Competitive Pricing Using Game Theory in the Next Generation Networks

Vesna Radonjić and Vladanka Aćimović Raspopović[1]

Abstract – **In this paper we present the game theory model for achieving Nash equilibrium in pricing Next Generation Networks services. We consider the competition between two Internet Service Providers offering the same service. Their competition is modelled as a simultaneous-play game in which the solution is obtained by Nash equilibrium. The proposed model is verified through numerous simulations performed by software that we developed for that purpose.**

Keywords – **Competitive Pricing, Quality of Service, Quality of Experience, Nash Equilibrium, Next Generation Networks.**

I. INTRODUCTION

Pricing is one of the important issues in Next Generation Networks (NGN). Through an appropriate pricing mechanism, a service provider offering NGN services tends to maximize his revenue, while users tend to achieve the highest satisfaction from service usage at the affordable price.

Two main factors influencing the price setting are users' demand and competition among service providers. Price and demands for the same or even similar services are mutually dependent. If the demand is high, a high price can be charged by a service provider and his revenue will be increased. On the other hand, if the demand is low, the price must be reduced to attract more users. Competition among the service providers impacts the price setting. If the services are substitutable users buy a service that provides the highest satisfaction at the lowest price. If one service provider reduces its offered price to attract more users and gain higher revenue, this will impact the revenues of other providers who will try to compete by reducing their offering prices as well.

In this paper we present a model for service competition and pricing in a NGN where two Internet Service Providers (ISPs) compete with each other to detain existing and attract new users in a particular service area. They try to achieve that goal with trade-off between Quality of Service (QoS) and price. In our model price and bandwidth consumption are optimized for users which bandwidth demand is in certain range of interest, named as partially elastic users. We consider the case where both ISPs offer their prices at the same time, i.e. simultaneous-play game. The solution of this competition is given by Nash equilibrium for which both ISPs are satisfied the solution in terms of prices.

The paper is organized in the following way. In Section 2 we explain the meaning of pricing, charging and billing processes and the pricing role in a QoS differentiation especially in NGN. In Section 3 both users' and ISPs optimization problems are presented and the model for solving these problems is proposed. In Section 4 simulation results are presented and analyzed. Conclusion is given in the Section 5.

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II. PRICING FOR THE NEXT GENERATION NETWORKS

A. Pricing Issue in the Next Generation Networks

Pricing is the process of determining tariffs, i.e., cost per unit. It is based on particular pricing model and controlled by a pricing policy. Charging combines the tariffs and the results of metering needed for the charge of users. The output of charging process is the charge per party (customer, service provider, content provider). The billing process produces an invoice on the basis of the charge per party [1]. The process can be configured by means of the billing policy, e.g., how often a bill is sent to a user. The payment process results in the actual transfer of money, based on an invoice as input.

QoS differentiation introduces a clear need for incentives to be offered to users and encourage them to choose the service that is most appropriate for their needs. In commercial networks, this can be most effectively achieved through pricing. Price discrimination of services is appropriate for encouraging service differentiation with the associated revenues that should be paid for any needed network expansions. NGN must be flexible enough to enable the use of different pricing models [2], [3]. Pricing model should fulfil a trade-off between providing satisfying user's utility and provider's revenue, still preserving implementation efficiency and feasibility. User's utility can be expressed as a function of available network resource offered to a user which indicates a user's sensitivity to changes in QoS.

It is suggested for NGN that the basic best-effort architecture should be left intact with QoS schemes solely reserved for resource intensive high quality real-time services [4].

B. Description of Quality of Service, Network Performance and Quality of Experience in the NGN

The QoS paradigm requires a network that could carry out service differentiation with packets serviced depending upon their value. QoS is defined in Recommendation E.800 as follows: "Collective effect of service performance which determines the degree of satisfaction of a user of the service". This definition is a wide one encompassing many areas of work, including subjective user satisfaction. However, in [4] the aspects of QoS are restricted to the identification of parameters that can be directly observed and measured at the point at which the service is accessed by the user. Recommendation I.350 defines Network Performance (NP) as "NP is measured in terms of parameters which are meaningful to the network provider and are used for the purpose of system design, configuration, operation and maintenance. NP is defined independently of terminal performance and user actions". Quality of Experience (QoE) is defined as the overall acceptability of an application or service, as perceived subjectively by the end user. QoE includes the complete end-to-end system effects (client, terminal, network, services infrastructure, etc). Overall acceptability may be influenced by user expectations and context [4].

