TLM-Z Method Modelling of Microwave Cavity Loaded with Frequency-Dependent Dielectric Slab

Bratislav Milovanović, Nebojša Dončov, Jugoslav Joković, Tijana Dimitrijević

Abstract – **In this paper, Transmission Line Matrix method based on Z-transforms (TLM-Z) is presented as a tool for electromagnetic field study in linear frequency-dependent materials. On the example of the metallic cavity with rectangular cross section loaded with Debye water dielectric, this method is compared with both TRM and conventional TLM method. The main results are given, together with emphasized advantages of the TLM method based on Z-transforms.**

Keywords – **cavity, frequency-dependent load, resonant frequency, TLM, Z-transforms**

I. Introduction

Cylindrical metallic cavities loaded with homogeneous dielectric slabs represent a configuration very suitable for good modeling of some practical microwave applicators used in the processes of dielectric material heating and drying. The knowledge of the mode tuning behavior in a cavity under loading condition (i.e. physical and electrical parameters of the load) forms an integral part of the studies in microwave heating and it can considerably help in designing these applicators [1].

Transverse resonance method (TRM) [2] is a conventional approach for carrying a theoretical analysis of such partially loaded cylindrical metallic cavities. Representing a loaded cavity as a cascade connection of the equivalent transmission lines, characteristic equation for resonant frequency calculation can be derived from the transverse resonance condition. In a general case, this equation is complex transcendental and its solution requires an appropriate numerical technique and an efficient mode identification procedure [3]. Such complex mathematical calculations are hardware and time consuming representing a main disadvantage of TRM approach.

In last few decades, several computational electromagnetic techniques, among them the Finite Difference Time Domain (FD-TD) [4] and Transmission Line Matrix (TLM) [5] are most popular in the field; emerge as an invaluable numerical tool providing a computationally more efficient cavity design solution than TRM. TLM time-domain method is a general, electromagnetically based numerical method that has been developed and applied by a number of research groups in the world. Authors of this paper have been used the TLM method in the past to investigate the influence of irregular dielectric slab shapes and its inhomogeneity to the resonant frequencies of the cylindrical metallic cavity [6,7]. In all these applications, cavity load was treated as a constant parameter dielectric material whose complex relative permittivity was calculated usually at the central frequency of range of interest. However, such conventional TLM method approach is not adequate in a number of practical microwave heating and drying applications where dielectric materials exhibit properties with significant frequency dependence. Recent improvement in TLM method, based on introducing Z-transforms [8], allows for accurate time-domain description of general frequency-dependent properties in isotropic, bi-isotropic, anisotropic and nonlinear materials.

In this paper, TLM method based on Z-transforms (TLM-Z) is applied for resonant frequency calculation of metallic cavity with rectangular cross-section loaded with either dielectric slab whose relative permittivity is complex and frequency independent or Debye water dielectric exhibiting frequency-dependent complex permittivity. Cavity dimensions are carefully chosen to illustrate advantages of TLM-Z method in comparison with TRM and conventional TLM approach in terms of inherent dielectric slab modeling especially at frequencies with significant changes of its electromagnetic properties.

II. Theoretical Analysis

Differential time-domain methods, such as FD-TD and TLM method, are now capabale for modelling of complex and non-uniform problems over a wide frequency range. TLM iteration procedure based on Z-transforms technique is first developed for 1-D, 2-D and 3-D modelling of electromagnetic wave propagation in constant parameter material. Then, it is extended to linear frequency-dependent materials [8].

A. 3-D TLM method based on Z-transforms

3-D TLM algorithm based on Z-transforms technique is developed from Maxwell's curl equations and the constitutive relations. Maxwell's curl equations in compact form are expressed by equation (1):

$$\begin{bmatrix} \nabla \times \underline{H} \\ \nabla \times \underline{E} \end{bmatrix} = \begin{bmatrix} \underline{J}_e \\ \underline{J}_m \end{bmatrix} + \frac{\partial}{\partial t} \begin{bmatrix} \underline{D} \\ \underline{B} \end{bmatrix} \tag{1}$$

Fig.1 shows 3-D TLM node. This node has 12 ports ($V_1,\ldots,V_{12}$) and 6 total field quantities (E_x, E_y, E_z, H_x, H_y, H_z) evaluated at the center of the cell. In TLM, the electric and magnetic fields and the current and voltage densities of Maxwell's equations are normalized so that the representations of these quantities have dimensions of volts. For simplicity, the space-steps are assumed to be regular, i.e. $\Delta x=\Delta y=\Delta z=\Delta l$, [4].

