Modeling of Step Discontinuity in Microstrip Structures by using Wave Digital Approach

Biljana P. Stošić[1], Miodrag V. Gmitrović[2]

Abstract – **Wave digital network is a model of the microstrip structure modeled by wave digital elements. Appropriate choice of a minimal section number in that model is very important because of the direct influence on the sampling frequency of that digital model, and on accuracy of the desired response. Also, very important in modeling microstrip structure is good compensation of the identified discontinuities. In this paper, two ways of modeling step discontinuities are given. Equivalent *L*-network of the discontinuity is modeled here by one equivalent transmission line and by cascade-connected two transmission lines. One test example, proving the response accuracy is given.**

Keywords – **Wave digital approach, step discontinuity, transmission lines, microstrip circuits**

I. Introduction

Modeling of the planar structures by wave digital elements, based on well known theory of wave digital filters [1]-[3], can be efficiently used for analysis of these structures in both the time and the frequency domains. Microwave planar structures can be modeled by one-dimensional [4]-[6] and by two-dimensional [7] wave digital elements.

A nonuniform structure has to be divided into cascade-connected uniform transmission lines (*UTL*). A lossless uniform transmission line is modeled by a two-port digital element with a delay appears in forward path. This wave digital two-port is called the unit element (*UE*) [2]. The port resistances of the *UE* are equal and correspond to the characteristic impedance of *UTL*. The connection of two *UE* with different port resistances is achieved by two-port adaptors (*TA*), [3].

In the complex microstrip structures, delays of the transmission lines vary from one another and because of this, each transmission line has to be represented as a cascade connection of a certain number of *UE*. A way of determination a minimal section numbers in wave digital network (*WDN*) of complex microstrip structure is given in the paper [8]. Number of section in *WDN* has direct influence on the sampling frequency of that digital model, and on accuracy of desired response.

Efficient and very simple algorithms for calculating transmission and input reflection coefficients of the microstrip structure modeled by wave digital elements are described in the papers [5]-[6]. The algorithms are very easily implemented in the *MATLAB* environment. The analysis of the wave digital structure is efficiently automated, which is inevitable when structure with large numbers of building blocks (*UE* and *TA*) are to be dealt with.

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In the previously published papers [4]-[6], nonuniform microstrip structures are observed as cascaded *UTL* segments, but effects of the step discontinuities have not been taken under consideration. But, once the step discontinuities have been identified in the structure, they must be corrected. One wave digital model of asymmetrical equivalent *T*-network of this discontinuity is described in the paper [9]. In this paper, two new ways of modeling equivalent circuit of this discontinuity are given. Modeling is based on the possible approximation of the reactive elements by transmission lines, which is well known from the electric circuit theory.

II. Approximation of Reactive Elements by Transmission Lines

From the electric circuit theory it is well known, that an inductance in series branch and a capacitance in parallel branch can be approximated by two-port transmission lines, [10]-[11]. For high frequencies, an inductance is replaced by transmission line of high characteristic impedance (Figure 1), and a capacitance by transmission line of low characteristic impedance (Figure 2). Electrical lengths of the transmission lines are $\theta < 45^o$.

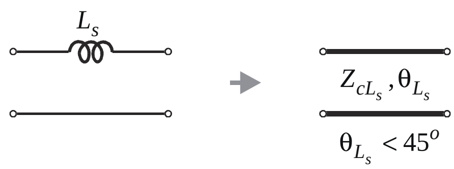

Fig.1. Synthesis of an inductance in series branch by transmission line of high characteristic impedance Z_{cL_s}

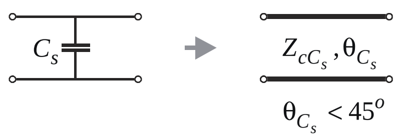

Fig.2. Synthesis of a capacitance in parallel branch by transmission line of low characteristic impedance Z_{cC_s}

An electrical length of a transmission line corresponding to the capacitance C_S in parallel branch can be found as

$$\theta_{C_s} = 2\pi f_C \cdot C_S \cdot Z_{cC_s} . \quad (1)$$

Also, an electrical length of a transmission line corresponding to the inductance L_S in series branch can be found as

$$\theta_{L_s} = 2\pi f_C \cdot L_S / Z_{cL_s} . \quad (2)$$

Parameter f_C is the cuttoff frequency for lowpass and highpass filters and the center frequency for bandpass and bandstop filters.

