# Statistics of the SSC Output SIR in the Presence of Correlated Weibull Fading and Interference

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*Abstract* - **In wireless communication systems, effects of fading and cochannel interference can be diminished using diversity reception. In those environments, the level of the cochannel interference is sufficiently high as compared to the thermal noise so the effect of the thermal noise can be ignored. In this paper SSC (switch and stay combining) diversity system, based on signal-to-interference ratio (SIR) in the presence of correlated Weibull fading and interference, is analyzed. The probability density function (pdf), cumulative distribution function (cdf) and moments of the SSC output SIR are analytically defined. The influences of threshold, parameters of fading and correlation coefficient to the first order statistics are also considered. Numerical results, based on proposed analytical analysis are confirmed by Monte Carlo simulations.**

*Keywords* – **diversity receivers, moments, Monte Carlo simulation, Weibull fading.**

## I. INTRODUCTION

There is great need for alleviating bad influences of fading and co-channel interference in wireless communication systems. Diversity reception is one of commonly used technique to combat all those bad influences and to upraise system performance gain [1-2]. There are several combining methods, which division can be performed by their dependence on complexity of communication system and amount of channel state information available at the receiver. Equal-Gain Combining (EGC) and Maximal-Ratio Combining (MRC) require all or some of the amount of the channel state information of received signal. So, there is need of one receiver for every branch, which means higher complexity for practical realization. In opposition to these techniques (MRC and EGC), there is Selection Combining (SC) and Switch and Stay Combining (SSC), which process only one of the diversity branches. SC receiver processes all branches continuously and chooses the best one, while SSC receiver processes selected branch and proceed to the other one, when the previous becomes unacceptable [3]. When a SSC diversity combiner is subject to co-channel interference, the combining decision algorithm is not unique. There are three different algorithms of determining: based on total (desired plus interference) power; based on desired signal power and based on signal-to-interference ratio (SIR) [2], [4]. The algorithm of determining based on SIR provides the best performance gain of system with cochannel interference [4-5]. In interference-limited systems, the level of cochannel is sufficiently high as compared to thermal noise so the effects of thermal noise can be ignored.

In some papers [5-8], the outage probability over uncorrelated fading channels in the presence of interference has been analyticaly determinated. However, uncorrelated fading assumes antennas to be placed sufficiently apart, which is not always realized in practice due to insufficient antenna spacing when diversity is applied in small terminals [3]. The outage probability in dual SIR-based SC and SSC with correlated and uncorrelated branches over Rayleigh fading channels has been evaluated and published in [9]. Moreover, analytical studies investigating outage probability and other performance measures of SC with the algorithm of determining based on SIR in correlated Nakagami-m and Weibull fading channels are reported in literature [10] and [11], respectively.

In opposition to analysis presented in [11], in this paper dual SSC diversity system in the presence of interference is obtained, instead of SC which processes both branches. The statistics of system's output SIR of correlated Weibull desired signal and interference envelopes is presented. The new form of formulae for the probability density function (pdf), cumulative distribution function (cdf) and average output SIR of the SSC are analytically defined. Numerical results are based on proposed analytical analysis and confirmed by Monte Carlo simulations.

## II. CHANNEL MODEL

We model fading and interference by Weibull distribution which is flexibile and often used for describing urban environments in cases for which Rayleigh distribution is inadequate [12-13]. The desired signal as well as interfering one received by $i$th antenna in Weibull fading environment can be written as [5]

$$y(t) = x(t)\exp\big(j(2\pi f_c t + \Phi(t) + \phi(t))\big) \qquad (1)$$

with $f_c$ being the carrier fraquency, $\Phi(t)$ the desired information signal, $\phi(t)$ the random phase uniformly distributed in $[0, 2\pi)$ and $x(t)$ a Weibull distributed random amplitude process [14]

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$$p_x(x) = \frac{\beta}{\Omega} x^{\beta-1} \exp\left(-\frac{x^\beta}{\Omega}\right) \tag{2}$$

where $\Omega=E[x^\beta]$ (E[.]) denoting expectation, parameter $\beta$ being Weibull fading parameter ($\beta>0$). When parameter $\beta$ increases, fading severity decreases. By setting $\beta=1$ Weibull distribution becomes exponential and by setting $\beta=2$ Weibull becomes Rayleigh [14].