Authors are with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia, E-mail: [bata, doncov, jugoslav, tijana]@elfak.ni.ac.yu

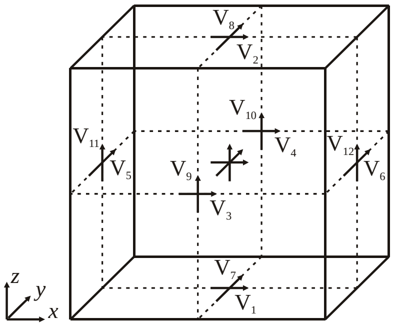

Fig.1. 3-D TLM node

The formulation of the 3-D TLM method based on Z-transform technique consists of three steps. First, the reflected fields are calculated from the incident voltages and free-sources:

$$2\begin{bmatrix} (V_1+V_2+V_3+V_4) \\ (V_5+V_6+V_7+V_8) \\ (V_9+V_{10}+V_{11}+V_{12}) \\ -(V_7-V_8-V_9+V_{10}) \\ -(V_{11}-V_{12}-V_1+V_2) \\ -(V_3-V_4-V_5+V_6) \end{bmatrix}^i - \begin{bmatrix} i_{fx} \\ i_{fy} \\ i_{fz} \\ V_{fx} \\ V_{fy} \\ V_{fz} \end{bmatrix} = 2\begin{bmatrix} V_x \\ V_y \\ V_z \\ -i_x \\ -i_y \\ -i_z \end{bmatrix}^r . \quad (2)$$

Assuming we have calculated the reflected fields, the total fields now need to be evaluated:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \\ i_x \\ i_y \\ i_z \end{bmatrix} = \begin{bmatrix} t_{ex} & & & & & \\ & t_{ey} & & & & \\ & & t_{ez} & & & \\ & & & t_{mx} & & \\ & & & & t_{my} & \\ & & & & & t_{mz} \end{bmatrix} \cdot \begin{bmatrix} V_x \\ V_y \\ V_z \\ -i_x \\ -i_y \\ -i_z \end{bmatrix}^r , \quad (3)$$

where $t_{ei} = 2/(4+g_e+\bar{s}2\chi_e)$ and $t_{mi} = 2/(4+r_m+\bar{s}\,2\chi_m)$, $i=\{x,y,z\}$ are transmission coefficients, g_e is normalized electric conductivity, r_m normalized magnetic resistivity, χ_e electric susceptibility and χ_m magnetic susceptibility.

Finally, the reflected transmission-line voltages are obtained using the following equation:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \end{bmatrix}^r = \begin{bmatrix} V_x - i_y - V_2^i \\ V_x + i_y - V_1^i \\ V_x + i_z - V_4^i \\ V_x - i_z - V_3^i \\ V_y - i_z - V_6^i \\ V_y + i_z - V_5^i \\ V_y + i_x - V_8^i \\ V_y - i_x - V_7^i \\ V_z - i_x - V_{10}^i \\ V_z + i_x - V_9^i \\ V_z + i_y - V_{12}^i \\ V_z - i_y - V_{11}^i \end{bmatrix} . \quad (4)$$

B. Linear isotropic material

Z-transform methods are used to incorporate frequency-dependent material properties into the TLM algorithm, leading to a formal technique applicable to all linear materials, including first-order dielectrics such as Debye material. 3-D TLM model for evaluating normalized EM field quantities in linear isotropic materials without magnetic effects is obtained using the bilinear transform $\bar{s} = 2(1-z^{-1})/(1+z^{-1})$.

