# Analysis of Smart Antennas with URA Based on Half -Wavelength Dipoles - Simple DOA and ABF Methods

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Abstract – In this paper, the principles of uniform rectangular array (URA) based on narrowband radio-frequency signals are introduced. An URA is composed of a number of uniformly distributed identical half-wavelength dipoles. Limited numerical examples and simulation results are presented to illustrate the direction of arrival (DOA) and adaptive beamforming (ABF) methods.

*Keywords* – smart antennas, uniform rectangular array, direction of arrival, adaptive beamforming.

### I. INTRODUCTION

Smart antennas have undergone enormous growth and become popular during the recent years. The central idea of smart antennas is spatial processing. Deployed at the base station of the existing infrastructure, adaptive arrays with an appropriate configuration can provide a substantial capacity improvement in the frequency-resource-limited radiocommunication system by an efficient frequency-reuse scheme.

The investigation of smart antennas suitable for wireless communication systems has involved primary uniform linear arrays (ULA) and uniform rectangular arrays (URA). ULA lack the ability to scan in 3-D space, and it is necessary for wireless devices to scan the main beam in any direction of elevation and azimuth, the URA is more attractive for mobile communications.

The DOA estimation involves a correlation analysis followed by signal/noise subspace formation and eigenstructure analysis. For the significant improvement in smart antenna resolution the 2-D unitary ESPRIT method is considered [1]. One of the most popular reference-based methods applicable to URA is the classical least mean squares (LMS) algorithm [2], [3].

# II. SMART ANTENNA WITH UNIFORM RECTANGULAR ARRAY STRUCTURE

The URA consisting N x M equally distributed identical half-wavelength dipoles (M, N – even), as illustrated in Fig. 1 is located symmetrical in x-y plane.

Let us assume that an incoming narrowband signal (plane wave with wavelength  $\lambda$ ) arrives at the array from elevation angle  $\theta$  and azimuth angle  $\phi$ . The origin of coordinate system is located at the center of the array.

As demonstrated in Fig. 1, the array factor (*AF*) of URA with its maximum along  $\theta_0$ ,  $\phi_0$  is given by [3]

$$\left[AF(\theta,\phi)\right]_{MxN} = 4\sum_{m=1}^{M/2} \sum_{n=1}^{N/2} A_{mn} \cos[(2m-1)u] \cos[(2n-1)v] \quad (1)$$

where

$$u = \frac{\pi d_x}{\lambda} \left( \sin \theta \cos \phi - \sin \theta_0 \cos \phi_0 \right)$$
(2)

$$\mathbf{v} = \frac{\pi \mathbf{d}_{y}}{\lambda} \left( \sin \theta \sin \phi - \sin \theta_{0} \sin \phi_{0} \right)$$
(3)

where  $A_{mn}$  is the amplitude excitation of the individual element, and  $d_x$ ,  $d_y$  are the interelement spacing along the x-axis and the y-axis, respectively.



Fig. 1. Geometry of (*N x M*) - element URA, along with an incoming plane wave.

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#### **III. DIRECTION OF ARRIVAL ESTIMATION**

After that the URA receives all incoming signals from directions of arrival, the DOA algorithm determines the directions of these signals based on the time delays. Let us assume that a narrowband plane wave impinges at an angle  $(\theta, \phi)$  on the URA. It produces time delays relative to the

other array elements. These time delays depend on array geometry, number of elements, and interelement spacing.

For the URA of Fig. 1, the time delay of the narrowband signal at the (m, n)th element with respect to the origin, is written as [4]

$$\tau_{mn} = \frac{md_x \sin\theta \cos\phi + nd_y \sin\theta \sin\phi}{c}$$
(4)

where *c* is speed of light in free space.

Two algorithms that fall into subspace-based method category for the azimuth and elevation estimation are MUSIC (Multiple Signal Classification) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique). In the paper the latter one is presented and used.

#### A. Classical ESPRIT – advantages in comparison with MUSIC

Classical ESPRIT is a robust method that exploits subarray structure for DOA estimation [3]. ESPRIT has become the method of choice because it has ability to offer a number of advantages over MUSIC, such as: a) does not require calibration of the antenna array; b) computationally less intensive and more efficient; c) does not involve search through all possible steering vectors to estimate the DOA.

#### B. 2-D Unitary ESPRIT algorithm for DOA estimation

The 2-D unitary ESPRIT algorithm is unique different from the classical ESPRIT, first of them provides closed-form automatically paired two dimensional estimation as long as the elevation and azimuth of each narrowband signal arrives at the URA [1]. This method provides closed form 2-D angle estimation in real time. This method gives several advantages in comparison with classical ESPRIT, such as: a) reduced computational complexity; b) lower SNR (signal-to-noise ratio) resolution thresholds; c) very accurate finds simultaneously both the elevation and azimuth angles of arrival for impinging signals at the antenna array.

