Investigations of Variable Fractional Delay Allpass Digital Filters

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Abstract – **In this paper a comparative study of two approaches to realize variable fractional delay allpass digital filters based on Thiran approximation is carried. After investigations of the structure and design complexity, the worst-case sensitivity, the tuning accuracy in real time and the behavior in a limited wordlength environment it is shown that the method proposed by the authors is considerably better. All the results are verified experimentally**

Keywords – **Digital allpass filter, Fractional phase delay, Variable fractional delay filters, Worst-case sensitivity.**

I. Introduction

Fractional delay (FD) filters are very useful in numerous applications [1] in digital signal processing and in telecommunications including time delay estimation, timing adjustment in digital modems, precise jitter elimination, asynchronous sample rate conversion and speech processing. In most of these applications the fractional delay value has to be tuned in real time and this is explaining the recent popularity of the variable FD filters [2]-[4].

Variable FD filters can be designed as both FIR and IIR realizations. The Farrow structure is the most popular variable FIR FD filters structure [2]. It allows control of the desired fractional delay by a single parameter and can be successfully used in low frequency applications where due to small coefficients changes the obtained accuracy is sufficient. But, in the most practical cases (high frequency applications) there is a need of much higher transfer function (TF) order to obtain the desired accuracy. The main disadvantage of all the FIR FD filters is that both the magnitude and the phase responses are varying from the desired ones when tuning the fractional delay.

The main advantage of the variable FD filters based on allpass structures is the reduced design complexity (the unity gain for all the frequencies allows concentrating only on phase response characteristics). Still, the TF order of a variable IIR structure satisfying the same phase delay requirements is considerably lower than that of the corresponding variable FIR filter. Although, the variable IIR filters have some disadvantages (complicated design procedure, higher round-off noise, possible instability and worst behavior in a limited wordlength environment) which must be taken into account, they are often preferred because of their lower complexity (less multipliers and adders) and their lower overall delay. The gathering structure [3][4] is the most popular variable allpass FD realization. It is based on Thiran approximation [5]. The design method is simple to use and gives a closed-form solution for the TF coefficients. The structure is derived from the direct form structure with transfer function coefficients represented as polynomials of the fractional delay parameter. The drawbacks of this structure are the long critical path, the large wordlength requirement for VLSI implementation (the transfer function coefficient differ in one or a few order) and the higher sensitivity (as of any direct-form structure).

It is well known that lower sensitivity to the multiplier coefficients will ensure higher accuracy and shorter word-length representation for a given accuracy. High accuracy of the tuning parameters, on the other hand, will speed up the computation and the update of the new tunable TF coefficients in real time. In order to obtain a higher FD time accuracy, we have proposed in [6] a new second order allpass section (called IS) which has lower sensitivity for the specific TF poles positions of Thiran-based allpass FD filters (for the most practical applications around z=0). In [7] we have proposed a new design method for variable allpass-based Thiran FD filters. It is based on truncated Taylor series expansion of the TF coefficients.

In this paper we compare variable FD filters obtained through both design procedures and structures by investigating the worst-case sensitivity, the hardware complexity and the accuracy of realization and tuning in a limited wordlength environment. All theoretical results obtained in this work are verified experimentally.

II. Variable FD with Gathering Structure

Let an N-th order allpass IIR filter has the following transfer function:

$$H_{AP}(z)=\frac{a_N+a_{N-1}z^{-1}+\ldots+a_1z^{N-1}+a_0z^{-N}}{a_0+a_1z^{-1}+a_2z^{-2}+\ldots+a_Nz^{-N}}. \qquad (1)$$

The coefficients a_N can be determined by Thiran formulas [5] as

$$a_k=(-1)^k\binom{N}{k}\prod_{n=0}^{N}\frac{d+n}{d+k+n}, for\ k=0,1,2\ldots N. \qquad (2)$$

In order to eliminate the division with a time-varying factor which is difficult to implement in hardware or software (due to too many multiplication and division operations) the authors in [3][4] transform the coefficients a_k to a polynomial form:

