The Performances of Generalized Selection Combiner in the Presence of Generalized-K Fading Channels

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Abstract **In this paper the performances of generalized selection combining (GSC) will be observed. Combiner has three inputs. The two strongest signals are isolated and added. For this combiner the probability density function for the output signal and the error probability will be determined. Generalized-K (K_G) fading is present at the input.**

Keywords – **Generalized selection combining, Generalized K fading, Probability density function, Bit error probability.**

I. Introduction

The shadow effect is a factor that degrades the system performances in mobile telecommunication systems at the most. It derogates the power of transmitted signal. When a received signal experiences shadow effect or fading during transmission, its envelope and phase fluctuate over time.

In wireless communication systems various techniques for reducing fading effect and influence of shadow effect are used. Such techniques are diversity reception, dynamic channel allocation and power control. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques.

Diversity reception, based on using multiple antennas at the receiver, (space diversity, with two or more branches), is very efficient methods used for improving system's quality of service, so it provides efficient solution for reduction of signal level fluctuations in fading channels. Multiple received copies of signal could be combined on various ways, and most popular of them are maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC).

Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining schemes whose SNR is the sum of the SNR's of each individual diversity branches. MRC is the optimal combining scheme, but its price and complexity are higher. Also, MRC requires cognition of all channel parameters and admit in the same phase all input signals, because it is the most complicated for realization ([1]-[3]).

With SC receiver, the processing is performed at only one of the diversity branches, which is selectively chosen, and no channel information is required. That is why SC is much simpler for practical realization. In general, selection combining, assuming that noise power is equally distributed over branches, selects the branch with the highest signal-to-noise ratio (SNR) that is the branch with the strongest signal ([4]-[6]).

The fading influence to the system performances is considered in [7]. The most often Rayleigh, Rice, Nakagami, Weibull and log-normal fading are considered.

Relatively new models in communications over fading channels are *K* and Generalized-*K* distributions [8], which in the past have been widely used in radar applications [9], [10]. The Generalized-*K* distribution has two shaping parameters, and as a consequence includes the *K* distribution as a special case. Moreover, it is sufficiently generic as it is able to incorporate most of the fading and shadowing effects observed in wireless communication channels and hence seems to be appropriate for the generic modeling of fading channels [11].

In [8], the generalized-K distribution was presented as analytically simpler than the Nakagami-lognormal or Suzuki distributions and general enough to approximate them, as well as several others including Rayleigh and Nakagami-*m*. Moreover, in the same work, the Amount-of-Fading (AF) and the Average Bit-Error-Rate (ABER) for the special case of the binary phase swift keying (BPSK) modulation, were derived. However, detailed performance analysis for the SNR statistics of a receiver operating over Generalized-*K* fading channel has not yet been published in the open technical literature and this is the topic of the current work.

A detailed performance analysis of generalized selection combining GSC (2, *L*) diversity receivers operating over generalized-gamma fading channels is presented in [12]. For this class of receiver a novel closed-form expression for the moment's output signal-to-noise ratio is derived. Furthermore, infinite series representation for the moments-generating and the cumulative distribution function are obtained. The proposed mathematical analysis is accompanied by various performance evaluation results. These theoretical results are complemented by equivalent computer simulated results, which validate the accuracy of the proposed analysis.

In the paper[13] a simple closed form expression for the average SNR of this generalized selection combining for independent identically distributed diversity channels is found, which is upper bounded by the average SNR of maximal ratio combining and lower bounded by the average SNR of conventional selection combining. In this paper the

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presence of log-normal fading at the input of generalized selection combiner will be observed.

We will derive the expressions for the probability density function for the output signal, characteristic function for the output signal, the moments for the output signals and the error probability. We will present the error probability results graphically for different signal and fading parameters values

II. System Performances

The model of the complex GSC combiner with three inputs, considered in this paper, is shown in Fig. 1.