Normalized electric field quantity is determined from the following equations:

$$V_i = T_e(2V_i^r + z^{-1}S_{ei}) \quad (5)$$

$$S_{ei} = V_i^r + k_e V_i - \bar{g}_e(z)V_i + 2\bar{\chi}_e(z)V_i , \quad (6)$$

for $i=\{x,y,z\}$, where S_{ei} is the main electric field accumulator , and coefficients T_e and k_e are calculated knowing electric properties of used material:

$$T_e = (4+g_{e0}+4\chi_{e0})^{-1} \quad (7)$$

$$k_e = -(4+g_{e1}-4\chi_{e1}) . \quad (8)$$

Similarly, normalized magnetic field quantity is obtained:

$$i_i = T_m(2i_i^r + z^{-1}S_{mi}) \quad (9)$$

$$S_{mi} = i_i^r + k_m i_i - \bar{r}_m(z)i_i + 2\bar{\chi}_m(z)i_i \quad (10)$$

for $i=\{x,y,z\}$, where S_{mi} is the main magnetic field accumulator, and coefficients T_m and k_m are calculated knowing magnetic properties of used material:

$$T_m = (4+r_{m0}+4\chi_{m0})^{-1} \quad (11)$$

$$k_m = -(4+r_{m1}-4\chi_{m1}) . \quad (12)$$

The frequency-domain electric susceptibility of a first-order (Debye) dielectric is:

$$\chi_e(s) = \chi_{e\infty} + \frac{\Delta\chi_e}{1+s\tau_e} , \quad (13)$$

where $\chi_{e\infty}$ is the optical susceptibility, $\Delta\chi_e$ is the susceptibility contrast of the dispersion, and τ_e is the dielectric relaxation time. Transforming to the Z domain using $\beta_e = e^{-\Delta t/\tau_e}$ yields:

$$\chi_e(z) = \chi_{e\infty} + \frac{\Delta\chi_e(1-\beta_e)}{1-z^{-1}\beta_e} . \quad (14)$$

Frequency dependence may be presented as the function of the field values from the previous time-step [4]:

$$(1-z^{-1})\chi_e(z) = \chi_{e0} - z^{-1}(\chi_{e1} + \bar{\chi}_e(z)) \quad (15)$$

$$\chi_{e0} = \chi_{e\infty} + \Delta\chi_e(1-\beta_e) , \; \chi_{e1} = \chi_{e\infty} \quad (16)$$

$$\bar{\chi}_e(z) = \frac{\alpha_e/2}{1-z^{-1}\beta_e} \quad (17)$$

where $\alpha_e/2 = \Delta\chi_e(1-\beta_e)^2$. Substitution of (17) in (6) leads to procedure for evaluating normalized electric field components in a Debye dielectric:

$$V_i = T_e(2V_i^r + z^{-1}S_{ei}) \quad (18)$$

$$S_{edi} = 2\alpha_e V_i + z^{-1}\beta_e S_{edi} \quad (19)$$

$$S_{ei} = 2V_i^r + k_e V_i + S_{edi} , \quad (20)$$

for $i=\{x,y,z\}$, where the coefficients T_e i k_e are calculated using (7) and (8) for frequency-independent normalized conductivity g_e:

$$T_e = (4+g_e+4\chi_{e0})^{-1} \text{ and } k_e = -(4+g_e-4\chi_{e1}) \quad (21)$$

Normalized magnetic field component in a Debye dielectric is evaluated using (9) and (10), where the coefficients T_m and k_m for Debye dielectric are $T_m = 1/4$ and $k_m = -4$.

III. Numerical Analysis

3-D TLM method based on Z-transform (3-D TLM-Z) was applied to the metalic rectangular cavity loaded with one dielectric slab. A water, representing a linear isotropic dielectric material with frequency-dependent relative premittivity, is often used as a load in the processes of dielectric material heating and drying.

Relative permittivity of water is evaluated using Debye's equation [4]:

$$\varepsilon_r(\omega) = \varepsilon'_r(\omega) - j\varepsilon''_r(\omega) \tag{22}$$

$$\varepsilon'_r(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2 \tau_e^2} \tag{23}$$

$$\varepsilon''_r(\omega) = \frac{(\varepsilon_s - \varepsilon_\infty)\omega\tau_e}{1 + \omega^2 \tau_e^2}, \tag{24}$$

where ε_s is the static relative premittivity, ε_∞ is the optic relative premittivity, τ_e is the electrical relaxation time, and ω radial frequency. Dependence of both real and imaginary part of the relative permittivity on frequency is shown in Fig. 2.