III. MODELING OF THE STEP DISCONTINUITY EQUIVALENT NETWORK

Discontinuity in the width of a microstrip line is a very often used in the microstrip circuits in order to change characteristic impedance of the line. This is important for design of the filters and the impedance matching networks. In practice, accuracy of the discontinuity models depends on their physical dimensions [10].

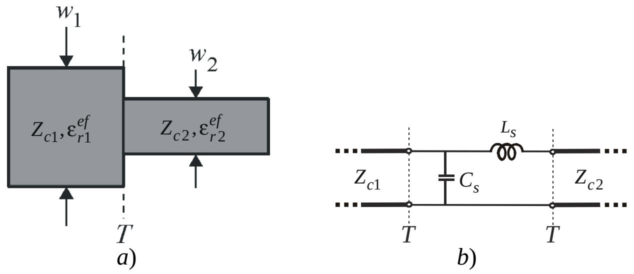

Fig.3. *a*) Step discontinuity and
b) Equivalent network of the discontinuity

Equivalent network of the discontinuity has a parallel capacitance C_S placed near the wide line, and the series inductance L_S placed near the narrow line, as shown in Figure 3*b*.

A. Modeling by One Transmission Line

Discontinuity network shown in Figure 3*b*, can be aproximated with one equivalent transmission line. Characteristic impedance of the transmision line is

$$Z_{cvs} = \sqrt{L_S / C_S}\ , \tag{3}$$

and a delay on the transmission line is

$$T_S = \sqrt{L_S C_S}\ , \tag{4}$$

where L_S is the inductivity of the series inductance and C_S is the capacitivity of the parallel capacitance. Electrical length of the transmission line is

$$\theta_S = 2\pi f_C T_S \tag{5}$$

and physical length can be found as

$$d_S = cT_S / \sqrt{\varepsilon_r}\ , \tag{6}$$

where $c = 3 \cdot 10^8\, m/s$ is light velocity in free space and ε_r is relative dielectric constant of the substrate.

Planar microstrip structure shown in Figure 3*a*, can be aproximated by cascade connection of 3 transmission lines as shown in Figure 4. Each *UTL* segment from Figure 3*a* is aproximated by one transmission line (Z_{c1} and Z_{c2}). Equivalent network of discontinuity is aproximated by one transmission line given in the middle (Z_{cvs}).

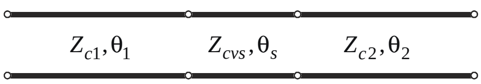

Fig.4. Cascade connection of 3 transmission lines

Then, each transmision line is modeled by certain number of *UE*. A nonuniform planar microstrip structure with step discontinuity shown in Figure 3*a*, is modeled by a *WDN* composed only of two types of building blocks (*UE* and *TA*) as shown in Figure 5. This type of *WDN* is very easily and simply analyzed by using wave transfer matrices and algorithm given in the paper [5].

