An Analysis of the Possibilities for Using Lattices for the Interpolation of the Exact Signal Vector Position of QAM Constellations

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Abstract – **In this paper it is made an analysis of using rectangular and hexagonal lattices to define the exact vector position of height order QAM signal super constellation, passed trough to the cable communication channel with nonlinearity.**

Keywords – **Signal vector position estimation, Height order QAM modulation, and Super constellation deformation.**

I. INTRODUCTION

In wide band multi channel hybrid optical and coaxial cable communication systems there are some modules which causes nonlinearity [1], [2]. In the optical part the most significant is external MZ modulator [3], [4]. In the coaxial one it is non the ideal transfer function of the cable amplifiers [5]. Both of them are the reason for nonlinear distortions – generation intermodulation contributions, which are fallow in the spectrum of transmitted signal. In another way, the nonlinearity causes deformation of QAM super constellation. In this paper we will consider the static effect of transfer nonlinearity. A relatively small deformation of a high order QAM constellation can reduce the Euclidian, noise protection distance between some signal vector position and this is the reason for degradation of the symbol error ratio (SER), for the corresponding signal points – fig.1. Bearing in mind the above the signal becomes more vulnerable to the channel noise process, which can be modelate by normal distribution [3], [4], [5], [6]. For this reason estimating exact error vector (EV) magnitude, e.g. its modulation components must be made by a representative ensemble of realizations of the corresponding vector position to eliminate noise influence. Literal estimation EV for each symbol will be taking relatively a lot of time and system resource.

A special interest is to obtain some of the exact vector positions by the interpolation of another statistical estimated. This will be reducing calculation time and system resource.

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II. INTERPOLATION BY USING RECTANGULAR LATTICE

The simplest case is two dimensional rectangular lattice. The

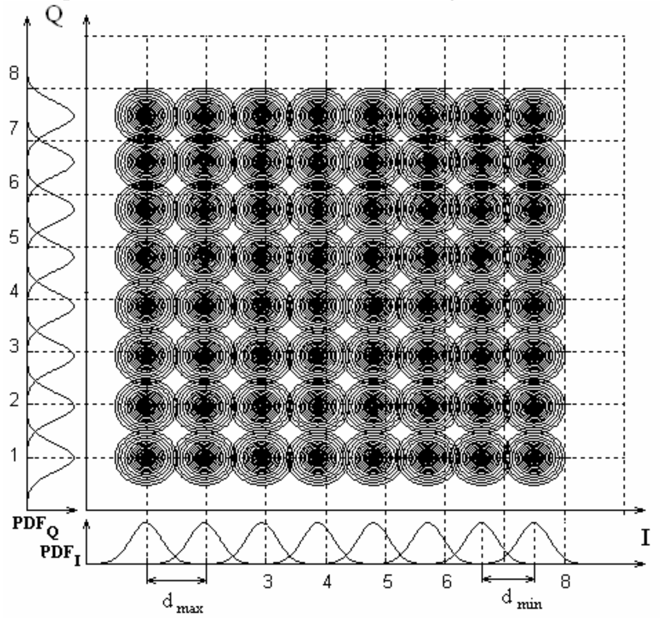

Fig.1. 256 QAM super constellation passed through to the Gaussian channel with nonlinearity – first quadrant

position of the interpolated point is practically is the geometric middle between neighbor points, for a rectangular regular lattece. The polar coordinates can be calculated by:

$$\varphi_k = arctg\frac{I_{k-1}+I_{k+1}}{Q_{k-1}+Q_{k+1}} \quad (1)$$

$$M_k = \sqrt{\frac{(I_{k-1}+I_{k+1})^2}{4}+\frac{(Q_{k-1}+Q_{k+1})^2}{4}} \quad (2)$$

Relatively to the origin it can be written:

$$\overrightarrow{V_X} = \frac{1}{4}\sum_{k=1}^{4} w_k.(I_k\ Q_k) \quad (3)$$

Where w_k is the weigh of corresponding vector I_k and Q_k are the vector inphase and the quadrature modulation components.