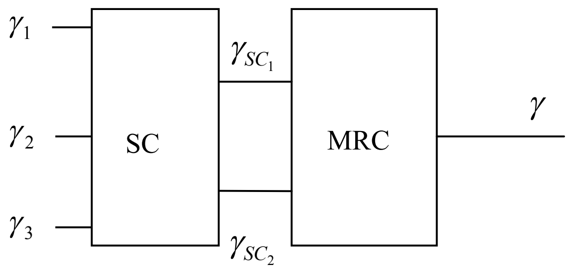

Fig. 1. Model of the GSC combiner with three inputs

The signals at the combiner input are γ_1, γ_2 and γ_3. The combiner chooses first the two strongest input signals from their three input signals. The SC combiner output signals are γ_1 and γ_2. These signals are added in the MRC combiner. The MRC combiner output signal is γ. That is the complex (GSC) combiner output signal and it is:

$$\gamma = \gamma_{SC_1} + \gamma_{SC_2} \tag{1}$$

For this complex GSC combiner we derive the probability density for the output signal, characteristic function, moments for the output signal and the error probability.

The probability density of the input signals, γ_1, γ_2 and γ_3, are:

$$p_{\gamma_1}(\gamma_1) = \frac{2\left(\frac{k_1 m_1}{\bar{\gamma}_1}\right)^{(k_1+m_1)/2} \gamma_1^{(k_1+m_1-2)/2}}{\Gamma(m_1)\Gamma(k_1)} K_{k_1-m_1}\left[2\left(\frac{k_1 m_1}{\bar{\gamma}_1}\gamma_1\right)^{\frac{1}{2}}\right], \quad \gamma_1 \geq 0 \tag{2}$$

$$p_{\gamma_2}(\gamma_2) = \frac{2\left(\frac{k_2 m_2}{\bar{\gamma}_2}\right)^{(k_2+m_2)/2} \gamma_2^{(k_2+m_2-2)/2}}{\Gamma(m_2)\Gamma(k_2)} K_{k_2-m_2}\left[2\left(\frac{k_2 m_2}{\bar{\gamma}_2}\gamma_2\right)^{\frac{1}{2}}\right], \quad \gamma_2 \geq 0 \tag{3}$$

$$p_{\gamma_3}(\gamma_3) = \frac{2\left(\frac{k_3 m_3}{\bar{\gamma}_3}\right)^{(k_3+m_3)/2} \gamma_3^{(k_3+m_3-2)/2}}{\Gamma(m_3)\Gamma(k_3)} K_{k_3-m_3}\left[2\left(\frac{k_3 m_3}{\bar{\gamma}_3}\gamma_3\right)^{\frac{1}{2}}\right], \quad \gamma_3 \geq 0 \tag{4}$$

where $m_i \geq 1/2$ is the Nakagami-m shaping parameter, k_i is shadowing shaping parameter and $\bar{\gamma}_i$ is average SNR for input channels [14].

The joint probability density of the SC combiner output signals γ_{SC_1} and γ_{SC_2} is:

$$\begin{aligned} p_{\gamma_{SC_1}\gamma_{SC_2}}(\gamma_{SC_1}\gamma_{SC_2}) = \\ = p_{\gamma_1}(\gamma_{SC_1}) \cdot p_{\gamma_2}(\gamma_{SC_2}) \cdot F_{\gamma_3}(\gamma_{SC_2}) + \\ + p_{\gamma_2}(\gamma_{SC_1}) \cdot p_{\gamma_1}(\gamma_{SC_2}) \cdot F_{\gamma_3}(\gamma_{SC_2}) + \\ + p_{\gamma_1}(\gamma_{SC_1}) \cdot p_{\gamma_3}(\gamma_{SC_2}) \cdot F_{\gamma_2}(\gamma_{SC_2}) + \\ + p_{\gamma_3}(\gamma_{SC_1}) \cdot p_{\gamma_1}(\gamma_{SC_2}) \cdot F_{\gamma_2}(\gamma_{SC_2}) + \\ + p_{\gamma_2}(\gamma_{SC_1}) \cdot p_{\gamma_3}(\gamma_{SC_2}) \cdot F_{\gamma_1}(\gamma_{SC_2}) + \\ + p_{\gamma_3}(\gamma_{SC_1}) \cdot p_{\gamma_2}(\gamma_{SC_2}) \cdot F_{\gamma_1}(\gamma_{SC_2}) \end{aligned} \tag{5}$$

where: $\gamma_{SC_1} \geq \gamma_{SC_2}$.

