Marginal Loss Coefficients and Nodal Factor Method for Loss Allocation in Distribution Systems with DG

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Abstract – **The main idea of this paper is to elaborate and evaluate the efficiency of two already proposed methods for loss allocation in distribution systems with dispersed generation (DG). This method's are: marginal loss coefficients and nodal factor pricing methods. This method's will be implemented on real distribution network. Several useful conclusion and problems with the efficiency of this method's will be presented.**

Keywords – **Dispersed Generation, Loss allocation, Marginal loss coefficients, Nodal factors pricing.**

I. Introduction

This paper is primarily concerned with the allocation of variable network losses in distribution network with DG. According to structural changes in power systems and introducing DG into distribution networks, the problem of allocation of losses become very important. In literature [1], requirements for ideal loss allocation method are summarized as follows: 1) *Economic efficiency:* Losses must be allocated in a way to reflect the true cost that each user imposes on the network; 2) *Accuracy, consistency and equity:* Loss allocation method must be accurate and equitable i.e. must avoid or minimize cross subsidies between users and between different time of use; 3) *Utilization of metered data:* From a practical standpoint it is desirable to base allocation of losses on actual metered data; 4) *Simplicity of implementation:* For any proposed method to find favor it is important that the method is easy to understand and implement. The main idea of this paper is to elaborate and evaluate the efficiency of two already proposed methods for loss allocation in distribution systems with dispersed generation. This methods are: marginal loss coefficients (MLCs) method [1] and nodal factor (NFs) pricing method [2]. By definition marginal loss coefficients measure the change in total active power losses due to a marginal change in consumption/generation of active and reactive power at each node in the network. The nodal factor pricing method determines the prices at different nodes in the distribution networks using nodal factors. These prices are short-run economically efficient and allocate losses on location. Paper begins with short theoretical elaboration of MLCs and NF methods. After that, this methods are implemented on real distribution network, which is a part of the Distribution company in Bitola, Republic of Macedonia. Results of the implementation will be presented and discussed according to the requirements for ideal loss allocation. Several useful conclusion and problems with the efficiency of this method's are presented. Power flow results will be used as input data for loss allocation with the elaborated methods.

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II. Marginal Loss Coefficients and Nodal Factors Method

A. Marginal Loss Coefficients

By definition MLCs measure the change in total active power losses L due to a marginal change in consumption/generation of active power P_i and reactive power Q_i at each node i in the network.

$$\tilde{\rho}_{Pi} = \frac{\partial L}{\partial P_i} \qquad \tilde{\rho}_{Qi} = \frac{\partial L}{\partial Q_i} \tag{1}$$

where $\tilde{\rho}_{Pi}$ and $\tilde{\rho}_{Qi}$ represent the active and reactive power related MLCs. If a user, i.e. generator, takes part in voltage control by injecting required reactive power (PV node); there are no loss-related charges for the reactive power to be allocated. This is reflected by

$$\tilde{\rho}_{Qi} = \frac{\partial L}{\partial Q_i} \overset{def}{=} 0 \qquad i \text{ is a PV node} \tag{2}$$

Since in load flow calculations, losses are deemed to be supplied from the slack node, the loss-related charges for this node are zero. In other words, total power losses are insensitive to changes in active and reactive injections at the slack node i.e.

$$\frac{\partial L}{\partial P_s} = \frac{\partial L}{\partial Q_s} = 0 \qquad \text{s is the slack node} \tag{3}$$

MLCs are a function of a particular system operating point. As there is no explicit relationship between losses and power injections the standard chain rule is applied in the calculations of MLCs using intermediate state variables, voltage magnitudes and angles. Therefore only a load flow solution for a particular system operating point is required to compute MLCs.