Let us consider the system where the fading amplitude of the desired $R(t)$ and interfering $r(t)$ signal have Weibull distribution. Then the PDF for instantaneous SIR, denoted by $z=R^2/r^2$, has the form

$$p_z(z) = \frac{\beta\Omega_d}{2\Omega_i} \frac{z^{\beta/2-1}}{\left(1+\frac{\Omega_d}{\Omega_i} z^{\beta/2}\right)^2} \tag{3}$$

where $\Omega_d=E[R^\beta]$, $\Omega_i=E[r^\beta]$. The cdf of instantaneous SIR is:

$$F_z(z) = \frac{z^{\beta/2}}{\frac{\Omega_d}{\Omega_i} + z^{\beta/2}} \tag{4}$$

Since $E[R^2]=\Omega_d^{2/\beta}\Gamma(1+2/\beta)$, $E[r^2]=\Omega_i^{2/\beta}\Gamma(1+2/\beta)$ [14], and $S=E[R^2]/E[r^2]=(\Omega_d/\Omega_i)^{2/\beta}$ ($\Omega_d/\Omega_i=S^{\beta/2}$) are the average SIRs equal at two input branches of the SSC, then the dual SIR-based SSC output pdf (3) and cdf (4) can be respectively obtained as:

$$p_z(z) = \frac{\beta}{2S^{\beta/2}} \frac{z^{\beta/2-1}}{\left(1+\left(\frac{z}{S}\right)^{\beta/2}\right)^2} \tag{5}$$

and

$$F_z(z) = \frac{1}{1+\left(\frac{S}{z}\right)^{\beta/2}} \tag{6}$$

In diversity system where the anntennas are unsufficiently appart, the envelopes of desired signals and interference are correlated. In this paper dual diversity system, commonly used in practise, is considered. Also, similarly to some already published papers [3], [9], the balanced SIRs at two input branches are assumed.

The joint probability density function of instantaneous input SIRs $z_1 = R_1^2/r_1^2$ and $z_2 = R_2^2/r_2^2$, for unbalanced case, $\Omega_{d_1}/\Omega_{i_1} = \Omega_{d_2}/\Omega_{i_2} = S^{\beta/2}$ with $\Omega_{d_1} = E\left[R_1^\beta\right]$, $\Omega_{i_1} = E\left[r_1^\beta\right]$, $\Omega_{d_2} = E\left[R_2^\beta\right]$, $\Omega_{i_2} = E\left[r_2^\beta\right]$ can be finally expressed as [11]

$$p_{z_1z_2}(z_1,z_2) = \sum_{i_1,i_2=0}^{\infty} \frac{\rho^{i_1+i_2}(1-\rho)^2\beta^2}{4(i_1!i_2!)^2 S^{\beta(1+i_1)}} \Gamma^2(2+i_1+i_2) z_1^{\beta(1+i_1)/2-1}$$
$$z_2^{\beta(1+i_1)/2-1}\left(\left(1+\left(\frac{z_1}{S}\right)^{\beta/2}\right)\left(1+\left(\frac{z_2}{S}\right)^{\beta/2}\right)\right)^{-(2+i_1+i_2)}, \tag{7}$$

where $\Gamma(.)$ is the Gamma function [15, eq. (8.310/1)] and $\rho$ correlation coefficient.