In this manner all possible combinations of input signals values, γ_1, γ_2 and γ_3, are taken into consideration.

For: $\gamma_i \geq \gamma_j \geq \gamma_k$ SC combiner output signals are:

$$\gamma_{SC_1} = \gamma_i; \quad \gamma_{SC_2} = \gamma_j$$

where $i=1,2,3;$ $j=1,2,3;$ $k=1,2,3;$ $i \neq j \neq k$

We obtain the cumulative probability density of the input signals, γ_1, γ_2 and γ_3 as [14]:

$$\begin{aligned} F_{\gamma_i}(\gamma_i) = \pi \csc[\pi(k_i - m_i)] \cdot \\ \cdot \left[\frac{(k_i m_i \gamma_i / \bar{\gamma}_i)^{m_i} \, {}_1F_2(m_i; 1+m_i-k_i, 1+m_i; m_i k_i \gamma_i / \bar{\gamma}_i)}{\Gamma(k_i)\Gamma(1+m_i-k_i)\Gamma(1+m_i)} - \right. \\ \left. - \frac{(k_i m_i \gamma_i / \bar{\gamma}_i)^{k_i} \, {}_1F_2(k_i; 1-m_i+k_i, 1+k_i; m_i k_i \gamma_i / \bar{\gamma}_i)}{\Gamma(m_i)\Gamma(1-m_i+k_i)\Gamma(1+k_i)} \right] \end{aligned} \tag{6}$$

$i=1,2,3.$

If we replace the expressions (2)-(4) and (6) into (5) we have the joint probability density of the SC combiner output signals γ_{SC1} and γ_{SC2}, (for $\gamma_{SC_1} \geq \gamma_{SC_2}$), in too long form for this place.

The probability density of the GSC combiner output signal γ is:

$$p_\gamma(\gamma) = \frac{1}{\pi}\int_0^\gamma p_{\gamma_{SC_1}\gamma_{SC_2}}(\gamma - \gamma_{SC_2}, \gamma_{SC_2}) d\gamma_{SC_2} \tag{7}$$

Finally, the error probability is [7]:

$$P_b(e) = \frac{1}{\pi}\int_0^{\infty}\int_0^{\pi/2} \exp\left(-\frac{g\gamma}{\sin^2\phi}\right) p_y(\gamma)\, d\phi\, d\gamma \qquad (8)$$

III. Numerical Results

After some substitutions in the (8) the error probability becomes:

$$P_b(e) = \frac{1}{\pi}\int_0^{\infty}\int_0^{y_1}\int_0^{\pi/2} \exp\left(-\frac{gy_1}{\sin^2\phi}\right)\exp\left(-\frac{gy_2}{\sin^2\phi}\right) p_{y_1y_2}(y_1 y_2)\, d\phi\, dy_1\, dy_2 \qquad (9)$$

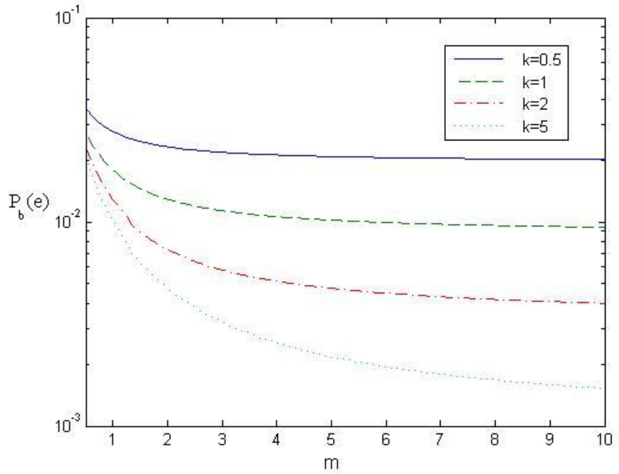

Fig. 2. *Pb(e)* versus parameter *m* for *k =0.5; 1; 2;5,* $\bar{\gamma} = 5$ for GSC combiner.