Applying the standard chain rule, the following general system of linear equations can be established for calculating MLCs

$$\left[\begin{array}{ccc|ccc} \frac{\partial P_1}{\partial \theta_1} & \cdots & \frac{\partial P_N}{\partial \theta_1} & \frac{\partial Q_1}{\partial \theta_1} & \cdots & \frac{\partial Q_N}{\partial \theta_1} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial P_1}{\partial \theta_N} & \cdots & \frac{\partial P_N}{\partial \theta_N} & \frac{\partial Q_1}{\partial \theta_N} & \cdots & \frac{\partial Q_N}{\partial \theta_N} \\ \hline \frac{\partial P_1}{\partial U_1} & \cdots & \frac{\partial P_{N-1}}{\partial U_1} & \frac{\partial Q_1}{\partial U_1} & \cdots & \frac{\partial Q_N}{\partial U_1} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial P_1}{\partial U_N} & \cdots & \frac{\partial P_N}{\partial U_N} & \frac{\partial Q_1}{\partial U_N} & \cdots & \frac{\partial Q_1}{\partial U_N} \end{array}\right] \cdot \left[\begin{array}{c} \frac{\partial L}{\partial P_1} \\ \vdots \\ \frac{\partial L}{\partial P_N} \\ \hline \frac{\partial L}{\partial Q_1} \\ \vdots \\ \frac{\partial L}{\partial Q_N} \end{array}\right] = \left[\begin{array}{c} \frac{\partial L}{\partial \theta_1} \\ \vdots \\ \frac{\partial L}{\partial \theta_N} \\ \hline \frac{\partial L}{\partial U_1} \\ \vdots \\ \frac{\partial L}{\partial U_N} \end{array}\right] \tag{4}$$

Eqn. (4) can be written in a more compact form as follows:

$$\mathbf{A} \cdot \tilde{\boldsymbol{\rho}} = \mathbf{b} \tag{5}$$

Matrix A is the transpose of the Jacobian in the Newton-Raphson load flow and can be calculated on the basis of load flow results for a particular system operating point. The vector $\tilde{\boldsymbol{\rho}}$ represents MLCs whereas the right-hand vector $\mathbf{b}$ represents sensitivities of total losses with respect to voltage angle and magnitude (θ, U). Total system active loss L is given by:

$$L = \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij} \left[U_i^2 + U_j^2 - 2U_i U_j \cos(\theta_i - \theta_j) \right] \tag{6}$$

Therefore the entries of vector b in eqn. (5) are

$$\frac{\partial L}{\partial \theta_i} = 2 \sum_{j=1}^{N} G_{i-j} U_i U_j \sin(\theta_i - \theta_j) \qquad i = 1,....N \tag{7}$$

$$\frac{\partial L}{\partial U_i} = 2 \sum_{j=1}^{N} G_{i-j} \left[U_i - U_j \cos(\theta_i - \theta_j) \right] \quad i = 1,....N \tag{8}$$

Note that there are no equations for any voltage-controlled node as by definition the MLC with respect to reactive power for any such node is zero. The result of applying MLCs calculated in accordance with the procedure outlined yields approximately twice the amount of losses. That is:

$$\sum_{i=1}^{N-1} \left[\tilde{\rho}_{Pi} \cdot P_i + \tilde{\rho}_{Qi} \cdot Q_i \right] \approx 2 \cdot L \tag{9}$$

Therefore there is a need of reconciliation. Constant multiplier reconciliation factor k_0 is introduced in order to obtain vector of reconciled MLCs ρ. The factor k_0 is calculated as follows:

$$k_0 = \frac{L}{\sum_{i=1}^{N-1} \left[\tilde{\rho}_{Pi} \cdot P_i + \tilde{\rho}_{Qi} \cdot Q_i \right]} \tag{10}$$

The vector of reconciled MLCs ρ is then calculated as follows:

$$\rho = k_0 \cdot \tilde{\rho} \tag{11}$$

Reconciled MLCs enable the allocation of the total system active power losses to individual users such that:

$$\sum_{i=1}^{N-1} \rho_{P_i} \cdot P_i + \sum_{i=1}^{N-1} \rho_{Q_i} \cdot Q_i = L \tag{12}$$