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$$\hat{a}_k = (-1)^k \binom{N}{k} \frac{\prod_{n=0}^{k-1}(d+n)}{\prod_{n=1}^{k}(d+N+n)} \prod_{n=1}^{N}(d+N+n)$$
$$= (-1)^k \binom{N}{k} \prod_{n=0}^{k-1}(d+n) \prod_{n=k+1\le N}^{N}(d+N+n) \qquad (3)$$
$$= \sum_{l=1}^{N} \hat{e}_{lk} d^l, for\ k = 1, 2 \ldots N.$$

Then an N-th order allpass IIR filter can be expressed as

$$H_{AP}(z) = \frac{g(d)[\hat{a}_N + \ldots + \hat{a}_1 z^{-(N-1)}] + z^{-N}}{1 + g(d)[\hat{a}_1 z^{-1} + \ldots + \hat{a}_N z^{-N}]}. \qquad (4)$$

The normalized coefficients $g(d)$ are approximated using the truncated Maclaurin series as [3][4]

$$g(d) = \frac{1}{\prod_{n=1}^{N}(d+N+n)}$$
$$\cong \frac{N!}{(2N)!} \prod_{n=1}^{N}\left[1 + \sum_{k=1}^{I}(-1)^k \left(\frac{d}{N+n}\right)^k\right], \qquad (5)$$

where I is the order of the approximating polynomial in terms of d.

The obtained through this method structure (called "gathering structure") in case of second order Thiran-based FD allpass filter ($N = 2$) and second order Maclaurin approximation ($I = 2$) is given in Fig. 1. Third order Maclaurin approximation ($I = 3$) adds to the structure the elements given in the figure with dashed lines. The corresponding TF coefficients are given in Table I.

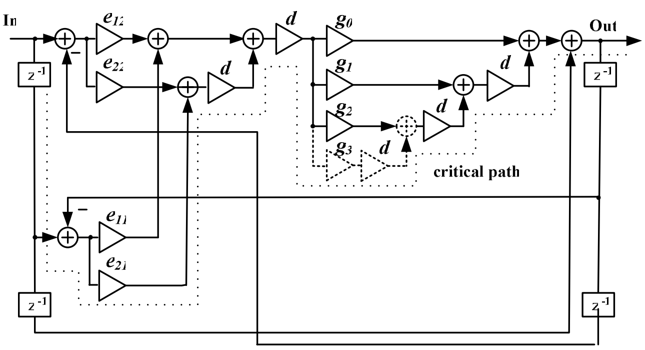


Fig. 1. Gathering structure for the variable FD allpass filter.

TABLE I
TF COEFFICIENTS OF GATHERING STRUCTURE

$\hat{a}_k$		$g(d)$
$\hat{e}_{11} = -8$	$\hat{e}_{12} = 1$	$g_0 = 0.083333$
$\hat{e}_{21} = -2$	$\hat{e}_{22} = 1$	$g_1 = -0.048611$
		$g_2 = 0.021412$
		$g_3 = -0.0084394$

As it can be seen the required wordlength is large because of the significant difference (about 10^2 for $N = 2$, $I = 2$ and about 10^3 for $N = 2$, $I = 3$) between the orders of the coefficients. The higher transfer function order (N) leads to a bigger difference between the smallest and the biggest coefficient (for example: the difference is about 10^5 for $N = 3$, $I = 3$).

III. Variable FD Realization with the Proposed Allpass Structure

In order to eliminate the division with a time-varying factor we propose in [7] to express each transfer function coefficients a_k (2) as a truncated Taylor series expansion with respect to d and it was shown that a truncation after the quadratic term was improving considerably the accuracy. Here we expand this method to the following steps:

1. Select a proper order of the transfer function ensuring a required flat part of the delay response.
2. Obtain an allpass FD TF using Thiran approximation.
3. Depending on the TF poles positions, proper allpass sections are selected in a way to ensure minimal sensitivities.
4. Depending on the required tuning accuracy and the initial value of FD, expand each TF coefficient in Taylor series and truncate it after the linear (for adjustment of the phase delay), square or cubic (for tuning of the phase delay) term.
5. Realize all the multiplier coefficients as composite multiplier realizations (see Figs. 3, 4).