# III. SWITCH AND STAY COMBINING

In this paper dual SSC diversity system with correlated envelopes of desired and interfering signals are observed with the algorithm of determining based on SIR. We assume that the first branch is selected. If the instantaneous SIR falls below a given threshold, the switch selects the second branch. So, the system processes the selected branch until the instantaneous SIR falls below a given threshold when switch selects the second one. Cumulative distribution function (cdf) of output instantaneous SIR can be obtained as:

$$F_{ssc}(z) = \Pr(z_T \le z_1 \le z) + \Pr(z_2 < z_T \wedge z_1 \le z) \tag{8}$$

Differentiating previous equation, the output SIR's probability density function (pdf) is expressed as [16]:

$$p_{ssc}(z) = \begin{cases} r_{ssc}(z), & z \le z_T \\ r_{ssc}(z) + p_z(z), & z > z_T \end{cases} \tag{9}$$

where $z_T$ is previously determinated threshold, $p_z(z)$ defined by equation (5) and $r_{ssc}(z)$

$$r_{ssc}(z) = \int_0^{t_T} p_{z_1z_2}(z,z_2)\mathrm{d}z_2 =$$
$$\sum_{i_1,i_2=0}^{\infty} \frac{\rho^{i_1+i_2}(1-\rho)^2\beta}{2(i_1!i_2!)^2 S^{\beta(1+i_1)}} \Gamma^2(2+i_1+i_2) z^{\beta(1+i_1)/2-1}$$
$$\times z_T^{\beta(1+i_1)/2}\left(1+\left(\frac{z}{S}\right)^{\beta/2}\right)^{-(2+i_1+i_2)} \tag{10}$$
$$\times {}_2F_1\left(1+i_1, 2+i_1+i_2, 2+i_1, -\left(\frac{z_T}{S}\right)^{\beta/2}\right),$$

with ${}_2F_1(a, b; c; x)$ being the Gaussian hypergeometric function [15, eq. (9.100)].

The cumulative distribution function, after some manipulations of equation (8) can be written as [3]:

$$F_{ssc}(z) = \begin{cases} F_{z_1z_2}(z,z_T), & z \le z_T \\ F_z(z) - F_z(z_T) + F_{z_1z_2}(z,z_T), & z > z_T. \end{cases} \tag{11}$$

with $F_z(z)$ defined by equation (6) while cumulative distribution function is $F_{z_1z_2}(z,z_T)$

$$F_{z_1z_2}(z,z_T) = \sum_{i_1,i_2=0}^{\infty} \frac{\rho^{i_1+i_2}(1-\rho)^2\Gamma^2(2+i_1+i_2)}{(i_1!i_2!)^2 S^{\beta(1+i_1)}(1+i_1)^2} z^{\beta(1+i_1)/2} z_T^{\beta(1+i_1)/2}$$
$$\times {}_2F_1\left(1+i_1, 2+i_1+i_2, 2+i_1, -\left(\frac{z}{S}\right)^{\beta/2}\right) \tag{12}$$
$$\times {}_2F_1\left(1+i_1, 2+i_1+i_2, 2+i_1, -\left(\frac{z_T}{S}\right)^{\beta/2}\right).$$

# IV. AVERAGE OUTPUT SIR

Average output SIR is useful parameter for system performances evaluation in enviromentes of wireless communication in the presence of cochannel interference. Average output SIR, $\overline{S}_{out}$, at the SC output may be evaluated as [10], [3]

$$\overline{S}_{out} = \int_0^\infty z p_{ssc}(z)\mathrm{d}z = \int_0^\infty z r_{ssc}(z)\mathrm{d}z + \int_{z_T}^\infty z p_z(z)\mathrm{d}z = I_1 + I_2 \,. \quad (13)$$

Substituting (5) and (10) in (13), the infinite-range integrals can be obtained in the form of infinite series:

$$I_1 = \sum_{i_1,i_2=0}^{\infty} \frac{\rho^{i_1+i_2}(1-\rho)^2\,\Gamma(2+i_1+i_2)\Gamma(1+i_1+2/\beta)\Gamma(1+i_2-2/\beta)}{(i_1!i_2!)^2\,S^{\beta(1+i_1)/2-1}(1+i_1)}$$

$$\times z_T^{\beta(1+i_1)/2}\,{}_2F_1\left(1+i_1, 2+i_1+i_2, 2+i_1, -\left(\frac{z_T}{S}\right)^{\beta/2}\right), \quad (14)$$

$$I_2 = \frac{\beta}{\beta-2} S^{\beta/2} z_T^{-\beta/2+1}\,{}_2F_1\left(2, 1-\frac{2}{\beta}, 2-\frac{2}{\beta}, -\left(\frac{S}{z_T}\right)^{\beta/2}\right). \quad (15)$$

