The second-order statistical measures of SC Macrodiversity System over independent Weibull Fading Channels

Dragana S. Krstic[1], Stefan R. Panic [2], Jelena A. Anastasov [3], Goran Lj. Stamenovic [4] and Dusan M. Stefanovic [5]

Abstract – **The average level crossing rate and average fade duration of SC (Selection Combining) macrodiversity system's output signal in the presence of independent Weibull fading are analyzed. The SC macrodiversity system consists of two microdiversity systems and selection is based on their output signal's average power values. Both microdiversity systems are composed of SC diversity systems with uncorrelated branches in the presence of Weibull fading. Average power of the output signal is modeled by Gamma distribution. Analytical results for LCR (Level crossing rate) and AFD (Average Fading Duration) of the output signal envelope are graphically presented.**

Keywords – **Average level crossing rate, Average fade duration, Selection Combining, Weibull fading.**

I. Introduction

The effect of fading on the performance of wireless communications systems has received a great deal of research interest due to the possible use of space diversity technique [1], [2]. The rapid and random fluctuations of the signal envelope and phase in a radio channel are caused with two propagation phenomena: multipath scattering and shadowing. The multipath components include a LOS component and many scattered components, while the effects of shadowing can be modeled as LOS shadow fading or multiplicative shadow fading. The amplitude of the LOS is often assumed to be a lognormal random variable or a gamma random variable. On the other hand, there are many researches based on analyzing fast fading. Fast fading can be modeled by several distributions including Weibull distribution [2], [3].

[1]Dragana S. Krstic is with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, e-mail: dragana.krstic@elfak.ni.ac.rs

[2]Stefan R. Panic is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia, e-mail: stefanpnc@yahoo.com

[3]Jelena A. Anastasov is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia, e-mail: anastasovjelena@gmail.com

[4]Goran Lj. Stamenovic is with Tigar, Pirot, 18300 Pirot, Serbia, e-mail: goran.stamenovic@tigar.com

[5]Dusan M. Stefanovic is with the Faculty of Electronic Engineering, A. Medvedeva 14, 18000 Nis, Serbia, e-mail: dusan.stefanovic@itcentar.co.yu

For all diversity techniques: equal gain combining (EGC), maximal ratio combining (MRC), selection combining (SC), and generalized selection combining (GSC), the receiver processes the obtained diversity signals to maximize the system's power efficiency. Among these diversity techniques, SC is the least complicated [4].

All of mentioned techniques can be applied in the presence of fast fading. When there is shadowing (slow fading), the use of macrodiversity system is suggested. Macrodiversity system simultaneously combines signals from the outputs of microdiversity systems and thus reduces the effect of shadowing.

Other than the outage probability which is essential to determine several systems' performance measures, there are some the second-order statistical measures also used in describing system's performance. The average level crossing rate (LCR) and the average fade duration (AFD) are the system second-order statistical measures, which reflect the correlation properties of its input/output signals. The average LCR can be used to estimate the rate of occurrence at which the envelope of the received signal crosses a specified level.

The level crossing rate and average fading duration are used in modeling and designing wireless communication systems. Actually, these second-order statistical measures are related to criterion used to assess error probability of packets of distinct lenght and to determinate parameters of equivalent channel, modeled by a Markov chain with defined number of states [5].

In this paper the second-order statistical measures (AFD and LCR) at the output of macrodivesity system over the independent Weibull fading channels are presented. Two microdiversity systems with SC receivers are obtained. The selection is based on picking maximal signal's average power value at the output of each SC receiver. Average powers of the output signals are modeled by Gamma distribution. Some numerical results of the average level crossing rate and the average fade duration are presented.

I. Model of the System

Let r be the received sampled envelope and $\dot{r}$ its derivative with respect to time, with joined probability density function (pdf) $p_{\dot{r},r}(\dot{r},r)$. The average LCR of the envelope ratio at threshold R represents the average number of times, per time unit, the stationary fading process $R(t)$ crosses threshold R in the positive direction, and is mathematically defined by formula

$$N_r(r)=\int_0^\infty \dot{r}\,p_{\dot{r},r}(\dot{r},r)\,d\dot{r} \tag{1}$$

The AFD is defined as the average time that the envelope ratio remains below the threshold R, and is determinated as ratio of output signal's envelope cumulative distribution function (cdf) and the average LCR:

$$T_r(r)=\frac{F_r(r)}{N_r(r)} \tag{2}$$

Weibull fading channel model has been often used to analyze performance in mobile communication environment. Weibull distribution is an adequate distribution in describing multipath waves propagating in the nonhomogenous environments in which Rayleigh distribution is not an adequate choice.