In the general case the value of all weighs is one. Without nonlinearity this lattice gives exact results. In the practical case there are two errors. First is the statistic estimation error. It does not depend of the vector position or of nonlinearity. Second is the error of interpolation. It depends not only of both of them, but also of estimation error of primary neighbor points – (4).

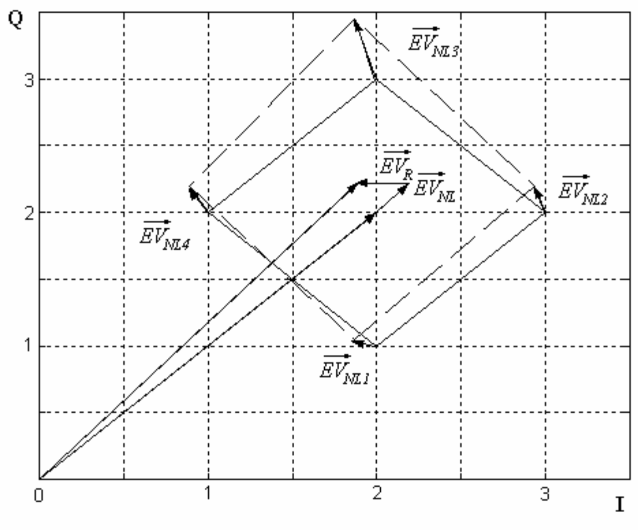

Fig. 2 The Error Vector of interpolation of one signal vector position in nonlinear sheet

$$\Delta(i, j) = \Delta_L(i, j, \Delta_S) + \Delta_S \tag{4}$$

For this reason interpolation error computing is a very important problem. In fig. 2 is shown the interpolation of one signal vector point.

To analyze the interpolation error first we will assume that the statistical error is zero, e.j. the primary signal points are exact. Second, the analisys will be made relatively to the modulation components. Thus the absolute interpolation error of each modulation component will depends only of nonlinearity and vector position.

$$\Delta_I = I(i) - \frac{I(i-1) + I(i+1)}{2} \tag{5}$$

But if the nonlinearity is represented by third order power series expansion [7] and $I=i$ it can be written as follows:

$$\Delta_I = i + a_2.i^2 + a_3.i^3 - \frac{i-1+a_2(i-1)^2 + a_3(i-1)^3}{2} + \frac{i+1+a_2(i+1)^2 + a_3(i+1)^3}{2} \tag{6}$$

After the required mathematical operations, the expression (6) becomes:

$$\Delta_I = -a_2 - 3a_3.i \tag{7}$$

As seen in equation (7) the dependence is linear. It is more convenient to accept the module of (7). The relatively error can be calculated:

$$\delta_I = \frac{a_2 + 3a_3.i}{i + a_2.i^2 + a_3.i^3}.100[\%] \tag{8}$$

In the worst practical case the numerical values of polynomial coefficients, for example can be: $a_2 = -4{,}472.10^{-3}; a_3 = 1{,}186.10^{-6}$. It is shown in fig.1. In this case the maximal relatively error is δ_I =0,44%. The absolute module error vector can be calculated:

$$\Delta_{M_{i,q}} = \sqrt{(a_2 + 3a_3.i)^2 + (a_2 + 3a_3.q)^2} \tag{9}$$

The relatively error will be:

$$\delta_{M_{i,q}} = \sqrt{\frac{(a_2 + 3a_3.i)^2 + (a_2 + 3a_3.q)^2}{(i + a_2.i^2 + a_3.i^3)^2 + (q + a_2.q^2 + a_3.q^3)^2}} \tag{10}$$

For the considered case δ_M =0,32%.