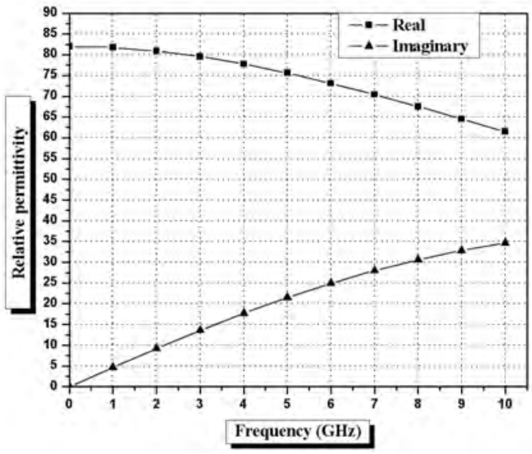

Fig.2. Real and imaginary part of relative permittivity versus frequency

First, metalic cavity with rectangular cross-section of dimensions 35×37 cm and height of 27cm, loaded with dielectric slab of thickness t=[0÷5]cm placed at the cavity bottom, was considered. It was assumed that relative permittivity of water is: 1) complex and constant with value ε_r = 81.725 - j4.66 calculated at f = 1 GHz, while using TLM conventional method and TRM and 2) complex and frequency dependent as described by Eqs.(22-24) while using TLM-Z method. Appropriate numerical results are presented in Fig.3.

From Fig.3. it is obvious that resonant frequencies of modes TM_{11p}, p={0,1,2,3,4}, obtained using TLM-Z method are in excellent agreement with the values evaluated by other two methods. In fact, sush results are expected since, when it comes to water, real and imaginary parts of relative permittivity vary with frequency inconsiderably in the analyzed frequency range up to 2 GHz. Because of that, when TLM-Z method is used, there is very small or none difference between results corresponding to the real case and results when losses and relative permittivity dependence on frequency are disregarded. Table I shows comparative results of resonant frequencies of the given cavity modelled by TLM-Z method for both constant ε_r and frequency-dependent complex ε_r.

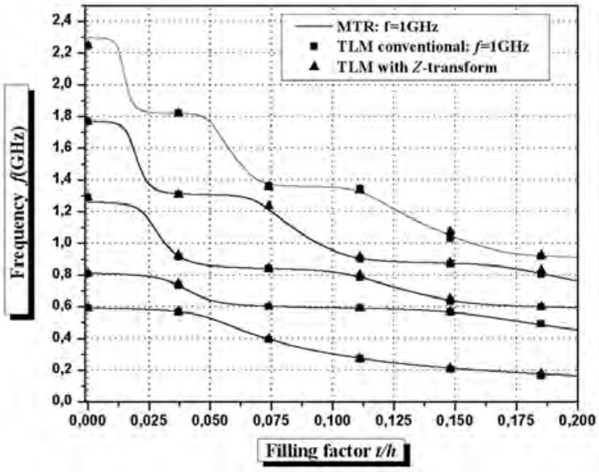

Fig.3. Resonant frequencies versus filling factor for the case of dielectric slab with compex relative permittivity

TABLE I
RESONANT FREQUENCIES OF THE CAVITY (35X37X27)CM FOR CONSTANT ε_r AND FREQUENCY-DEPENDENT ε_r

Filling factor	TM_{mnp}	Resonant frequency [GHz]		$\frac{\Delta f}{f}$ [%]
		TLM-Z: ε_r = 81.725	TLM-Z: $\varepsilon_r = \varepsilon_r(f)$	
t/h=0.037	TM_{110}	0.5789	0.5709	1.38
	TM_{111}	0.7465	0.7466	0.01
	TM_{112}	0.9295	0.9259	0.39
	TM_{113}	1.3028	1.3065	0.28
	TM_{114}	1.8151	1.8188	0.20
t/h=0.074	TM_{110}	0.3989	0.3989	0.00
	TM_{111}	0.6002	0.6002	0.00
	TM_{112}	0.8380	0.8417	0.44
	TM_{113}	1.2369	1.2369	0.00
	TM_{114}	1.3540	1.3577	0.27
t/h=0.111	TM_{110}	0.2745	0.2745	0.00
	TM_{111}	0.5892	0.5892	0.00
	TM_{112}	0.8014	0.8014	0.00
	TM_{113}	0.9149	0.9149	0.00
	TM_{114}	1.3357	1.3357	0.00
t/h=0.148	TM_{110}	0.2123	0.2123	0.00
	TM_{111}	0.5672	0.5672	0.00
	TM_{112}	0.6477	0.6477	0.00
	TM_{113}	0.8746	0.8783	0.42
	TM_{114}	1.0906	1.076	1.34
t/h=0.185	TM_{110}	0.1756	0.1757	0.06
	TM_{111}	0.4904	0.4904	0.00
	TM_{112}	0.6002	0.6002	0.00
	TM_{113}	0.8307	0.8307	0.00
	TM_{114}	0.9257	0.9259	0.02