B. Nodal factor pricing method

MLCs are used for defining nodal factors [2]. The prices which in the same time are optimizing the global system and individual user of the system are defined as follows:

$$pa_{k_g} = \lambda \cdot \left(1 - \frac{\partial L}{\partial P_{k_g}} \right) \tag{13}$$

$$pr_{k_g} = -\lambda \cdot \left(\frac{\partial L}{\partial Q_{k_g}} \right) \tag{14}$$

$$pa_{k_p} = \lambda \cdot \left(1 + \frac{\partial L}{\partial P_{k_p}} \right) \tag{15}$$

$$pr_{k_p} = \lambda \cdot \left(\frac{\partial L}{\partial Q_{k_p}} \right) \tag{16}$$

where P_{k_g}, Q_{k_g} respectively is the active and reactive power injected by generator in bus k_g; P_{k_p}, Q_{k_p} respectively is the active and reactive power consumed by demand in bus k_p; pa_{k_g} is the price that a generating type network user will offer for one unit of active energy at bus k_g; pa_{k_p} is the price that a demand type network user will pay for one unit of active energy at bus k_p; pr_{k_g}, pr_{k_p} similar definitions but for the reactive energy; λ is the price of electric energy on the wholesale market at the connection bus between the distribution and transmission network.

These prices define the economic dispatch and correspond to what is widely known as nodal pricing. As seen before, the active energy marginal prices result from the product of λ by the factor:

$\left(1 - \frac{\partial L}{\partial P_{k_g}} \right)$ in the case of a generator bus

$\left(1 + \frac{\partial L}{\partial P_{k_p}} \right)$ in the case of a demand bus

If we make the following change of variables, $P_k = P_{k_p}$ and $P_k = -P_{k_g}$, it results $pa_k = \lambda \cdot \left(1+\frac{\partial L}{\partial P_k}\right)$. Therefore we define $f_{n_k} = \left(1+\frac{\partial L}{\partial P_k}\right)$ as the active NF corresponding to bus k ($pa_k = \lambda \cdot f_{n_k}$). In the same way, it is possible to define the reactive NF for bus k as $f'_{n_k} = \frac{\partial L}{\partial Q_k}$ ($pr_k = \lambda \cdot f'_{n_k}$).

III. Application and Evaluation of the Elaborated Methods

Let us consider real radial 10 kV distribution network of Fig. 1, which is a part of the distribution network of Distribution Company-Bitola. There are different types of loads in the nodes of this network and to provide realistic load variations, several customer types are included in the analysis, using typical daily load profiles. Application of the elaborated methods is performed on hourly basis for two extreme cases: typical winter working day and summer Sunday. As input data for allocation of losses with the methods, power flow calculations with Newton-Raphson are used. DG is working with constant output and constant power factor during the considered typical days. It is important to mention that in bus TS Pumpi Vodovod where HEC Dovledzik is placed, there is a consumer also and DG is supplying energy to the consumer and the rest of the energy injects into the network. In order better to obtain the influence of DG on network losses, power flow calculations are performed for two scenarios for each case: base scenario (without DG) and scenario with DG.

On Fig. 2 variation of network losses is shown for typical winter working day for the two scenarios. It is obvious that in scenario with DG, network losses are increased only in the period of low load conditions between 2 and 4 hour, until in the rest of the day DG is significantly decreasing network losses. On Fig. 3 variation of network losses is shown for typical summer Sunday for the two scenarios. From this results it can be concluded that DG decreases network losses during hall day. These analysis is done in order better to evaluate the results from loss allocation obtained with MLCs and NFs method.

On Fig. 4 loss allocation profiles in kW for network busses are shown with MLCs method, for a typical winter working day. On Fig. 5 active energy (power) NFs variation for a typical winter working day is illustrated. The connection bus (10 kV Bitola 4) between the distribution and transmission network is considered referent bus with active $NF = 1$. Because of the lack of space reactive NFs variation is not shown. From loss allocation profiles with MLCs shown on Fig. 4, it can be concluded that DG HEC Filternica in periods of low load between 0-7 and 15-24 hours has positive allocation of losses, what means that it should pay for increasing network losses. In the same period the bus TS Filetrnica which is a consumer bus, has a negative allocation of losses, what means it should be rewarded for decreasing