In fact δ_M is the value of interpolation error vector (IEV). In dB it can be expressed:

$$\overrightarrow{IEV} = 10.\lg\left[\frac{(i + a_2.i^2 + a_3.i^3)^2 + (q + a_2.q^2 + a_3.q^3)^2}{(a_2 + 3a_3.i)^2 + (a_2 + 3a_3.q)^2}\right] \tag{11}$$

For the current example IEV=49,9dB. But this is not a real value of error. To make this analysis we first neglect the error of statistical processing. Thus we assumed that the primary vector positions are exact, but it is not precisely. In this way the real error has to be computed. It can be done by considering the case which primary points that the definition is with maximal error, e.j. it must to be considered the worst case.

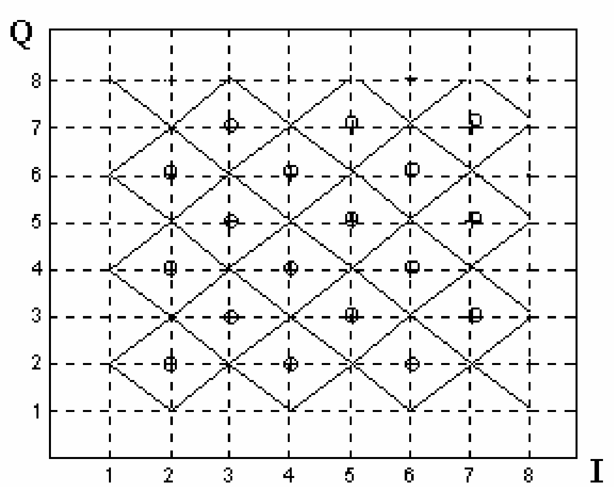

Fig.3 Signal point interpolation with rectangular lattice – first quadrant of 256 QAM super constelation

Using formula (6) it can be expressed the absolute inphase (quadrature) component error, given of non-literal estimation of primary points:

$$\Delta_{Is} = \frac{1}{4}\sum_{k=1}^{4}\Delta_{Ik} \qquad (12)$$

For a current analysis we will use the regular rectangular lattice and the statistical estimated error will be simulated by nonlinear processing over the primary signal points. Supposing that three of the points are exact and one is determined with error – fig.4. Thus the relatively module error

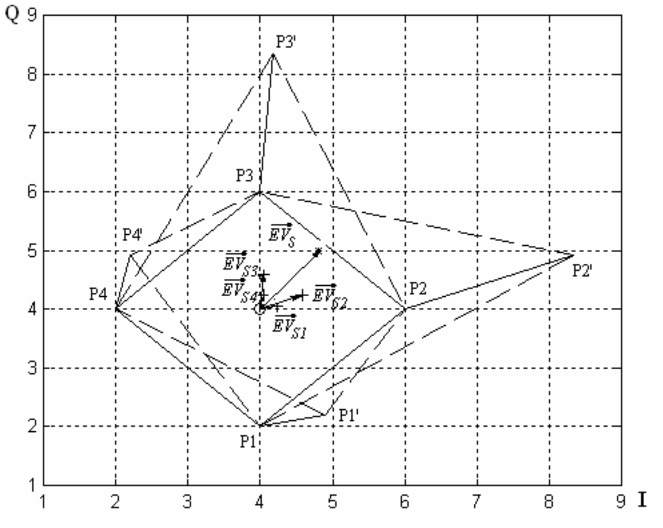

Fig. 4. Reflection an error estimation of primary into interpolated point

will be:

$$\delta_{Ms_{i,q}} = \frac{1}{2}.\sqrt{\frac{\Delta_I^2 + \Delta_Q^2}{\left(i + a_2.i^2 + a_3.i^3\right)^2 + \left(q + a_2.q^2 + a_3.q^3\right)^2}} \qquad (13)$$

Practically with formula (13) it can be calculated the module of relatively error caused by absolute error estimation of one signal point. It can be seen that the lattice decreases the statistical estimation error, reflecting in to the interpolated point. The module of full statistical error vector is a sum of four vector errors. Thus the formula (13) becomes:

$$\delta_{Ms_{i,q}} = \frac{1}{2}.\sqrt{\frac{\left(\sum_{k=1}^{4}\Delta_{I_k}\right)^2 + \left(\sum_{k=1}^{4}\Delta_{Q_k}\right)^2}{\left(i + a_2.i^2 + a_3.i^3\right)^2 + \left(q + a_2.q^2 + a_3.q^3\right)^2}} \qquad (14)$$