Adaptor coefficients in the *WDN* are

$$\begin{aligned} \alpha_G &= (R_g - Z_{c1})/(R_g + Z_{c1})\,, \\ \alpha_{V1VLsCs} &= (Z_{c1} - Z_{cvs})/(Z_{c1} + Z_{cvs})\,, \\ \alpha_{VLsCsV2} &= (Z_{cvs} - Z_{c2})/(Z_{c2} + Z_{cvs})\,, \\ \alpha_P &= (Z_{c2} - R_p)/(Z_{c2} + R_p)\,. \end{aligned} \tag{7}$$

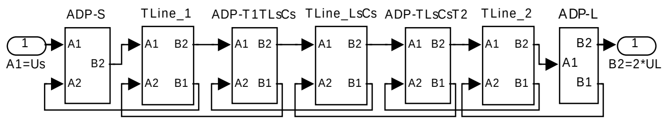

Fig.5. *WDN* of the structure with modeled discontinuity elements with one equivalent transmission line

B. Modeling by Two Cascaded Transmission Lines

Each element of the equivalent discontinuity network, L_S and C_S, is aproximated by one transmission line. So, equivalent network of the discontinuity is aproximated by cascade connection of two transmission lines.

A physical length of a transmission line corresponding to the capacitance C_S in parallel branch can be found as

$$d_{C_s} = c \cdot C_S \cdot Z_{cC_s} / \sqrt{\varepsilon_{r1}^{ef}}\ , \tag{8}$$

where ε_{r1}^{ef} is effective dielectric constant of the wide line in the structure shown in Fig.3*a*.

Also, a physical length of a transmission line corresponding to the inductance L_S in series branch can be found as

$$d_{L_s} = c \cdot L_S \Big/ \left(Z_{cL_s} \cdot \sqrt{\varepsilon_{r2}^{ef}} \right), \tag{9}$$

where ε_{r2}^{ef} is effective dielectric constant of the narrow line in the structure shown in Fig.3*a*.

These physical lengths depend on the characteristic impedances of the transmission lines. They are chosen in the next way: characteristic impedance of the transmission line used for aproximation of the series inductance is chosen to be $Z_{cL_s} = 150\,\Omega$, and characteristic impedance of the transmission line used for aproximation of the parallel capacitance is $Z_{cC_s} = 5\,\Omega$. This is a result of the well known fact that typical values of these impedances are $\leq 10\,\Omega$ and $\geq 150\,\Omega$, respectively.

Equivalent representation of the microstrip structure with step discontinuity, shown in Figure 3*b*, can be here aproximated by cascade connection of four transmission lines as shown in Figure 6. Equivalent network of discontinuity is aproximated by two transmission lines given in the middle (Z_{cC_s} and Z_{cL_s}).

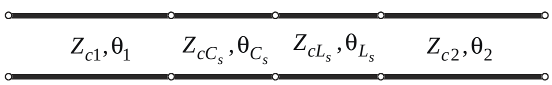

Fig.6. Cascade connection of 4 transmission lines

Cascade connection of 4 transmission lines shown in Figure 6, is modeled by *WDN* as shown in Figure 7. A total number of building blocks in this network is bigger then in that one shown in Figure 5, because of the equivalent discontinuity network approximation by two cascaded transmisison lines.

Adaptor coefficients in the *WDN* are

$$\alpha_{V1VCs} = (Z_{c1} - Z_{cC_s})/(Z_{c1} + Z_{cC_s}),$$
$$\alpha_{VCsVLs} = (Z_{cC_s} - Z_{cL_s})/(Z_{cL_s} + Z_{cC_s}), \qquad (10)$$
$$\alpha_{VLsV2} = (Z_{cL_s} - Z_{c2})/(Z_{c2} + Z_{cL_s}).$$

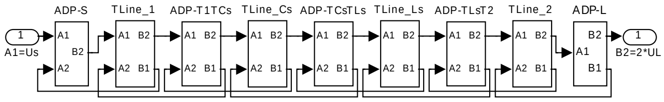

Fig.7. *WDN* of the structure with modeled discontinuity elements with separate transmission lines

IV. Analysis Example

A microstrip stepped-impedance 7th order Chebyshev lowpass filter with passband ripple of $0.0137\,dB$ and cutoff frequency of $900\,MHz$ [11], is used for verification of the proposed method. The layout is shown in Figure 8, and this circuit is also analyzed in [5].