The proposed design procedure is simple to use and the obtained structures have no critical path. The method can be applied for an arbitrary TF order and in the cases of first and second order TFs it is possible to implement structures different from direct form and to minimize the sensitivity of the realizations.

The real applications require small FD parameter values (in the range $-0.5 < d < 0.5$) corresponding to TF poles situated in the area around $z = 0$. Our previous investigations for second order allpass FD transfer functions [6] demonstrate that for the poles situated in that area the IS structure proposed by us in [6] and shown in Fig. 2 is the best choice (with the lowest worst-case sensitivity and the best behavior in a limited wordlength environment). Its transfer function is

$$H_{IS}(z) = \frac{b + (-a - 2b + ab)z^{-1} + z^{-2}}{1 + (-a - 2b + ab)z^{-1} + bz^{-2}}. \qquad (6)$$

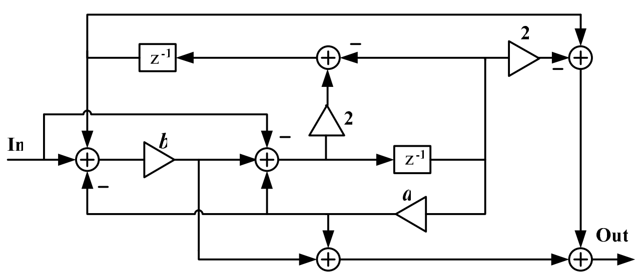


Fig. 2. IS allpass section.

The transfer function coefficients of the IS section realizing fractional delay with maximally flat group delay response can be expressed as

$$a = \frac{d}{d+2} \quad (7) \qquad b = \frac{d(d+1)}{(d+3)(d+4)}. \quad (8)$$

The representations of the coefficients a and b after using second and third order Taylor approximation are correspondingly:

$$a = \frac{1}{2}d - \frac{1}{4}d^2 \quad (9) \qquad b = \frac{1}{12}d + \frac{5}{144}d^2, \quad (10)$$

$$a = \frac{1}{2}d - \frac{1}{4}d^2 + \frac{1}{8}d^3 \quad (11) \quad b = \frac{1}{12}d + \frac{5}{144}d^2 - \frac{47}{1728}d^3. \quad (12)$$

All these coefficients have homogenous structure and can be realized as composite multipliers containing fixed and variable multipliers. The composite multiplier realizations for second and third order Taylor approximation are shown in Fig. 3 and Fig. 4.

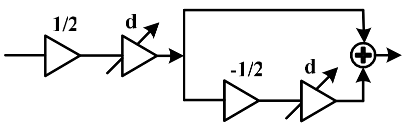


Fig. 3. Composite variable multiplier realization of a (7) after a second order Taylor approximation.

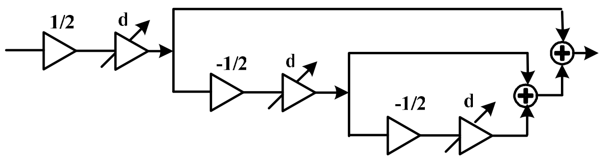


Fig. 4. Composite variable multiplier realization of a (7) after a third order Taylor approximation.

IV. Comparative Studies

We have investigated the phase sensitivity (using the package PANDA [8]) of the gathering and the proposed variable IS structure for first, second and third order Taylor approximation and for different FD parameter values in the range $-0.5 < d < 0.5$. The results for third order Taylor approximation ($I = 3$) and for some FD parameter values are given in Fig. 5. As it can be seen the sensitivity of the IS structure is approximately two times lower than that of the gathering structure for positive values of *d.* The same behavior of the sensitivity (as it is shown in the Fig. 5) is maintained for any of the cases (approximation order and FD parameter values) not given here.

