control signal (overall).

VI. CONCLUSION

This paper offers a DSM based design of robust position control of IM for high performance applications. The proposed control structure, relying on simplified IFOC scheme, encloses ADE to further improve accuracy and robustness to parameter perturbations and external disturbances. Chattering free DSM tracking controller, designed for nominal plant, is employed as position and ADE controller as well.

The experimental results have confirmed analytically predicted performances, demonstrating excellent dynamic response and high accuracy under action of considerable parameter perturbations, un-modeled dynamics and external disturbances. Verified high efficiency of the proposed positional servo-system has justified the introduction of ADE.

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