Synthesis and Analysis of Equal Power Division Reduced-Size Branch-line Hybrids

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Abstract: This paper proposes a method for synthesis and analysis of equal power division reduced size branch-line hybrids. Based on dispersion equation of the capacitively loaded transmission line, closed form synthesis formulae are derived. Using the fourfold symmetry of the reduced size branch-line hybrid, analysis formulae are derived. The output ports' phase difference is analyzed for different synthesis configurations. The fractional bandwidth of the reduced size hybrid is about 10% narrower than the conventional branch-line hybrid. The results presented in the paper are applicable to uniplanar structure designs.

Keywords: Reduced size, branch line hybrid, capacitively loaded transmission line.

I. Introduction

Quadrature branch-line hybrids play important role as power divider, or power combiner, image rejection mixers, and balanced amplifiers. The classic form of the quadrature branch-line hybrid is shown on Fig.1.

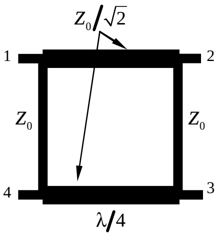

Fig.1 Conventional branch-line coupler

For a given input power at port 1, there will be 90^o phase difference between the waves at ports 2 and 3 for the centre frequency. This phase difference varies over $\pm 5^o$ for a 10% fractional bandwidth. The usable bandwidth of the branch-line hybrid is constrained by the change in the isolation. Such branchline couplers can be designed for 3-10dB coupling.

Main arms of the coupler are of quarterwave length. This size is unacceptable large especially for the lower microwave bands and MMIC applications. The branch-line hybrid occupies large area on the printing circuit board or on the chip.

The use of lumped-element hybrid, which uses spiral inductors and lumped capacitors, is one possible solution to this problem [1,2]. However, the design of spiral inductors is requires precise empirical models based on measurement of test elements including parasitics.

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This design approach becomes difficult for frequencies above 20GHz [3]. Another method for the hybrid coupler size reduction is proposed in [4]. The branch line is realized as lumped element Π or T equivalent networks. Each lumped element value is determined by equating the ABCD matrices of both structures – transmission line and lumped element. The coupler involves shunt inductors, which are inconvenient for fabrication. However, this method eliminates the uncertainty caused by the lumped inductors.

The method presented in [1,4], the branchline hybrid utilizes high impedance lines loaded by shunt lumped capacitors. The authors present a special case for the theory of this coupler.

In this paper, we present a general theory of reduced size branch-line hybrid, composed of capacitively loaded transmission lined. With the proposed method, a 3dB branch-line coupler can be designed for various impedance and capacitor values. The reduction of the size is connected with increase of the characteristic impedance of the transmission line. The phase difference between the output ports for various characteristic impedances and capacitors is studied. Because of the slow-wave effect of the capacitive loaded line, the phase difference is not symmetrical along the 90^0 value.

II. Reduced Size hybrid

The main element in the conventional hybrid coupler is the quarter-wavelength transmission line. In order to obtain reduced-size coupler, shown on Fig.2, in [1,4] is proposed to use capacitive loaded transmission line instead of quarter-wavelength line. To assure equivalent electrical parameters, it is examined both circuits shown on Fig.3a and Fig.3b. The circuit on Fig.3b may be considered as a unit element of periodic slow wave structure [4-7].

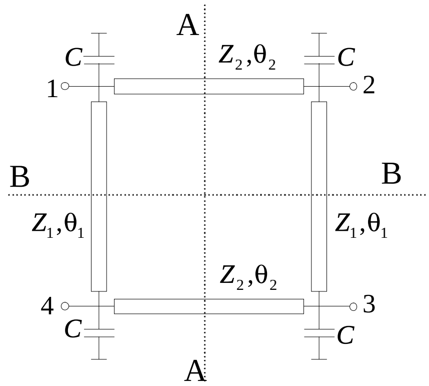

Fig.2 Reduced size branch-line coupler

The slow-wave line consists of a transmission line loaded on both sides by lumped capacitors C. The main parameters of the transmission line are the characteristic impedance Z_c, length of the line l, and the propagation constant k.