Fig.8. Layout of lowpass filter

Here, $50\ \Omega$ leader lines at the ends of the structure are not included during the analysis of the structure modeled by *UE*. These leader lines affect to the total delay in the *WDN* and have effect of shifting response characteristics. So, the microstrip lowpass filter is observed as a cascade connection of seven *UTL* segments. Their delays vary from one another because of their dependence on the effective dielectric constant. In order to have delays in the wave digital models as possible equal to these delays, each transmission line has to be represented as a cascade connection of a certain number of *UE*.

A. Modeling of the Discontinuity Equivalent Network by One Transmission Line

Cascade connection of the equivalent network elements L_S and C_S, is aproximated by one transmission line with characteristic impedance $Z_{cvs} = 38.3220\,\Omega$ according to (3), and physical length $d_{vs} = 0.6489\,mm$ according to (6).

Microstrip structure given in Figure 8 is aproximated by cascade connection of 13 transmission lines with parameters given in Table I. *WDN* has the structure given in Figure 5, where a number of blocks corresponding to transmission lines is 13 and a number of blocks corrsponding to two-port adaptors is 14.

A minimal number of section for the given error, can be found as described in the paper [8] where multiple factor $q \geq 1$ is used. For given error $n\_er = 0.01\,\%$, first relative error of delay with absolute value less then given error is for $q = 5$, [8]. Then, a total minimal number of sections in *WDN* is $n_t = \sum_{k=1}^{13} n_k = 851$. For individual *UTL* segments, a number of sections n_k is 116, 5, 55, 5, 205, 5, 69, 5, 205, 5, 55, 5 and 116, respectively. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min}/q = 901.7895\,ps$ where $T_{\min} = \min\{T_1, T_2, ..., T_{13}\} = 5.2984\,ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{13} T_k = 901.7682\,ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 943.6792\,GHz$. A relative error of delay is $er = \dfrac{T_\Sigma - T_t}{T_\Sigma} \cdot 100\,\% = -0.0024\,\%$.

TABLE I
TRANSMISSION LINE PARAMETERS

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	18.1149	68.8833	122.6590
2	0.6489	38.3220	5.2984
3	7.8345	15.9611	58.7383
4	0.6489	38.3220	5.2984
5	32.0420	68.8833	216.9616
6	0.6489	38.3220	5.2984
7	9.7713	15.9611	73.2599
8	0.6489	38.3220	5.2984
9	32.0420	68.8833	216.9616
10	0.6489	38.3220	5.2984
11	7.8345	15.9611	58.7383
12	0.6489	38.3220	5.2984
13	18.1149	68.8833	122.6590

B. Modeling of the Discontinuity Equivalent Network by Two Cascaded Transmission Lines

Each element of equivalent circuit of the discontinuity, L_S and C_S, is aproximated by one transmission line with characteristic impedances $Z_{cL_s} = 150\,\Omega$ and $Z_{cC_s} = 5\,\Omega$, respectively.

Microstrip structure given in Figure 8 is aproximated by cascade connection of 19 transmission lines with parameters given in Table II. *WDN* has the structure given in Figure 7, where a number of blocks corresponding to transmission lines is 19 and a number of blocks corrsponding to two-port adaptors is 20.

For given error $n\_er = 0.01\,\%$, a total minimal number of sections in *WDN* is $n_t = \sum_{k=1}^{19} n_k = 16590$, [8]. For individual *UTL* segments, a number of sections n_k is 2307, 25, 13, 1105, 13, 25, 4080, 25, 13, 1378, 13, 25, 4080, 25, 13, 1105, 13, 25 and 2307, respectively. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min}/q = 882.2080\,ps$ where $q = 13$ is a multiple factor and $T_{\min} = \min\{T_1, T_2, ..., T_{19}\} = 0.6913\,ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{19} T_k = 882.2474\,ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 18805.0889\,GHz$. In this case, a relative error of delay is $er = 0.0045\,\%$.