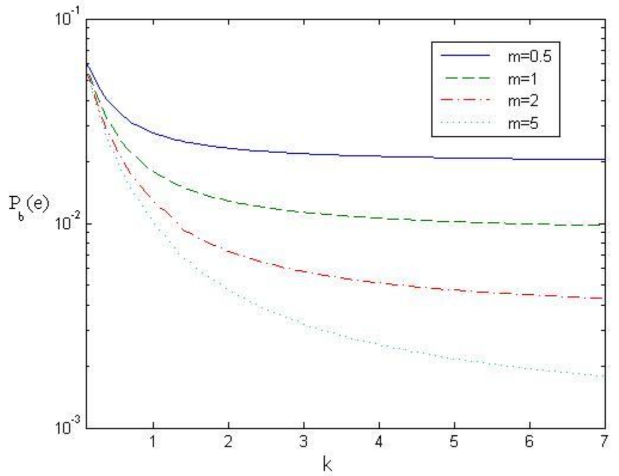

Fig. 3. *Pb(e)* versus parameter *k* for *m=0.5; 1; 2;5,* $\bar{\gamma} = 5$ for GSC combiner

In this section we considered BPSK modulation scheme. For this case parameter *g* in above equation is equal to 1.

The main values of the input signals amplitudes are:

$$p_\gamma(\gamma) = \frac{1}{\pi}\int_0^{\gamma} p_{\gamma_{SC_1}\gamma_{SC_2}}(\gamma - \gamma_{SC_2}, \gamma_{SC_2})\, d\gamma_{SC_2} \qquad (10)$$

where $\gamma_{SC_1} \geq \gamma_{SC_2}$.

Figures 2, 3 and 4 show *Pb(e)* versus parameters *m*, *k* and $\bar{\gamma}$, respectively, for different values of other parameters.

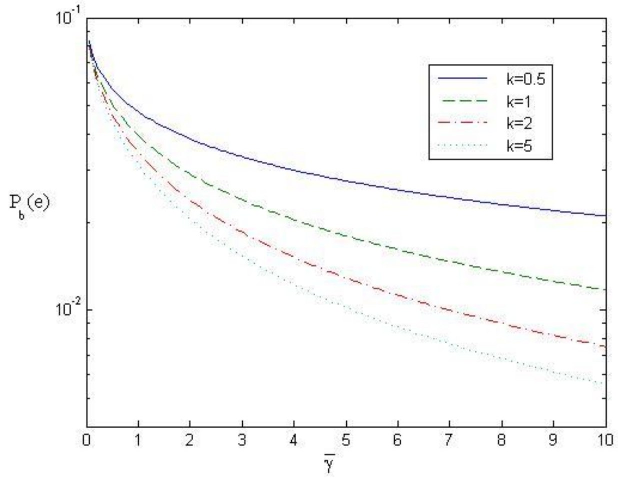

Fig. 4. *Pb(e)* versus parameter $\bar{\gamma}$ *for m=1, k =0.5; 1; 2;5,* for GSC combiner