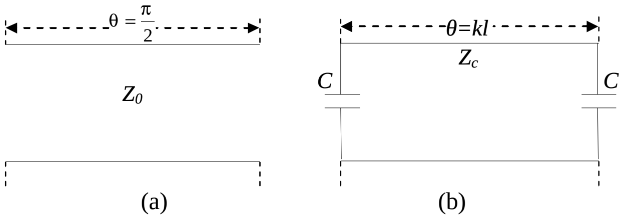

Fig.3 (a) Quarter wavelength line, (b) capacitive loaded line

The electric length of the unloaded line is $\theta = kl$. The electrical characteristics of the line are described by the ABCD matrix. The overall matrix of the resonator is a product of the three ABCD matrices – the lumped capacitors C and the unloaded transmission line.

$$[ABCD] = [ABCD]_1 [ABCD]_2 [ABCD]_3 \qquad (1),$$

where

$$[ABCD]_1 = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}, \ [ABCD]_3 = \begin{bmatrix} \cos\theta & jZ_c \sin\theta \\ j\dfrac{1}{Z_c}\sin\theta & \cos\theta \end{bmatrix},$$

$$[ABCD]_3 = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}.$$

After the multiplication, the elements of the ABCD matrix are derived as:

$$A = \cos\theta - \omega C Z_c \sin\theta \qquad (2a)$$

$$B = jZ_c \sin\theta \qquad (2b)$$

$$C = 2j\omega C\cos\theta + j\left(\frac{1}{Z_c} - (\omega C)^2 Z_c\right)\sin\theta \qquad (2c)$$

$$D = \cos\theta - \omega C Z_c \sin\theta \qquad (2d)$$

It is easily checked out the main property of the ABCD matrix: $AD - BC = 1$.

The ABCD matrix of the quarter wavelength line is:

$$[ABCD] = \begin{bmatrix} 0 & jZ_0 \\ j\dfrac{1}{Z_0} & 0 \end{bmatrix} \qquad (3)$$

Comparing the ABCD matrices of both circuits (2) and (3) it is obtained the main design equations:

$$\begin{aligned} &\cos\theta - \omega C Z_c \sin\theta = 0 \\ &Z_c \sin\theta = Z_0 \end{aligned} \qquad (4)$$

It is obvious from Eq.4 that the characteristic impedance of the slow-wave line Z_c is greater than the characteristic impedance Z_0 of the conventional hybrid coupler.

There are two main approaches to design a reduced-size branch-line coupler. The first approach starts with the choice of the characteristic impedances Z_1 and Z_2 of the transmission lines (Fig.2) and calculation of their electrical lengths θ_1 and θ_2. On the final stage the value of the lumped element capacitor is calculated. The design equations become:

$$\begin{aligned} &\theta_1 = \arcsin\frac{Z_0}{Z_1} \\ &\theta_2 = \arcsin\frac{Z_0}{\sqrt{2}Z_2} \\ &\omega C Z_0 = \cos\theta_1 + \sqrt{2}\cos\theta_2 \end{aligned} \qquad (5).$$

The second approach starts with the choice of the electrical lengths of the transmission lines θ_1, θ_2 and calculation of the characteristic impedances Z_1 Z_2 and the value of the lumped capacitor. The design equations are as follows:

$$\begin{aligned} &Z_1 = \frac{Z_0}{\sin\theta_1} \\ &Z_2 = \frac{Z_0}{\sqrt{2}\sin\theta_2} \\ &\omega C Z_0 = \cos\theta_1 + \sqrt{2}\cos\theta_2 \end{aligned} \qquad (6)$$

The analysis of the reduced size hybrid is carried out using the fourfold symmetry of the schematic shown on Fig.2. Using proper excitation of the structure both symmetry planes AA and BB can be either magnetic (open circuit) or electric walls (short circuit). The equivalent schematics for the four different excitations are shown on Fig.4.