Finally the module of full relatively error can be obtained by the sum between (14) and (10):

$$\delta_{M_{i,q}} = \sqrt{\frac{\left(a_2 + 3.a_3.i\right)^2 + \left(a_2 + 3.a_3.q\right)^2 + 0,25.\left[\left(\sum_{k=1}^{4}\Delta_{I_k}\right)^2 + \left(\sum_{k=1}^{4}\Delta_{Q_k}\right)^2\right]}{\left(i + a_2.i^2 + a_3.i^3\right)^2 + \left(q + a_2.q^2 + a_3.q^3\right)^2}} \qquad (15)$$

III. Interpolation by using hexagonal lattece s

To apply hexagonal lattice for rectangular QAM, first we must make a shifting of odd or even vector positions of I or Q modulation component of QAM super constellation in half of Euclidian distance (noise protection distance). It must to be noted that it are necessary six neighbor points to calculate interpolated position – fig.5.

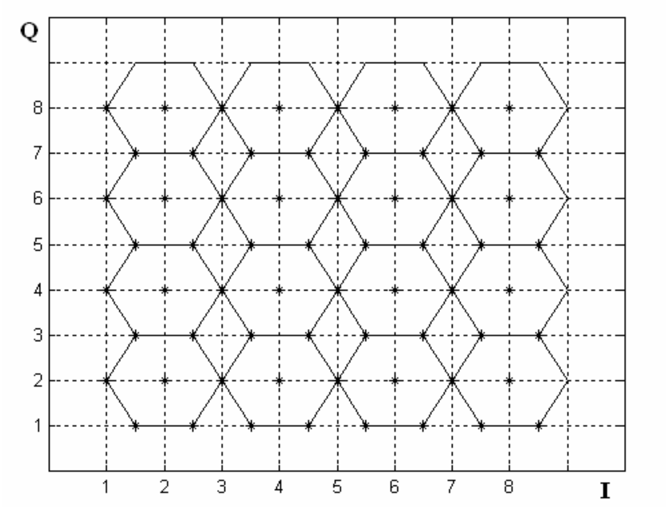

Fig.5. Interpolation signal vector position, using hexagonal lattice.

Using expression (3), for hexagonal lattice can be written:

$$\overrightarrow{V_X} = \frac{1}{6}\sum_{k=1}^{6} w_k.\left(I_k\, Q_k\right) \qquad (16)$$

Without nonlinearity, this lattice is also exact but in nonlinearity space the interpolation error is smallest in compare with rectangular lattice, because interpolated point depends of six neighbor positions instead of four.

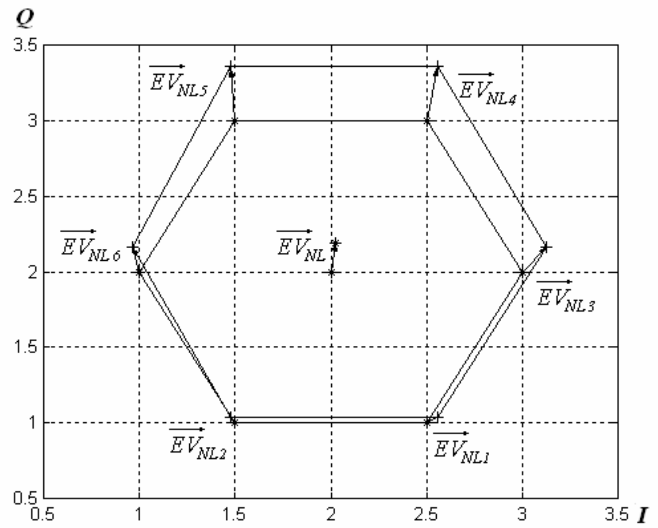

Fig.5. Interpolation signal vector position, using hexagonal lattice in nonlinearity space