#### **IV. ADAPTIVE BEAMFORMIG ESTIMATION**

Two classes of adaptive beamforming (ABF) algorithms are represented in literature : 1) DOA-based adaptive beamformig algorithms that utilizes information for angles of arrival of incoming signals to ideally steer the maximum of the antenna radiation pattern toward the desired signal and place nulls toward the unwanted signals or interferences; 2) referencebased ABF algorithms does not need DOA information but instead uses the reference signal to adjust weights of correlation array matrix to match the created time delays. In this section, we consider one of the most popular referencebased ABF algorithms - least mean squares (LMS) algorithm that uses previous samples when estimating the gradient at the nth iteration.

The LMS algorithm is applicable mainly when weights are updated utilizing reference signal. This algorithm uses an estimator of the gradient instead of the real value of the gradient because the real value estimation requires DOA information.

The expression of optimal weights for half-wavelength dipoles is given by [5], [6], [7]

$$w(n+1) = w(n) - \mu g(w(n))$$
(5)

where  $\mathbf{w}(n+1)$  denotes a new computed weights vector at the (n+1)th iteration,  $\mu$  is the gradient step size, and the array output is given by

$$y(\mathbf{w}(n)) = \mathbf{w}^{H}(n)\mathbf{x}(n+1)$$
(6)

where  $\mathbf{x}(n+1)$  is array signal vector computed at the (n+1)th iteration, and  $y(\mathbf{w}(n))$  is output signal.

In its standard form it uses an estimate of the gradient by replacing array correlation matrix **R** and correlation between array signals and reference signal **r** by their noisy estimates at the (n+1)th iteration [5]

$$\mathbf{g}(\mathbf{w}(n)) = 2\mathbf{x}(n+1)\mathbf{x}^{H}(n+1)\mathbf{w}(n) - 2\mathbf{x}(n+1)r^{*}(n+1)$$
(7)

where **g** is the gradient vector.

The error between array output and the reference signal is given by [5]

$$\varepsilon(\mathbf{w}(n)) = r(n+1) - \mathbf{w}^{H}(n)\mathbf{x}(n+1)$$
(8)

and

$$\mathbf{g}(\mathbf{w}(n)) = -2\mathbf{x}(n+1)\varepsilon^{*}(\mathbf{w}(n))$$
(9)

The estimated gradient is a product of the error between the reference signal and the output of the array and the signals after the nth iteration.

This algorithm provides several advantages: the gradient estimate is unbiased, and the low complexity.

## V. NUMERICAL EXAMPLES AND SIMULATION RESULTS

We investigate the DOA estimation under the conditions of a URA structure with half-wavelength dipoles. The 2-D unitary ESPRIT method is used to perform the estimation [1]. The signal of interest (SOI) impinges from  $(\theta = 50^{\circ}, \phi = 100^{\circ})$ , while the three signals not of interest (SNOI) are directed from  $(\theta = 55^{\circ}, \phi = 105^{\circ})$ ,  $(\theta = 45^{\circ}, \phi = 95^{\circ})$ , and  $(\theta = 55^{\circ}, \phi = 95^{\circ})$ . Simulations were conducted employing: a) a N=6, M=6 elements uniform rectangular array with  $d_x = d_y = 0.5\lambda$ ; b) a N=8, M=8elements uniform rectangular array with  $d_x = d_y = 0.5\lambda$ ; c) a N=8, M=6 elements uniform rectangular array with  $d_x = d_y = 0.5\lambda$ . The URA is examined in the presence of the Additive White Gaussian Noise (AWGN) with the zero mean, and variance 0.1. The results demonstrate its great performance, accurate estimation ability, and robustness.