In order to compare the tuning accuracy of both structures we consider the phase delay behavior in a limited wordlength environment for first, second and third order Taylor approximation of the coefficients. It was found that using the first order approximation for the two structures is not applicable because of the quite limited range of tuning.

The phase delay error of the two structures is larger than 0.01 of a single sample time-interval T near dc for any $d > |\pm 0.2|$. The investigations for the second order Taylor approximation show the possibility of tuning in much wider FD range with reasonably good results. The phase delay error is larger near $d = \pm 0.5$. A higher tuning accuracy is obtained by using third order approximation with the price of improvement readily acceptable in many practical cases. The results for the tuning of the phase delay for FD parameter values in the range of $-0.5 < d < 0.5$ in case of third order Taylor approximation and the TF coefficients quantization to 3 significant bits (in CSD code) are shown in Fig. 6. As it can be seen the proposed variable IS structure is behaving better than the gathering structure. The phase delay error of the gathering structure (in the cases shown in Fig. 6) for the worst case $d = 0.5$ and for the TF coefficient quantization to 3 significant bits is -0.0135T at dc while that of the proposed FD allpass filter is only 0.0098T. The difference is more significant for the same case and TF coefficients quantization to 2 significant bits (in CSD code) where the phase delay error of the gathering structure is -0.05T at dc while that of the proposed variable IS structure remains the same 0.0098T.

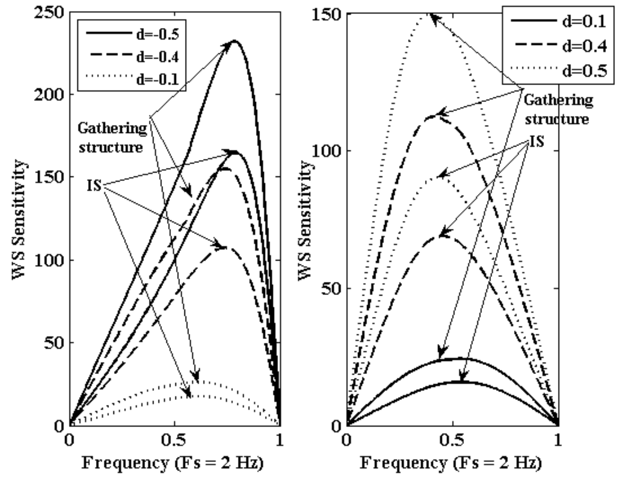
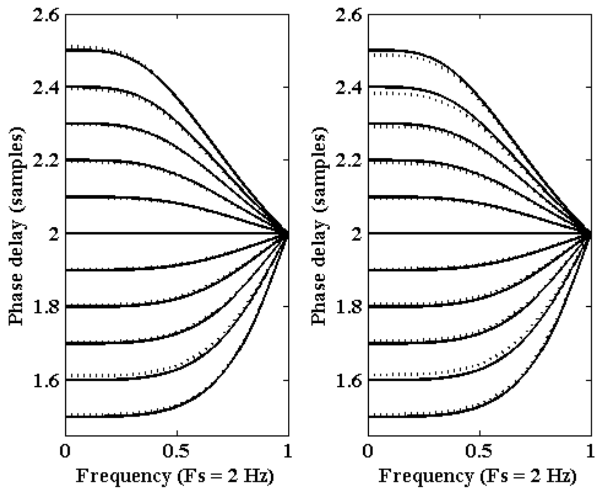


Fig. 5. Worst case sensitivity of the structures in case of third order Taylor approximation ($I = 3$).



a) variable IS structure b) gathering structure

Fig. 6. Tuning of the phase delay in case of third order Taylor approximation: ideal Thiran-based no variable non quantized FD filter (solid line); Thiran-based variable FD filters quantized to 3 bits (dotted line).