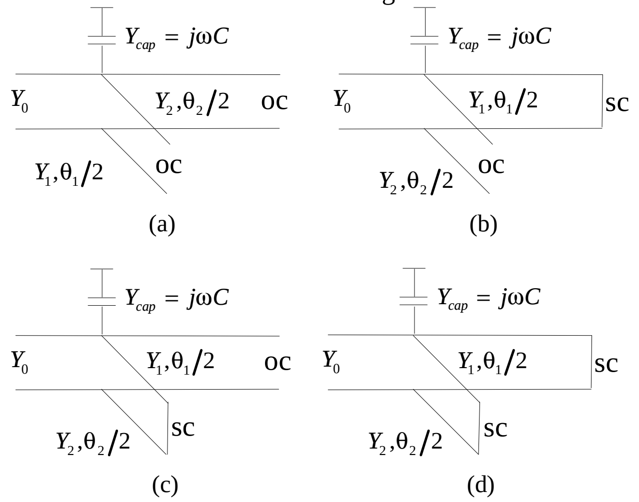

Fig.4. Equivalent schematics for one quarter section when (a) AA and BB are magnetic walls, (b) AA is electric wall, BB is magnetic wall, (c) AA is magnetic wall, BB is electric wall, (d) AA and BB are electric walls.

The reflection coefficients of each equivalent schematic are obtained as follows:

$$\Gamma_a = \frac{Y_c - j\omega C - jY_1 tg\frac{\theta_1}{2} - jY_2 tg\frac{\theta_2}{2}}{Y_c + j\omega C + jY_1 tg\frac{\theta_1}{2} + jY_2 tg\frac{\theta_2}{2}} \qquad (7a)$$

$$\Gamma_b = \frac{Y_c - j\omega C - jY_1 tg\frac{\theta_1}{2} + jY_2 cotg\frac{\theta_2}{2}}{Y_c + j\omega C + jY_1 tg\frac{\theta_1}{2} - jY_2 cotg\frac{\theta_2}{2}} \qquad (7b)$$

$$\Gamma_c = \frac{Y_c - j\omega C + jY_1 cotg\frac{\theta_1}{2} - jY_2 tg\frac{\theta_2}{2}}{Y_c + j\omega C - jY_1 cotg\frac{\theta_1}{2} + jY_2 tg\frac{\theta_2}{2}} \qquad (7c)$$

$$\Gamma_d = \frac{Y_c - j\omega C + jY_1 cotg\frac{\theta_1}{2} + jY_2 cotg\frac{\theta_2}{2}}{Y_c + j\omega C - jY_1 cotg\frac{\theta_1}{2} - jY_2 cotg\frac{\theta_2}{2}} \quad (7d).$$

The lengths of the branches are shorter than quarter wavelength. Consequently the electrical lengths are less than $\pi/8\ rad$. The following approximations are assumed $tg\theta \approx \theta$ and $cotg\theta \approx 1/\theta$. The previously derived formulae for the reflection coefficients are reduced to:

$$\Gamma_a = \frac{Y_c - j\omega C - jY_1\frac{\theta_1}{2} - jY_2\frac{\theta_2}{2}}{Y_c + j\omega C + jY_1\frac{\theta_1}{2} + jY_2\frac{\theta_2}{2}} \quad (8a)$$

$$\Gamma_b = \frac{Y_c - j\omega C - jY_1\frac{\theta_1}{2} + jY_2\frac{2}{\theta_2}}{Y_c + j\omega C + jY_1\frac{\theta_1}{2} - jY_2\frac{2}{\theta_2}} \quad (8b)$$

$$\Gamma_c = \frac{Y_c - j\omega C + jY_1\frac{2}{\theta_1} - jY_2\frac{\theta_2}{2}}{Y_c + j\omega C - jY_1\frac{2}{\theta_1} + jY_2\frac{\theta_2}{2}} \quad (8c)$$

$$\Gamma_d = \frac{Y_c - j\omega C + jY_1\frac{2}{\theta_1} + jY_2\frac{2}{\theta_2}}{Y_c + j\omega C - jY_1\frac{2}{\theta_1} - jY_2\frac{2}{\theta_2}} \quad (8d).$$