 TABLE I

 THE DOA ESTIMATIONS OBTAINED UTILIZING 2-D UNITARY ESPRIT

	Case 1	Case 2	Case 3
Number of elements	M=6 ,N=6	M=8, N=8	M=6, N=8
Interelement spacing	0.5λ	0.5λ	0.5λ
Number of			
incoming	1	1	1
signals			
Number of			
data	2000	2000	2000
samples			
Actual			
SOI	$\theta_1 = 50^0$ ,	$\theta_1 = 50^0$ ,	$\theta_1 = 50^0$ ,
	$\phi_1 = 100^0$	$\phi_1 = 100^0$	$\phi_1 = 100^0$
SNOI 1	$\theta_2 = 55^0$ ,	$\theta_2 = 55^0$ ,	$\theta_2 = 55^0$ ,
	$\phi_2 = 105^{\circ}$	$\phi_2 = 105^{\circ}$	$\phi_2 = 105^{\circ}$
SNOI 2	$\theta_{3}=45^{0}$ ,	$\theta_{3}=45^{0}$ ,	$\theta_{3}=45^{\circ}$ ,
	$\phi_3 = 95^{\circ}$	$\phi_3 = 95^{\circ}$	φ <sub>3</sub> =95 <sup>0</sup>
SNOI 3	$\theta_4 = 55^{\circ}$ ,	$\theta_4 = 55^{\circ}$ ,	$\theta_4 = 55^{\circ}$ ,
	$\phi_4 = 95^{\circ}$	$\phi_4 = 95^{\circ}$	$\phi_4 = 95^{\circ}$
DOA Estimations			
SOI	$\theta_1 = 50.021^{\circ}$ ,	$\theta_1 = 49.992^{\circ}$ ,	$\theta_1 = 49.999^{\circ},$
	$\phi_1 = 100.053^{\circ}$	$\phi_1 = 99.994^{\circ}$	$\phi_1 = 100.030^{\circ}$
SNOI 1	$\theta_2 = 55.057^0$ ,	$\theta_2 = 54.994^0$ ,	$\theta_2 = 55.001^{\circ}$ ,
	$\varphi_2 = 105.047^{\circ}$	$\varphi_2 = 104.993^{\circ}$	$\varphi_2 = 104.967^{\circ}$
SNOI 2	$\theta_3 = 44.970^\circ$ ,	$\theta_3 = 44.981^{\circ}$ ,	$\theta_3 = 45.001^{\circ}$ ,
	$\phi_3 = 94.964^{\circ}$	$\varphi_3 = 94.996^{\circ}$	$\phi_3 = 94.999^{\circ}$
SNOI 3	$\theta_4 = 55.021^{\circ}$ ,	$\theta_4 = 54.990^{\circ}$ ,	$\theta_4 = 55.003^{\circ}$ ,
	$\phi_4 = 94.971^{\circ}$	$\phi_4 = 94.998^{\circ}$	$\phi_4 = 94.966^{\circ}$

Simulation results, utilizing the LMS algorithm gave precise results when adapt the beamforming pattern. To illustrate the ABF algorithm applicability for URA with halfwavelength dipoles, we considered the three cases where LMS algorithm is used: a) a N=6, M=6 elements uniform rectangular array and interelement spacing  $d_x = d_y = 0.5\lambda$ ; b) a N=8, M=8 elements uniform rectangular array and interelement spacing  $d_x = d_y = 0.5\lambda$ ; c) a N=8, M=6 elements uniform rectangular array and interelement spacing  $d_x = d_y = 0.5\lambda$ . The results from simulations are depicted in figures. The URA is examined about following scenario: the signal of interest (SOI) impinges from  $(\theta = 50^\circ, \phi = 100^\circ)$ in the presence of the signal not of interest (SNOI) from direction ( $\theta = 75^{\circ}, \phi = 135^{\circ}$ ), and Additive White Gaussian Noise (AWGN) with the zero mean, and variance 0.1. All simulation results are based on 100 times Monte Carlo simulations. A stepsize  $\mu = 0.001$  and a signal is with uncoded BPSK modulation are used in the numerical examples to simplify the simulations. Figures illustrate the resulting beamforming pattern with respect to  $\theta_0 = 90^\circ$ . The results demonstrate its great performance, and accurate estimation ability.



Fig. 2. The beamforming pattern of the URA with N=6 and M=6 elements.



Fig. 3. The beamforming pattern of the URA with N=8 and M=8 elements.



Fig. 4. The beamforming pattern of the URA with N=8 and M=6 elements.

#### VI. CONCLUSION

This paper investigated uniform rectangular smart antennas with half-wavelength dipoles. Two main issues: estimation of direction of arrival (DOA) and adaptive beamforming (ABF) were examined. The main approach to DOA here was the algorithm 2-D unitary ESPRIT. The technique for ABF used here was the LMS algorithm. The URA antennas were exploited in order to obtain more efficient method for a calculation of accurate eigenvalues. Matlab programs are used for simulations.

The 2-D unitary ESPRIT is a method that provides closedform automatically-paired source azimuth and elevation estimates. These results are proved to be accurate enough.

Concerning beamforming the URA has shown to be accurate and stable enough regarding both: desired signal (maximum) and interfering signals (deep nulls). The figures have shown that the adaptive array puts the maximum of the beamforming pattern to the SOI and at the same time – deep nulls towards the SNOIs.

Numerical examples and simulation results have illustrated that the optimal scenario for the antenna geometry is URA with M=N=6 elements, because the DOA and ABF estimations are proved to be accurate and stable enough, and the ability of the smart antenna to reject SNOIs is affected by the size and geometry of the antenna array. Using a larger URA may even make the smart antenna costly and impractical to realize. Consequently, it is observed that the designs of the smart antenna impact on the overall wireless communication network efficiency.

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