QoS provides a valuable framework for network provider, but it is not necessarily usable in specifying performance requirements for particular network technologies (i.e. IP, MPLS, etc.). Similarly, NP ultimately determines the user observed QoS, but it does not necessarily describe that quality in a way that is meaningful to users. QoE is subjective in nature, i.e. depends upon user actions and subjective opinions. The definition of QoS, NP and QoE should make mapping clear in cases where there is not a simple one-to-one relationship among them. Table 1 shows some of the characteristics which distinguish QoS, NP and QoE.

TABLE 1.
DISTINCTION BETWEEN QOE, QOS AND NP [4]

QoE	**QoS**	**NP**
User oriented		Provider oriented
User behaviour attribute	Service attribute	Connection/Flow element attribute
Focus on user-expected effects	Focus on user-observable effects	Focus on planning, development (design), operations and maintenance
User subject	Between (at) service access points	End-to-end or network elements capabilities

NP definition includes transmitting time and response time. Transmitting time is the time interval during which a packet is transmitted between two network nodes. Response time is the time interval between the requirement sending and the receiving of required data. In this paper we did not particularly observed those parameters. The analyzed service model reflects partially elastic users for whom the QoS can be determined solely as a function of the average bandwidth. We defined QoE parameters through positive constants that regulate the sensitivity of users' satisfaction to the QoS/price trade-off.

C. Simultaneous-play Game and Nash equilibrium

Pricing problem can be modelled as a simultaneous-play game between ISPs in which all of them aim to maximize their corresponding objective functions. A strategy profile is the vector containing the strategies of all players. Each strategy profile yields the payoffs to each player. We assume that there are two players in the game. If the payoff function for the *k*th player is $T_k(x)$, where $x = (x_1, x_k)$ is the set of decision variables and x_k is the decision variable of the *k*th player, then each player $k = 1, 2$ in this game has to maximize his payoff function:

$$\max_{x_k} T_k(x) \quad (1)$$

The solution of this competition can be achieved as Nash equilibrium [5], [6].

Definition: A pure strategy Nash equilibrium is a strategy profile from which no player has a unilateral incentive to change his strategy.

In other words, Nash equilibrium is a state of the game where no player prefers a different action if the current actions of other players are fixed. Nash equilibrium can be interpreted as the best action that each player can play based on the given set of actions of the other players. Each player cannot profit from changing his action, and because the players are rational, this is a "steady state" [5].

III. SERVICE COMPETITION AND PRICING IN NGN

A. User's Utility and Bandwidth Demand

A utility function which best models user behaviour is a generalization of the logarithmic function employed, tailored for a connection oriented setting. QoS is defined by bandwidth obtained from the ISP. Depending upon the quality of service requested, each user would require a minimum bandwidth γ. Fewer bandwidth than γ on average are of no utility to the user [2]. The law of diminishing marginal utility ensures that the user derives the same amount of satisfaction from any bandwidth more than the maximum π (Figure 1).

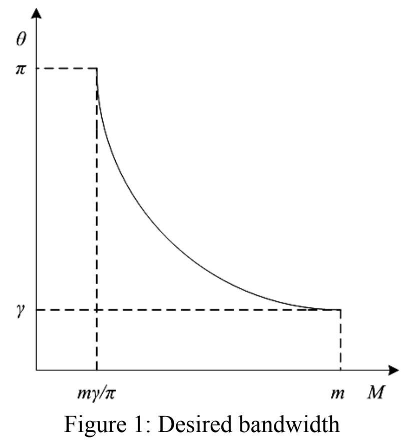

Figure 1: Desired bandwidth

It is considered that the user is willing to pay a maximum m per unit of bandwidth. When the ISP price M equals the maximal price m, the user will desire only the minimum acceptable bandwidth, γ. Any price beyond the maximal price reduces the user's desired bandwidth to zero. Over the interval $m\gamma/\pi \le M \le m$ the desired bandwidth θ decreases logarithmic with price with π. Dependence of desired bandwidth with price for arbitrarily chosen user is shown in Figure 1.

According to [2], user utility function can be shown to be:

$$U(\theta)=\begin{cases} m\theta, & \text{if } 0\le\theta\le\gamma \\ m\gamma\left(\log\left(\theta/\gamma\right)\right)+1, & \text{if } \gamma<\theta\le\pi \\ m\gamma\left(\log\left(\pi/\gamma\right)\right)+1, & \text{if } \pi<\theta \end{cases} \qquad (2)$$

This utility function (Figure 2) is concave and no decreasing. Also, U is strictly increasing on $[0,\infty)$ only when $\pi\to\infty$. The case of strictly elastic users can be obtained by setting $\gamma=0$ and $\pi\to\infty$, thereby rendering U strictly concave in $[0,\infty)$. Therefore, the utility function encompasses a wider spectrum of user behaviour by incorporating the range of user bandwidth requested [8].