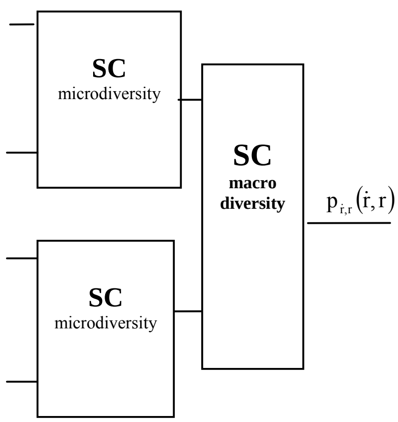

Fig. 1. Model of system

In Fig. 1 macrodiversity system contains of two microdiversity systems is presented. The independent Weibull fading at the input of both microdiversity systems is appeared. SC receivers with two input branches are obtained. The probability density function and cumulative distribution function of input signals' envelope [3]:

$$p_{r_{i1}}(r_{i1}/\Omega_1)=\frac{\alpha_1}{\Omega_1}r_{i1}^{\alpha_{i1}}\exp\left(-\frac{r_{i1}^{\alpha_1}}{\Omega_1}\right), r_{i1}\geq 0, i=1,2 \tag{3}$$

$$F_{r_{i1}}(r_{i1}/\Omega_1)=1-\exp\left(-\frac{r_{i1}^{\alpha_1}}{\Omega_1}\right) \tag{4}$$

$$p_{r_{i2}}(r_{i2}/\Omega_2)=\frac{\alpha_2}{\Omega_2}r_{i2}^{\alpha_{i2}}\exp\left(-\frac{r_{i2}^{\alpha_2}}{\Omega_2}\right), r_{i2}\geq 0, i=1,2 \tag{5}$$

$$F_{r_{i2}}(r_{i2}/\Omega_2)=1-\exp\left(-\frac{r_{i2}^{\alpha_2}}{\Omega_2}\right) \tag{6}$$

parameters α_1 and α_2 are Weibull fading parameters ($\alpha_1>0$, $\alpha_2>0$) and they express fading intensity measures. When values of those parameters increase the fading intensity decreases. For the special case when $\alpha_1=2$ and $\alpha_2=2$ proposed Weibull distributions become Rayleigh distribution, so our analysis has high level of generality. The mean powers from the input branches of each microdiversity system are defined as $\Omega_1=E(r_{i1}^{\alpha_1})$ and $\Omega_2=E(r_{i2}^{\alpha_2})$.

II. Analysis

SC combiners at the output of both microdiversity systems pick the maximal signal's mean power which can be mathematically presented as:

$$p_{r_1}(r_1/\Omega_1)=p_{r_{11}}(r_1/\Omega_1)F_{r_{21}}(r_1/\Omega_1)+p_{r_{21}}(r_1/\Omega_1)F_{r_{11}}(r_1/\Omega_1) \tag{7}$$

$$p_{r_2}(r_2/\Omega_2)=p_{r_{12}}(r_2/\Omega_2)F_{r_{22}}(r_2/\Omega_2)+p_{r_{22}}(r_2/\Omega_2)F_{r_{12}}(r_2/\Omega_2) \tag{8}$$

The probability density function of derivates $\dot{r}$ of the received signals r at the output of both microdiversity systems, with respect to time, are Gaussian pdfs [5]:

$$p(\dot{r}_j)=\frac{1}{\sqrt{2\pi}\hat{\sigma}_{r_j}}\exp\left(\frac{\dot{r}_j^2}{2\hat{\sigma}_{r_j}^2}\right), j=1,2 \tag{9}$$

where $\dot{r}_i$ presents derivatives of received microdiversity envelopes and can be expressed as [5]:

$$\dot{r}_j=\begin{cases}\dot{r}_{1j}, \dot{r}_{1j}>\dot{r}_{2j}\\ \dot{r}_{2j}, \dot{r}_{2j}>\dot{r}_{1j}\end{cases}, j=1,2 \tag{10}$$

For isotropic scattering, $\dot{r}$ is a Gaussian distributed random variable with zero mean and variance can be expressed as [6]:

$$\dot{\sigma}_{r_j}^2=\left(\frac{2\pi f_d}{\alpha_j}\right)\Omega_j r_j, j=1,2 \tag{11}$$

where f_d is a Doppler shift frequency.