The scattering matrix elements are derived as follows [6,7]:

$$\begin{aligned} S_{11} &= \frac{1}{4}(\Gamma_a + \Gamma_b + \Gamma_c + \Gamma_d) \\ S_{12} = S_{21} &= \frac{1}{4}(\Gamma_a - \Gamma_b + \Gamma_c - \Gamma_d) \\ S_{13} = S_{31} &= \frac{1}{4}(\Gamma_a - \Gamma_b - \Gamma_c + \Gamma_d) \\ S_{14} = S_{41} &= \frac{1}{4}(\Gamma_a + \Gamma_b - \Gamma_c - \Gamma_d) \end{aligned} \quad (9)$$

If the reduced size hybrid is designed according to Eq.5 or Eq.6, therefore the scattering matrix elements (Eq.9) will reduce to $S_{11} = S_{14} = 0$, $S_{13} = -1/\sqrt{2}$, $S_{12} = -j/\sqrt{2}$.

III. Numerical Results

In order to verify the applicability of the derived formulas (5,6) for design of reduced-size hybrid, it is examined the phase difference between the output ports for different electrical lengths and characteristic impedances of the branches.

It is convenient in some design cases to fix the characteristic impedances of the reduced-size hybrid branches. Utilizing Eq.(5) there are calculated the electrical lengths of the branches. The results are summarized in Table 1.

It is obvious that the increase in the characteristic impedance leads to shorter lines. The value of the lumped capacitance *C* is big enough and it could be realized in SMD as a trimmer for the low GHz range.

TABLE 1 Equal characteristic impedance of the branches

Number	Z_1, Ω	Z_2, Ω	θ_1, deg	θ_2, deg	$C, [pF]$
1	55	55	65.38	40	4.77
2	70.7	70.7	45	30	6.15
3	90	90	33.75	23.13	6.78

Fig.4 shows the phase difference between the output ports for the design examples summarized in Table 1. It is drawn the phase difference for conventional branch-line coupler for reference.

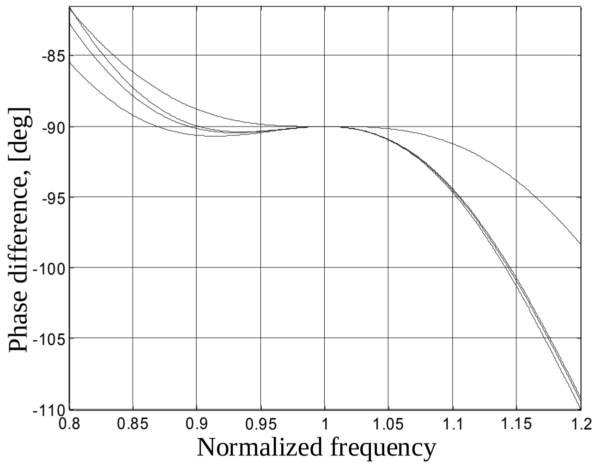

Fig.5 Phase difference between the output ports according Table1. (№1-dashed, №2- dotted, №3-dash-dotted). Conventional branch-line coupler-solid line

It is clearly seen from Fig.4 that the bandwidth of the reduced-size hybrid is narrower than the conventional branch-line coupler. The phase difference of reduced size hybrid is asymmetric against the central frequency. For the frequencies upper than the center, the phase difference increases very rapidly from the reference point of -90^{o}. For the frequencies lower than the center, the phase difference varies around -90^{o}. In contrast to the conventional branch-line coupler the output phase difference of the reduced size hybrid equals twice -90^{o}, while alternates around this value. This is due to the slow-wave effect observed in capacitively loaded transmission line. For lower frequencies the phase difference rises more rapidly than the conventional branch line coupler. Consequently as well-expressed is the slow-wave effect as narrower is the bandwidth.

In some design cases it is desirable to fix the electrical lengths of the branch lines. For the chosen lengths, it is computed the characteristic impedances of the lines and the values of the lumped capacitors (Eq.6). In Table2 are summarized the results for 6 different electrical lengths starting from *15 deg* to *75 deg*.