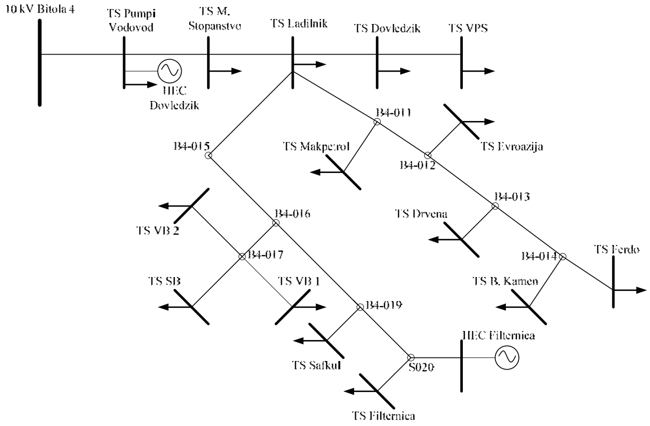

Fig. 1. Case studied 10 kV distribution network

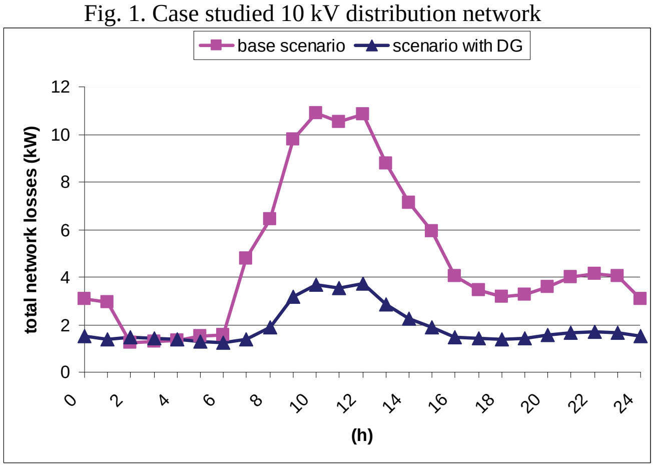

Fig. 2. Total network losses variation for typical winter working day

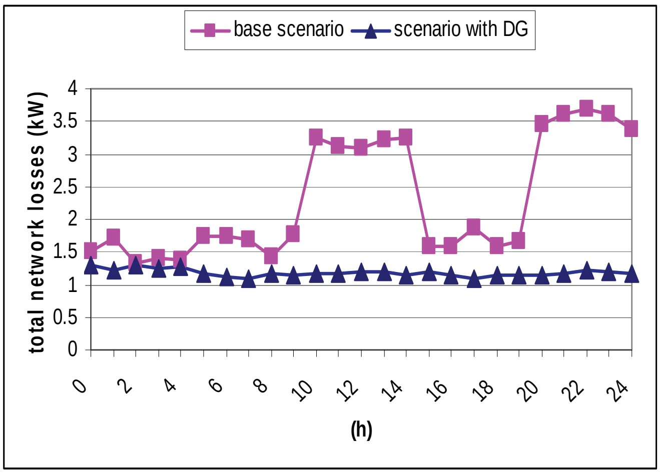

Fig. 3. Total network losses variation for typical summer Sunday

network losses. This is a classical example of cross subsidy, because according to the results for the total network losses for base scenario and scenario with DG it is obvious that network losses are decreased from the DG HEC Filternica. The same conclusions can be developed from Fig. 5 where active NFs are shown for a typical winter working day. In the period of low load between 0-7 and 15-24 hours, active NFs for DG HEC Filternica are less than 1, what means it gets low

-er price for injected active electric energy from the price in referent bus, because it increases losses in the network. In the same time bus TS Filternica which is a consumer bus and some other consumer buses receive lower price of active electric energy. This means that consumers are decreasing the losses in the network.

On Fig. 6 loss allocation profiles in kW for network busses are shown with MLCs method, for a summer Sunday. On Fig. 7 active NFs variation for a summer Sunday is illustrated. From loss allocation profiles with MLCs shown on Fig. 6, it can be concluded that DG HEC Filternica in period between 0-19 hours has positive allocation of losses, what means that it should pay for increasing network losses. In the period of 0-9 hours and 15-19 hours the bus TS Filetrnica which is a consumer bus, has a negative allocation of losses, what means it should be rewarded for decreasing network losses. This is also a classical example of cross subsidy, because according to the results for the total network losses for base scenario and scenario with DG for summer Sunday (Fig.3), it is obvious that network losses are decreased from DG HEC Filternica. The DG HEC Dovledzik in the bus TS P.Vodovod has positive allocation of losses during 24 hours and it is very close to zero because during summer Sunday this generator is used for supplying the consumer in the same bus. The same conclusions can be developed from Fig. 7 where active NFs are shown for a summer Sunday. In the period between 0-9 and 15-19 hours, active NFs for DG HEC Filternica are less than 1, what means it gets lower price for injected active electric energy from the price in referent bus, because it increases losses in the network. In the same time, bus TS Filternica which is a consumer bus and some other consumer buses receive lower price of active electric energy. This means that consumers are decreasing losses in the network.