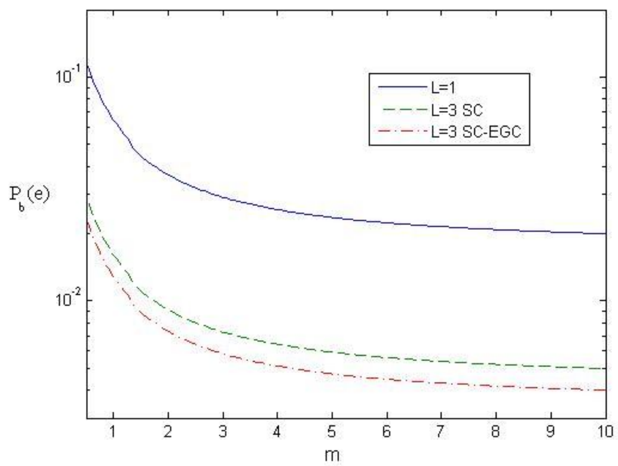

Fig. 5. *Pb(e)* versus parameter *m* for *k =2,* $\bar{\gamma} = 5$

In Figures 5, 6 and 7 we can see *Pb(e)* versus parameters *m*, *k* and $\bar{\gamma}$, respectively, for different values of other parameters, for *L=3* when we have only SC combiner, then when we have complex GSC combiner and the case for one channel transmission, *L=1*.

From all these figures we can note that excellent agreement exists between theoretical and computer calculated results.

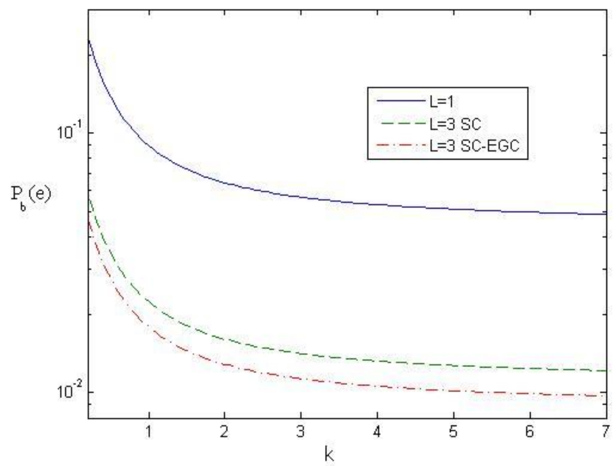

Fig. 6. *Pb(e)* versus parameter parameter *k* for *m =1,* $\overline{\gamma} = 5$.

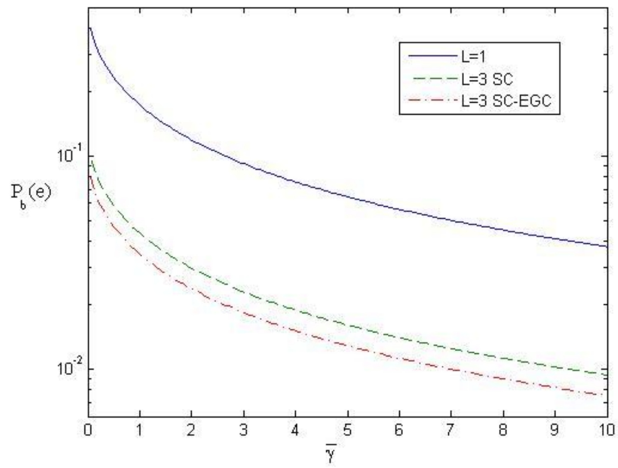

Fig. 7. *Pb(e)* versus parameter $\overline{\gamma}$ *for m=1, k =2*.

IV. CONCLUSION

In this paper diversity system with complex GSC combiner with three inputs is considered. The joint statistical characteristics of SC combiner output signal are derived, and then the probability density for the complex combiner output signal in the presence of generalized-*K* fading. The probability density can be used for the system error probability and for the outage probability calculating. Generalized selection combining (GSC) is considered as an alternative diversity scheme for bridging the performance emptiness between two classical diversity schemes, maximal ratio combining (MRC) and selection combining (SC). As compared to optimal MRC combining, GSC has reduced system complexity, while as compared to SC, which is one of the simplest diversity schemes, it has improved performance.

In the past the performance of GSC diversity receivers has been analyzed for various fading models, including Rayleigh, Nakagami-*m* and Weibull. Some attention is given to the practical important class of GSC (2, *L*), where among *L* available resolvable path the two strongest are adaptively combined. Since this class of receivers is an effective compromise achieving very good performance with reduced implementation complexity, it is considered also here in the presence of generalized-*K* fading and when number of branches is three: *L*=3. We can note the excellent agreement between theoretical and computer calculated results.

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