Whereas pdfs of signal envelopes r_i at the output of microdiversity systems and pdfs of its derivatives $\dot{r}_i$ are statistically independent, joint pdf of r_i and $\dot{r}_i$ is given by:

$$p_{r_j\dot{r}_j}(r_j,\dot{r}_j/\Omega_j)=p(r_j/\Omega_j)p(\dot{r}_j)=$$

$$=2\frac{\alpha_j}{\Omega_j}r_j^{\alpha_j}\exp\left(-\frac{r_j^{\alpha_j}}{\Omega_j}\right)\left(1-\exp\left(-\frac{r_j^{\alpha_j}}{\Omega_j}\right)\right)\cdot$$

$$\cdot\frac{1}{\sqrt{2\pi}\hat{\sigma}_{r_j}}\exp\left(\frac{\dot{r}_j^2}{2\hat{\sigma}_{r_j}^2}\right) \tag{12}$$

At the output of suggested macrodiversity system is also SC combiner. It picks the maximum microdiversity's output average signal power, which mathematically can be defined by formulae:

$$p_{r,\dot{r}}(r,\dot{r})=\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2\, p_{r_1\dot{r}_1}(r,\dot{r}/\Omega_1)P_{\Omega_1\Omega_2}(\Omega_1\Omega_2)+$$

$$+\int_0^\infty d\Omega_2\int_0^{\Omega_2} d\Omega_1\, p_{r_2\dot{r}_2}(r,\dot{r}/\Omega_2)P_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \qquad (13)$$

$$F_r(r)=\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2\, F_{r_1}(r/\Omega_1)P_{\Omega_1\Omega_2}(\Omega_1\Omega_2)+$$

$$+\int_0^\infty d\Omega_2\int_0^{\Omega_2} d\Omega_1\, F_{r_2}(r/\Omega_2)P_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \qquad (14)$$

There is shadowing at the input of microdiversity systems as well as at the input of macrodiversity system. The slow fading is statistically independent due to sufficient input antenna spacing. In this case we achieve the highest decrease of the fading influence on system's performances. The slow fading is modeled by joint Gaussian pdf [7]:

$$p_{\Omega_1,\Omega_2}(\Omega_1,\Omega_2)=p_{\Omega_1}(\Omega_1)p_{\Omega_{21}}(\Omega_2)=$$

$$=\frac{1}{\Gamma(c_1)}\frac{\Omega_1^{c_1-1}}{\Omega_{01}^{c_1}}\exp\left(\frac{-\Omega_1}{\Omega_{01}}\right)\frac{1}{\Gamma(c_2)}\frac{\Omega_2^{c_2-1}}{\Omega_{02}^{c_2}}\exp\left(\frac{-\Omega_2}{\Omega_{02}}\right) \qquad (15)$$

where Ω_{0i} present mean values at the inputs of macrocombiner, while c_1 and c_2 present orders of Gamma distributions, and they determine measures of the shadowing present in the channel.

III. NUMERICAL RESULTS

After substituting (12), (13) and (15) in (1), we obtain following expressions for normalized level crossing rate, LCR:

$$N_r(r)/f_d=\int_0^\infty H_1\gamma\left(c_2,\frac{\Omega_1}{\Omega_{02}}\right)\Omega_1^{c_1-1}\exp\left(-\frac{\Omega_1}{\Omega_{01}}\right)\dot{\sigma}_{r_1}^2 d\Omega_1+$$

$$+\int_0^\infty H_2\gamma\left(c_1,\frac{\Omega_2}{\Omega_{01}}\right)\Omega_2^{c_2-1}\exp\left(-\frac{\Omega_2}{\Omega_{02}}\right)\dot{\sigma}_{r_2}^2 d\Omega_2 \qquad (16)$$

with $\gamma(a,x)$ incomplete Gamma function [8] and

$$H_1=\frac{1}{\Gamma(c_1)\Omega_{01}^{c_1}\Gamma(c_2)\Omega_{02}^{c_2}}2\frac{\alpha_1}{\Omega_1}r^{\alpha_1}\exp\left(-\frac{r^{\alpha_1}}{\Omega_1}\right)\left(1-\exp\left(-\frac{r^{\alpha_1}}{\Omega_1}\right)\right) \qquad (17)$$

$$H_2=\frac{1}{\Gamma(c_1)\Omega_{01}^{c_1}\Gamma(c_2)\Omega_{02}^{c_2}}2\frac{\alpha_2}{\Omega_2}r^{\alpha_2}\exp\left(-\frac{r^{\alpha_2}}{\Omega_2}\right)\left(1-\exp\left(-\frac{r^{\alpha_2}}{\Omega_2}\right)\right) \qquad (18)$$

Normalized LCR for various values of system's parameters is presented in Fig.2 LCR is normalized by maximal Dopler shift frequency f_d. We have considered the case of balanced macrocobiner's inputs, $\Omega_{01}=\Omega_{02}$. Numerical results for LCR (and AFD) are presented in the function of normalized signal level. Signal level is normalized with the sqare root of mean power of Gamma distributed signal , $\rho=z/\sqrt{\Omega_{01}}$.