TABLE 2 Equal electrical lengths of the branches

Number	Z_1, Ω	Z_2, Ω	θ_1, deg	θ_2, deg	$C, [pF]$
1	193.18	136.6	15	15	7.42
2	118.31	83.66	25	25	6.96
3	87.17	61.64	35	35	6.29
4	70.71	50	45	45	5.43
5	57.73	40.82	60	60	3.84
6	51.76	36.6	75	75	2

Fig.5 shows the phase difference between the output ports for the design examples summarized in the first three rows in Table 2. For very small sized hybrids, it is impractical to realize in standard microstrip technology, because of the very high characteristic impedance of the lines and the necessity of high precision etching technology.

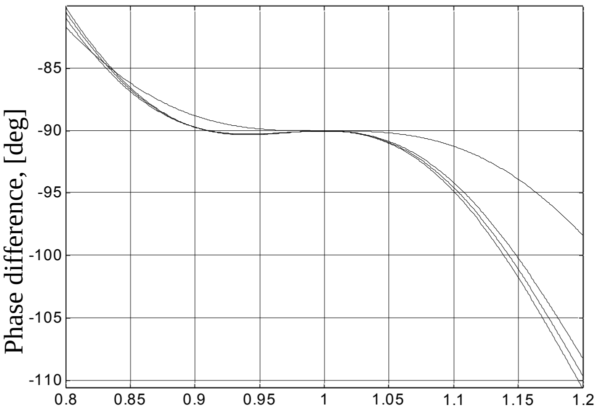

Fig.6 Phase difference between the output ports according Table2. (№1-dashed, №2- dotted, №3-dash-dotted). Conventional branch-line coupler-solid line

It is seen from Fig.5 that for highly dispersive structures the fractional bandwidth is much narrower than the conventional branch-line coupler. The ripple of the phase difference around the value of -90^{o} is under 0.8^{o} . For the frequencies upper the center frequency the phase difference decreases very rapidly and the output ports are not longer in quadrature.

Increasing of the branches electrical length leads to decreasing of their characteristic impedances. Therefore the lumped capacitance has lower value. Because of the very low value of the lumped capacitance a sensitivity analysis of the hybrid response is needed.

Fig.6 shows the phase difference for the design examples in the last three rows in Table2. The increasing of the electrical length of the branch lines causes their characteristic impedance to decrease.

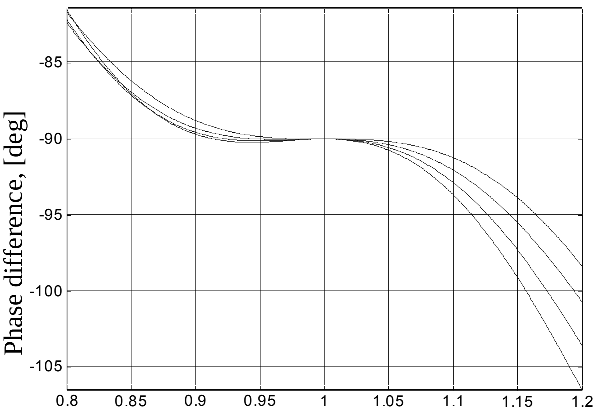

Fig.7 Phase difference between the output ports according Table2. (№4-dashed, №5- dotted, №6-dash-dotted). Conventional branch-line coupler-solid line

Comparing the results, shown on Fig.5 and Fig.6, the bandwidth of the reduced size hybrid is wider for longer branch lines. For electrical length $\theta_1 = \theta_2 = 75\deg$ and lumped capacitor 2pF, the phase difference is close to the conventional branch-line hybrid for lower frequencies than the center frequency. Due to the slow-wave effect the phase difference for higher frequencies lowers more rapid than the conventional branch-line hybrid.

V. Conclusion

The paper proposes a method for synthesis and analysis of equal power division reduced size branch line hybrids utilizing slow wave effect of capacitively loaded line. Using the fourfold symmetry of the hybrid, analysis formulae are obtained. Closed form synthesis formulae are derived for fixed characteristic impedance or fixed electrical length of the branch lines. The numerical results show that the fractional bandwidth of the reduced size hybrid is narrower than the conventional hybrid. The phase difference between the output ports is no longer symmetrical around -90^{o} due to the slow wave effect. The proposed synthesis method is suitable for low GHz range hybrids, balanced amplifiers on a microstrip technology.

References

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