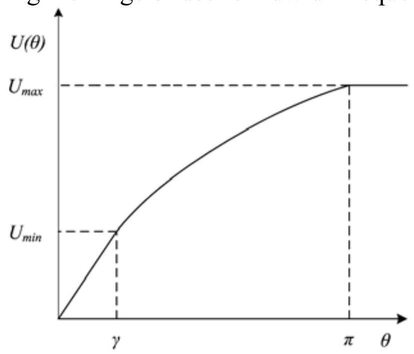

Figure 2: User utility as a function of bandwidth

We suppose that the shape of these functions (shown in Figures 1 and 2) is the same for all users, but parameters γ, π and m are different for different users.

In order to normalize user's utility function, we assume that for less bandwidth than γ user's utility is set to be zero and for more bandwidth than π user's utility is equal to 1. Then, this normalized user's utility function can be defined:

$$U_n=\begin{cases} 0, & \text{if } 0\le\theta\le\gamma \\ \dfrac{m\gamma\left(\log(\theta/\gamma)\right)+1}{m\gamma\left(\log(\pi/\gamma)\right)+1}, & \text{if } \gamma<\theta\le\pi \\ 1, & \text{if } \pi\le\theta \end{cases} \qquad (3)$$

The value of θ maximizing utility function (under conditions $0\le M\le m$ and $\gamma\le\theta\le\pi$) is:

$$\theta^*(M)=\frac{m\gamma}{M}. \qquad (4)$$

B. Competition model

User's demand D to accept a service is actually its satisfaction probability, which depends on the trade-off between QoS and price. Therefore, it is a function of normalized user utility U_n and price M. It can be defined as [8]:

$$D(M)=1-e^{-kU_n^A M^{-B}}, \qquad (5)$$

where k, A and B are positive constants that reflect the sensitivity of users' satisfaction to the QoS/price trade-off: k is a normalization constant, A indicates user's sensitivity to the QoS and B denotes user's sensitivity to the price. For example, increasing A makes the users more sensitive to the QoS, while increasing B does the same to the price. This equation is very general and it points the intuitive behaviour that the satisfaction of a user increases as the quality increases and/or the price decreases.

As distinct from parameters γ and π, which are QoS parameters, k, A and B are QoE parameters.

We consider the game in which two ISPs compete with each other to offer a NGN service to the users. We modelled this problem in a form of Nash game which is simultaneous-play game. We assume that the revenue of each ISP*k* is given by:

$$T_k=M_k\cdot\sum_{i=1}^{N}D_{ki}\left(M_1,M_2\right),\quad k=1,2. \qquad (6)$$

where M_1 and M_2 are prices offered by ISP_1 and ISP_2, respectively.

To include both service provider prices M_1 and M_2 in demand functions, we can formulate them in following way:

$$D_{1i}\left(M_1,M_2\right)=1-e^{-kU_{ni}^A M_1^{-B} M_2^{C}} \qquad (7)$$

$$\text{and } D_{2i}\left(M_1,M_2\right)=1-e^{-kU_{ni}^A M_2^{-2B} M_1^{C/2}} \qquad (8)$$

where C is a positive constant which indicates variation of the user's demand for the service offered by one ISP, depending of the price offered by the competing provider. We suppose that ISP_1 is a new provider in the market. Because of that users are more sensitive on change in price offered by ISP_1, comparing with the price offered by ISP_2. This is shown in Equations (7) and (8).

The range of partially elastic users interest for bandwidth is $\gamma\le\theta\le\pi$. Then, revenue functions for provider 1 and 2, respectively are given by:

$$\begin{aligned} T_1\left(M_1,M_2\right)&=M_1\cdot\left(1-\sum_{i=1}^{N}e^{-kU_{ni}^A M_1^{-B}M_2^{C}}\right)= \\ &M_1\cdot\left(1-\sum_{i=1}^{N}\exp\left(-k\left(\frac{m_i\gamma_i\left(\log(m_i/M_1)\right)+1}{m_i\gamma_i\left(\log(\pi_i/\gamma_i)\right)+1}\right)^{A}M_1^{-B}M_2^{C}\right)\right) \end{aligned} \qquad (9)$$

and

$$\begin{aligned} T_2\left(M_1,M_2\right)&=M_2\cdot\left(1-\sum_{i=1}^{N}e^{-kU_{ni}^A M_2^{-B}M_1^{C}}\right)= \\ &M_2\cdot\left(1-\sum_{i=1}^{N}\exp\left(-k\left(\frac{m_i\gamma_i\left(\log(m_i/M_2)\right)+1}{m_i\gamma_i\left(\log(\pi_i/\gamma_i)\right)+1}\right)^{A}M_2^{-2B}M_1^{C/2}\right)\right) \end{aligned} \qquad (10)$$