In practice it is usauall that treshold level at the receiver, r is set to the value, that is smaller than the root mean value Ω_{01}, in order to obtain reasonable indication of outage. Because of that, from practical point of view, more intrasting are results for the values of normalized signal level that follows ρ<0 dB [9]. We can observe from Fig. 2, that for the normalzed signal levels, which are ρ<0 dB, larger values of Weibull paremeters α_1 and α_2 lead to smaller LCR due to smaller fading severity caused by the increment of α_1 and α_2. Also we can observe that LCR has smaller values in observed range in the presence of shadowing with smaller values of parameters c_1 i c_2.

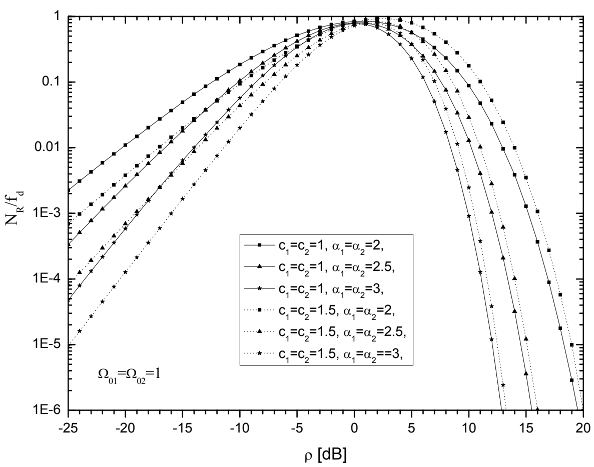

Fig. 2. Normalized LCR in the function of normalized signal level for various values of system's parameters

After substituting (14), (15) in (2), and following simillar mathematical procedure, we obtain following expressions for normalized average fade duration, AFD:

$$T_R(z)=\frac{\int_0^\infty\left(1-\exp\left(\frac{r^{\alpha_1}}{\Omega_1}\right)\right)\frac{1}{\Gamma(c_1)\Gamma(c_2)}\frac{\Omega_1^{c_1-1}}{\Omega_{01}^{c_1}}\exp\left(-\frac{\Omega_1}{\Omega_{01}}\right)\gamma(\Omega_1/\Omega_2,c_2)d\Omega_1}{N_z(z)}+$$

$$+\frac{\int_0^\infty\left(1-\exp\left(\frac{r^{\alpha_2}}{\Omega_2}\right)\right)\frac{1}{\Gamma(c_1)\Gamma(c_2)}\frac{\Omega_2^{c_2-1}}{\Omega_{02}^{c_2}}\exp\left(-\frac{\Omega_1}{\Omega_{01}}\right)\gamma(\Omega_2/\Omega_1,c_1)d\Omega_2}{N_z(z)} \qquad (19)$$

Normalized AFD for various values of systems' parameters is presented in Fig.3.

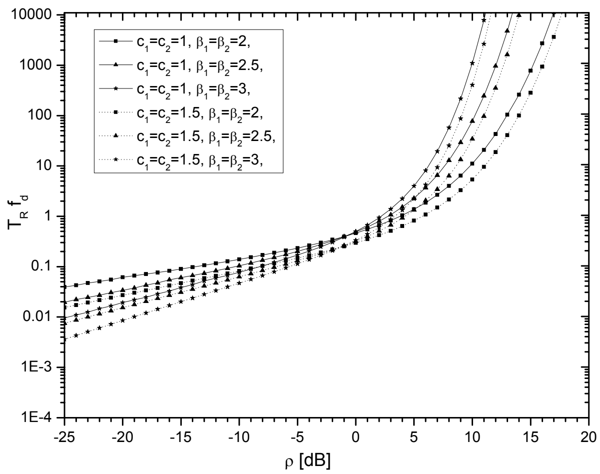

Fig. 3. Normalized AFD in the function of normalized signal level for various values of system's parameters

We can observe from Fig. 3, that for the normalzed signal levels, which are ρ<0 dB, larger values of Weibull paremeters α_1 and α_2 lead to higher values of AFD Also we can observe that AFD has higher values in observed range in the presence of shadowing with smaller values of parameters c_1 and c_2.

IV. CONCLUSION

Numerical results for LCR and AFD of SC macrodiversity system in the presence of independent Weibull fading at the microdiversity inputs are presented in this paper. Analysis has been performed for the case when selection on macro level is obtained considering mean power of the signlas at the input branches of macrocombiner. The shadowing was modeled by statistically independent Gamma distributions. Second order statistics has been analyzed for various values of systems' parameters.

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