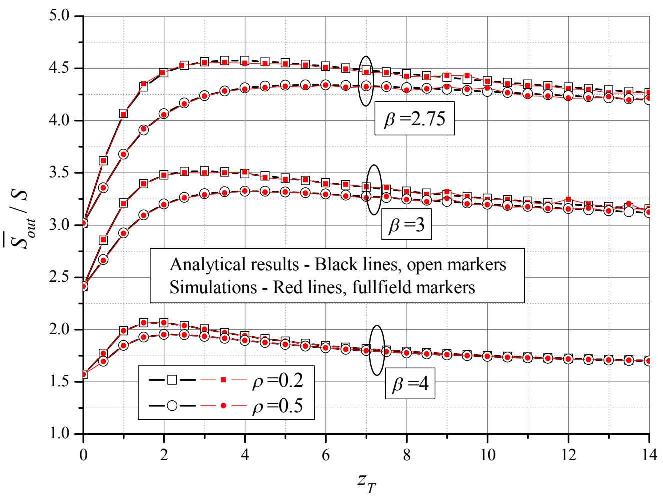

Fig. 2. Normalized output SIR versus threshold $z_T$

Fig.1 shows the first branch normalized output SIR $\overline{S}_{out}/S$, versus threshold for several values of Weibull paremeter ($\beta$) and correlation coefficient ($\rho$). It is obvious here that diversity gain decreases with an increase of the correlation coefficient. Also, when fading parameter $\beta$ increases (fading severity decreases), small increase or decrease in correlation coefficient does not have significant effect on $\overline{S}_{out}/S$. Simulations are performed using the C$^{++}$ programming language. The values of $\overline{S}_{out}/S$ are calculated based on over $10^8$ correlated Weibull fading samples. From Fig. 1 is evident good match between the theory and simulations.

Differentiating equation (13), the optimal value for threshold when output SIR has maximum, can be obtained. In this case, the optimal threshold can be evaluated numerically using *root-finding* method which is allowed in many software packages.

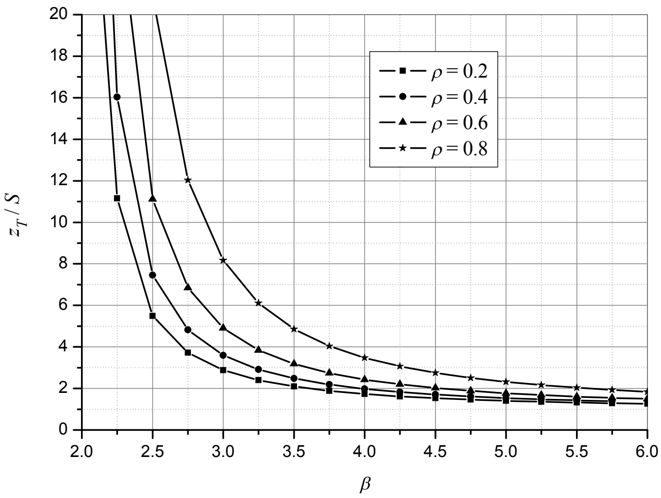

Fig. 2. Normalized optimal threshold versus Weibull feding parameter $\beta$

The effects of Weibull fading parameter and correlation coefficient on optimal evaluated threshold are described in Fig. 2. Actually, in Fig. 2 the normalized optimal threshold is plotted versus fading parameter $\beta$ for several values of correlation coefficient $\rho$. The optimal threshold decreases when fading parameter increases, as it is expected. At the other side, when correlation coefficient increases, optimal threshold also increases.

## V. OUTAGE PROBABILITY

Outage probability is a measure of quality in transmission over fading channels in wireless communication systems. This performance measure is commonly used in wireless communication systems especially in environments where co-channel interference is present.