The best response of the ISP_1 can be obtained from the optimal price M_1^* for which revenue $T_1\left(M_1^*,M_2\right)$ is maximized, given the price M_2 offered by the ISP_2. Similarly, the best response of the ISP_2 is the optimal price M_2^* for which revenue $T_2\left(M_1,M_2^*\right)$ is maximized given the price M_1 offered by the ISP_1. This best response is denoted by $B_k(M_p) = \arg\max_{M_k} T_k(M_k, M_p)$, where M_p is the price offered by the other ISP. Nash equilibrium gives the set of prices such that none of the service providers can increase the revenue by choosing a different price, with the given price offered by the other service provider. This is the point where $B_1(M_2^*) = M_1^*$ and $B_2(M_1^*) = M_2^*$.

IV. Simulation Results

For the purpose of carrying out simulations of the competitive pricing model, we developed software in C Sharp. In Figure 3, application for determining Nash equilibrium in a simultaneous-play game is presented. Best responses namely, the best prices offered by one ISP to a NGN user with the given prices offered by the other ISP are shown in Figure 4. For model parameters, as shown in Figure 3, Nash equilibrium is obtained for $M_1 = 0.64$ and $M_2 = 0.78$. With the given price $M_1 = 0.64$, ISP_2 cannot increase his revenue by choosing a different price than $M_2 = 0.78$. The same stands for ISP_1: his best response to price $M_2 = 0.78$ is $M_1 = 0.64$, i.e. the price given by Nash equilibrium.

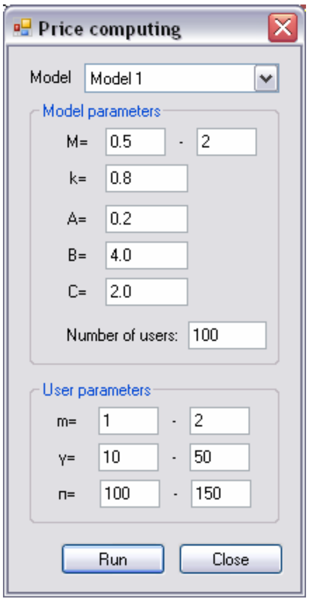

Figure 3: ISP computing prices interface

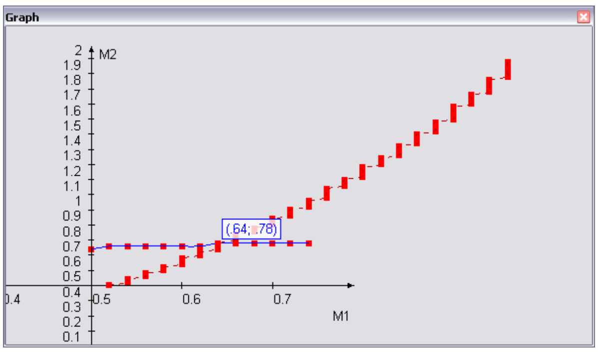

Figure 4: Best responses of both ISPs and Nash equilibrium

V. Conclusion

This paper considers one possibility for modelling the competition between two Internet Service Providers offering the same NGN service. We modelled this problem as a simultaneous-play game, assuming that both ISPs offer their prices at the same time. Both ISPs revenues depend on total users' demand for the service which is a function of acceptable QoS and QoE parameters. Furthermore, important contribution of this paper is defining distinction between QoS and QoE aspects. We supposed that reputations of ISPs are not the same, but the model also gives opportunity of changing market positioning of ISPs. The solution of this competition model is given by Nash equilibrium for which both ISPs are satisfied with the solution related to prices. We verified the proposed model through simulations with software solution especially developed for that purpose**.**

The important advantage of presented model is in stimulation of each user to choose the amount of available bandwidth to be charged for. At the same time, ISP considers users preferences, defined through QoS and QoE parameters. For solving the competition problem between ISPs, simultaneous-play game with Nash equilibrium proved to be a good scenario.

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