Even the advantage of TLM-Z method, accounted for the possibility of frequency-dependent property modelling, is not evident here, there is another advantage of TLM–Z method represented by unnecessity to use finer mesh for dielectric modelling. For instance, in our example mesh of 35×37×27 nodes was used for complete modelled space when TLM-Z method was applied, where as TLM conventional method demanded finer mesh in dielectric, increasing the simulation run-time significantly. In addition, TRM would failed to produce resonant frequency results if losses in dielectric slab were sligthy higher.

In order to illustrate TLM-Z capability of frequency-dependent dielectric materials modelling, another metalic rectangular cavity of cross-section dimensions 63×63 mm and height 69 mm, loaded with water dielectric of thikness t=[0÷15] mm, is considered. Resonant frequencies of such cavity are in higher frequency range where dielectric properties change significantly with frequency (Fig.2). For modelling purpose the mesh of 21×21×23 nodes was used. As in the previous analysis, resonant frequencies were obtained for two cases, that is for constant relative permittivity (ε_r = 75.647, at f=5 GHz,) and for frequency-dependent relative permittivity and losses included which corresponds to the real case. Comparative results are shown in Table II.

TABLE II
RESONANT FREQUENCIES OF THE CAVITY (63X63X69)MM FOR CONSTANT ε_r AND FREQUENCY-DEPENDENT ε_r

Filling factor	TM_{mnp}	Resonant frequency [GHz]		$\frac{\Delta f}{f}$ [%]
		TLM-Z: ε_r = 75.647	TLM-Z: $\varepsilon_r = \varepsilon_r(f)$	
t/h=0.0435	TM_{110}	2.7325	2.6715	2.23
	TM_{111}	3.4400	3.4278	0.35
	TM_{112}	4.1231	4.0987	0.59
	TM_{113}	5.6236	5.6235	0.00
	TM_{114}	7.4655	7.4655	0.00
t/h=0.087	TM_{110}	1.4516	1.4028	3.36
	TM_{111}	3.3546	3.3424	0.36
	TM_{112}	4.0133	4.0255	0.30
	TM_{113}	4.5257	-	-
	TM_{114}	5.7821	5.7699	0.21
t/h=0.1304	TM_{110}	1.0125	0.9759	3.61
	TM_{111}	2.8911	2.8667	0.84
	TM_{112}	3.4034	3.3912	0.36
	TM_{113}	4.1719	4.1597	0.29
	TM_{114}	4.8428	-	-
t/h=0.1739	TM_{110}	0.8051	0.7685	4.55
	TM_{111}	2.1958	2.1104	3.89
	TM_{112}	3.3302	3.3302	0.00
	TM_{113}	3.7328	-	-
	TM_{114}	4.2939	4.2695	0.57
t/h=0.2174	TM_{110}	0.6831	0.6587	3.57
	TM_{111}	1.7688	1.6956	4.14
	TM_{112}	2.9277	2.8301	3.33
	TM_{113}	3.3912	3.3668	0.72
	TM_{114}	4.1109	-	-

Table II shows significant difference between results corresponding to two analyzed cases, as opposed to the results presented in Table I. This is in accordance with expectations. Hence, TLM-Z method should be used for modelling of the cavity of small dimensions that have resonant frequencies in higher frequency range, since it allows accurate modelling of frequency-dependent properties of dielectric slab in the whole frequency range of interest.

IV. CONCLUSION

Inherent time domain modelling of frequency-dependent dielectric materials exposed often to the processes of heating and drying in microwave applicators, is done in the paper, by using TLM method based on Z-transforms. In comparison with transverse resonance method and conventional TLM approach, TLM-Z method takes into account frequency dependent complex permittivity providing a better accuracy especially in the areas with significant changes of permittivity with frequency. In addition, limitation that TRM can be applied to the dielectric slab with small losses and requirement to use a conventional TLM mesh of higher resolution in dielectric part of the cavity, are not imposed to TLM-Z method. It is clear that presented 3-D TLM-Z approach can be successfully applied to modelling of other microwave structures whose electromagnetic parameters are significantly changed with frequency.

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