Outage probability is defined as the probability which the instantaneous SIR at the output of SSC combiner falls below a certain given threshold $\lambda$ (protection ratio) and using equation (11) can be expressed as:

$$P_{out} = \int_0^\lambda p_{ssc}(z)\mathrm{d}z = F_{ssc}(\lambda)\,. \quad (16)$$

In Fig. 3, the outage probability is plotted versus normalized threshold $z_T/S$ for several values of fading parameter $\beta$. The normalized optimal threshold ($z_T$=$\lambda$), when outage probability has minimum, can be obtained from Fig. 3. In this case, for $z_T$=$\lambda$, SSC combining method becomes SC combining method. A very good match between analytical results and simulations is evident.

The outage probability versus normalized parameter $S/\lambda$ for different Weibull fading parameters $\beta$ and for optimal switching threshold value minimizing the outage probability is shown in Figure 4. When Weibull fading parameter increases (fading severity decreases), the outage probability also decreases. As it is expected, an increase in correlation coefficient does have effect on $P_{\mathrm{out}}$ (outage probability also increases).

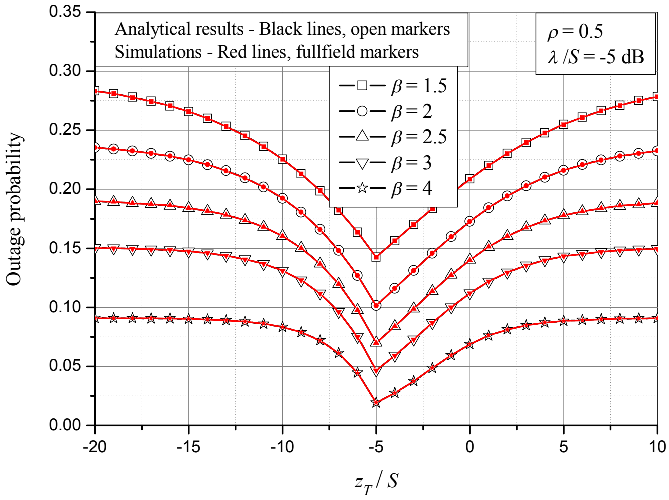


Fig. 3. Outage probability versus normalized threshold $z_T/S$

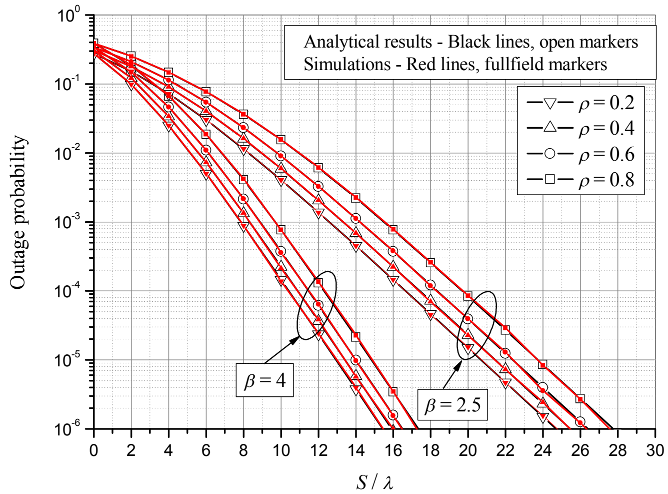


Fig. 4 Outage probability versus normalized input SIR

## VI. Conclusion

In this paper analytical results and required simulations of output SIR statistics of SSC diversity system in the presence of correlated Weibull fading were presented. Useful formulae for the probability density function (pdf), cumulative distribution function (cdf) of SSC output SIR and average output SIR in the form of infinite series were derived. Using these new formulae and by observing dual SSC diversity system, the outage probability and other performance measures were graphically presented. The effects of the fading severity, optimal threshold and level of correlation to the performance measures were observed. Also, a very good match between analytical results and simulations in this paper was confirmed.

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