For this reason in current analysis it can be neglect. Using formula (14) can be written:

$$\delta_{Ms_{i,q}} = \sqrt{\frac{0{,}17.\left[\left(\sum_{k=1}^{4}\Delta_{I_k}\right)^2 + \left(\sum_{k=1}^{4}\Delta_{Q_k}\right)^2\right]}{\left(i + a_2.i^2 + a_3.i^3\right)^2 + \left(q + a_2.q^2 + a_3.q^3\right)^2}} \quad (17)$$

It can be seen that the hexagonal lattice give more exact result, but it is necessaries six neighbor points. Also it mast be done primary shifting of some of vector positions in a half of Euclidian noise protection distance, which is additional processing. For this reasons the hexagonal lattice is not efficient for considering goal. Another important problem is interpolation a points, located in the end of constellation. The required primary points can be recovered not exactly, e.g. it will be done with significant error. The problem is very complicate and has to be discussed.

IV. MULTI LEVEL INTERPOLATION

Of Fig. 2 and Fig. 5 can be seen that the resultant error vector *EV* of interpolated point is smallest than maximal same of primary points. This fact can be used for two purposes. Firs it can be done a reducing the statistical error, e.g. decreasing. Second, by a lot of iterations the space can be made relatively linear, because the lattice seeks to equalize the point distance. Both of these purposes can be achieved by multi level interpolation. An example is depicted in Fig.6.

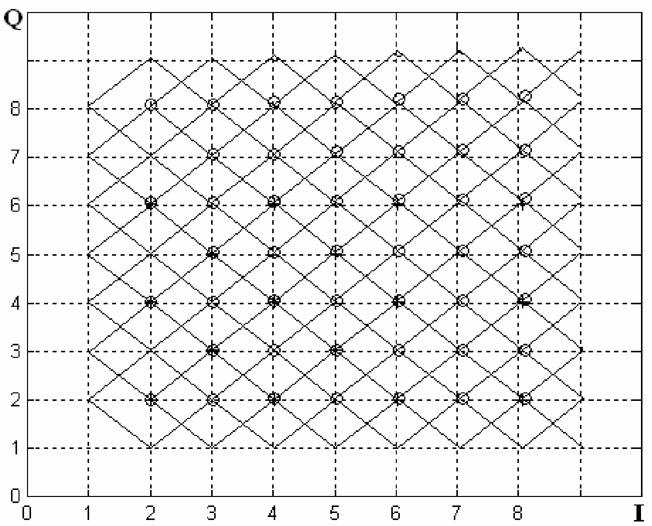

Fig.6. An example of two level interpolation, using rectangular lattice in first and second level

In first level it must be selected a primary and then will be obtained interpolated positions. In second level the interpolated points becomes primary. In this way it can be calculated another positions, which were a primary in first iteration, so called second interpolation level. Thus they can be obtained more exact in comparison with statistical estimation only, because the interpolated vector position depends of four neighbor points. Also it can be use a combination between rectangular and hexagonal lattice. For example first interpolation level can be done with a rectangular lattice. After shifting of any determinate vector positions how it was considered earlier, the second interpolation level can be done by using a hexagonal lattice. The effect of this processing will depends of each concrete case, of iterations number and the choice of the lattices kind.

V. CONCLUSION

In this paper an analysis has been done of the possibilities for using rectangular and hexagonal lattices for the interpolation of the vector position of square QAM super constellation after the transition of a signal trough to the Gaussian channel with nonlinearity. We have worked out relations for estimating the full error after interpolation for rectangular (15) and hexagonal lattice (17). A practical case has been considered where the nonlinearity is represented by a third order power series expansion. It has also been shown that the value of the error depends not only on nonlinearity, but also on the current signal vector position. Finally has given an idea for decreasing a statistical estimation error by using multi level interpolation.

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