TABLE II
COMPARISON OF THE COMPLEXITY OF THE STRUCTURES

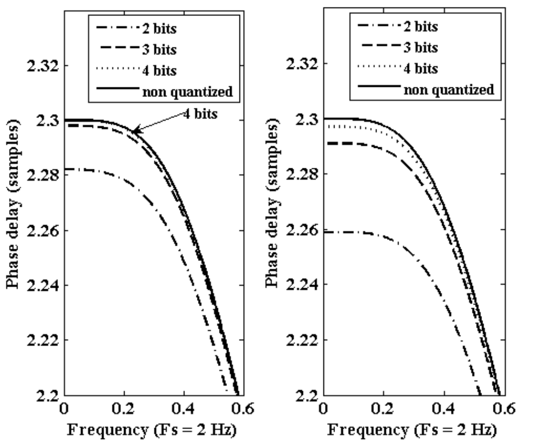
	Variable IS structure			Gathering structure		
	$I=1$	$I=2$	$I=3$	$I=1$	$I=2$	$I=3$
Multiplier	4	8	12	6	11	13
Adder	10	12	14	5	8	9
Delay element	2	2	2	4	4	4

The proposed variable IS FD structure has no critical path while the gathering structure has long critical path requiring 7 consecutive multiplications in case of second order TF and third order TF coefficients approximation. On the other hand, the complexity of the structure of the proposed variable IS FD filter is reduced, as it can be seen from Table II.

All these advantages of the proposed variable IS FD filter are very significant and they allow more precise tuning of the phase delay response in a limited wordlength environment obtained with much less elements (multipliers and delays).

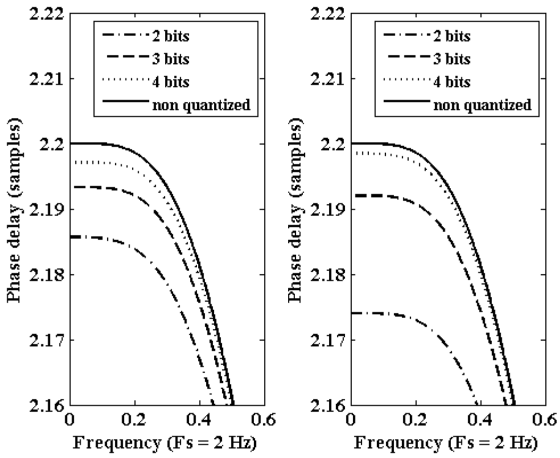
V. EXPERIMENTS

In order to observe and compare the tuning accuracy of the proposed variable IS FD and the gathering structure we have designed two second order allpass FD filters with third order TF coefficients approximation and a given fractional delay parameter value $d = 0.3$. The results are given in Fig. 7. Because of the low sensitivity of the IS structure the tuning accuracy is more precise than that of the gathering structure even when the TF coefficients are quantized to 2 significant bits (in CSD code). As it can be seen the deviations from the desired phase delay (0.3 samples) of variable IS FD filter near dc for 4, 3 and 2 bits are correspondingly smaller than 10^{-5}, -0.002 and -0.0179. The corresponding deviations of gathering structure are -0.0029, -0.009 and -0.289. The same behavior is observed for second order TF coefficients approximation as it is shown in Fig. 8 for a given fractional delay parameter value $d = 0.2$. Because of the lower sensitivities of the IS structure, its accuracy for different word-length is obviously better with exception of the case B=4 bit, due to the influence of the second-order Taylor coefficient approximation.



a) variable IS structure b) gathering structure

Fig. 7. Wordlength dependence of the accuracy of the phase delay for $d = 0.3$ in case of third order Taylor approximation.



a) variable IS structure b) gathering structure

Fig. 8. Wordlength dependence of the accuracy of the phase delay for $d = 0.2$ in case of second order Taylor approximation.

VI. CONCLUSION

A comparison of variable allpass FD filters obtained through two design procedures based on Thiran approximation is proposed in this paper. It was shown that the proposed method can be applied in the case of different from direct form first and second order structures and thus allows minimizing the sensitivity of the variable allpass FD filters. The obtained structure complexity is reduced and moreover, the obtained tuning accuracy is higher even in case of severe coefficients word-length representation.

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