IV. Conclusion

This paper has investigated the efficiency of two already proposed methods for loss allocation in distribution systems with dispersed generation: MLCs method and nodal factor NFs pricing method. Results of this analysis illustrates that MLCs and NFs methods are producing temporal and spatial cross subsidies. According to requirements for ideal loss allocation this two methods does not fulfill two basic conditions for ideal allocation: economic efficiency and eliminating cross subsidies.

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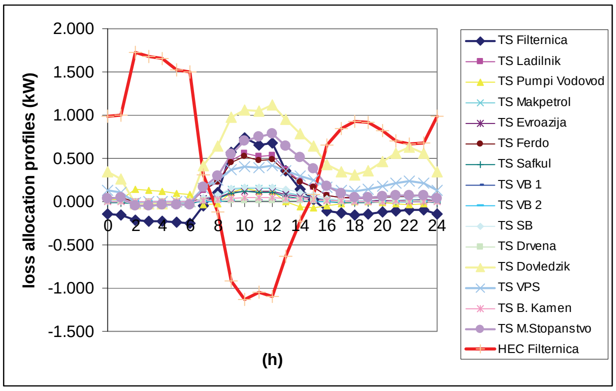

Fig. 4. Network buses loss allocation profiles in (kW) with MLCs for typical winter working day

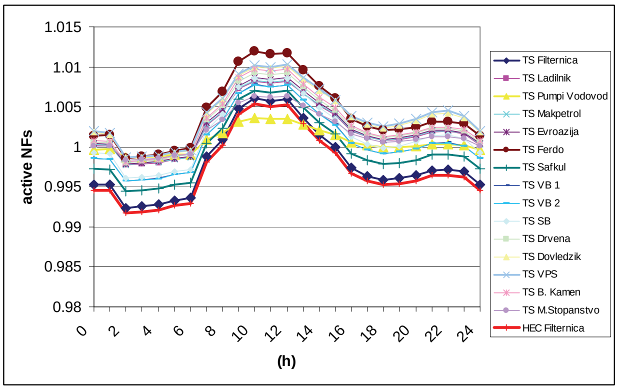

Fig. 5. Network buses active NFs profiles for typical winter working day

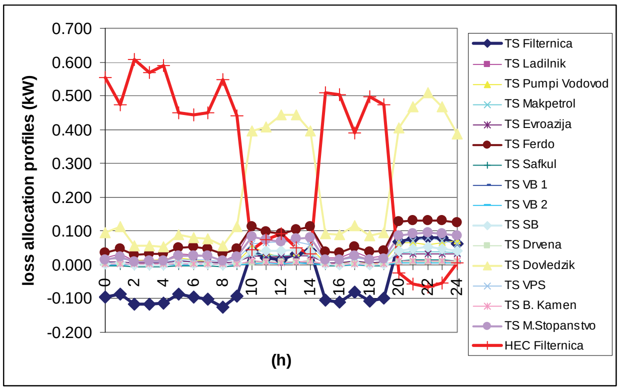

Fig. 6. Network buses loss allocation profiles in (kW) with MLCs for typical summer Sunday

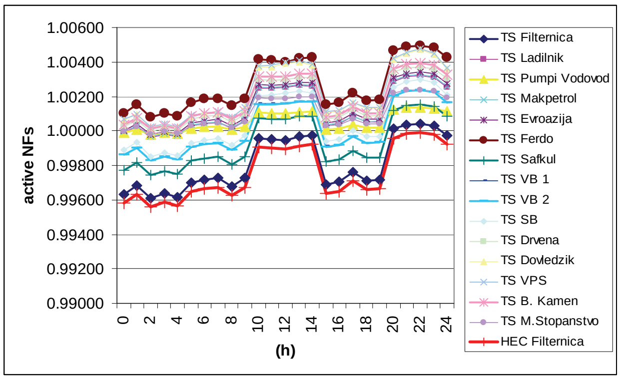

Fig. 7. Network buses active NFs profiles for typical summer Sunday