TABLE II
TRANSMISSION LINE PARAMETERS

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	18.1149	68.8833	122.6590
2	0.1999	150.0000	1.3536
3	0.0922	5.0000	0.6913
4	7.8345	15.9611	58.7383
5	0.0922	5.0000	0.6913
6	0.1999	150.0000	1.3536
7	32.0420	68.8833	216.9616
8	0.1999	150.0000	1.3536
9	0.0922	5.0000	0.6913
10	9.7713	15.9611	73.2599
11	0.0922	5.0000	0.6913
12	0.1999	150.0000	1.3536
13	32.0420	68.8833	216.9616
14	0.1999	150.0000	1.3536
15	0.0922	5.0000	0.6913
16	7.8345	15.9611	58.7383
17	0.0922	5.0000	0.6913
18	0.1999	150.0000	1.3536
19	18.1149	68.8833	122.6590

C. Result Comparison

Response comparison for the two presented ways of modeling step discontinuity is shown in Figures 9 and 10.

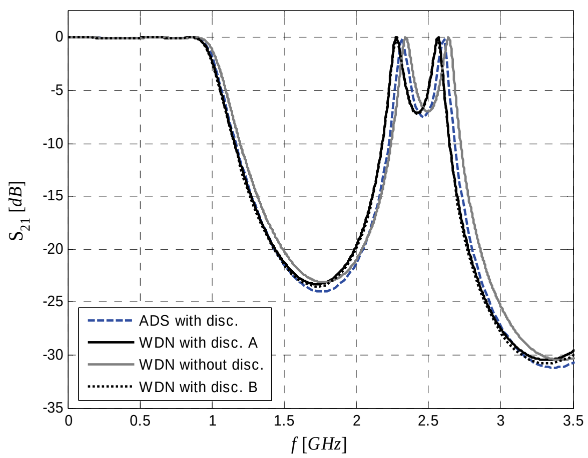

Fig.9. Response comparison.

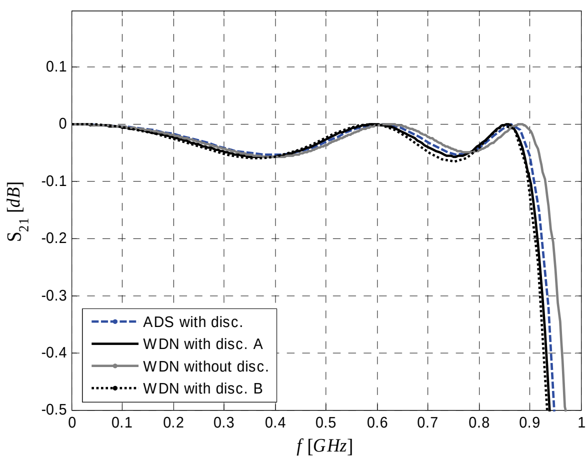

Fig.10. Comparison of the bandpass responses.

It is clear that curves corresponding to the results obtained by modeling structure and its discontinuities in the described ways (approaches A and B) are very close one another and practically identical.

In the region below $1.5\,GHz$, the agreement between the results calculated by using *WDN* with modeled discontinuities and the results obtained in *ADS* is very good. For the region above $1.5\,GHz$ the curve is shifted slightly to the left. A curve for *WDN* without modeled discontinuities is shifted to the right in whole frequency band.

V. CONCLUSION

To prove the accuracy of the proposed modeling of the step discontinuity, the computer simulated results obtained by *WDN* where discontinuities are modeled with one and two transmission lines are compared to those obtained in *ADS* (Advanced Design Systems). We can see that results obtained by described approaches of modeling step discontinuity are practically identical and have very good agreement with *ADS* data in whole frequency band. If approaches A and B are compared, it can be seen that the approach of modeling discontinuities with one transmission line (approach A) is better because it requires less *UE* and simpler digital *WDN*.

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