Theory Approach and Method for Linear Antenna Array Design with Improved Selectivity

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Abstract – **In the article a theory for design of linear antenna array, based on approximation of "shifted" delta function, is shown. An appropriate basis function, which leads to obtaining narrow beam pattern, is offered. An analysis of antenna array's parameters and comparison with such of Dolph-Chebyshev's and Riblet's is done. An idea for practical realization of antenna array is given. With offered method, antenna array with better selectivity than those of Dolph-Chebyshev's and Riblet's, can be obtained.**

Keywords – **Approximation, Polynomial, Antenna array, Pattern, Phase response.**

I. Introduction

Antenna array are used for direct transmitting or receiving of electromagnetic power to definite direction. They consist of *n* equal uniform set elements. Basic parameters of antenna array are:

- general array shape (linear, circular, planar, etc.);
- element spacing *d* normalized against wave length λ ;
- element excitation amplitude I_n ;
- element excitation phase φ_n ;
- pattern of array element.

The radiation of antenna array is defined by the pattern multiplication theorem [1]:

Array pattern = Array element pattern x Array Factor.

The array Factor depends of general array shape, element excitation amplitude and element excitation phase. For linear antenna array it has the form of the function $\sin(x)/x$. The synthesis procedure reduces to obtaining the array factor.

Object of investigation in this article are linear array antennas with narrow-beam, low-sidelobe pattern. The most popular methods [2] are Schelkunoff's, Dolph–Chebyshev's Taylor's, Villeneuve's, Orchard's etc. Dolph–Chebyshev's method and its Riblet's modification [3] are with best properties.

In this article a method for linear antenna array design with odd number elements, which has better parameters then above mentioned is offered.

II. Theoretical Establishment

The theoretical base of the method is the Alternation Theorem:

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- If a function $f(x)$ is defined and continuous in the closed definition area it can be approximated by trigonometric polynomial $P_n(x)$ by power n, with basis function $\cos(x)$.
- The polynomial is the unique and best approximation, if the error function $w(x) = f(x) - P_n(x)$ has least $n+2$ extremes in the definition area.
- All extremes are alternatively and theirs modules are equal at positive number ε .

The idea of the method is an approximation of "shifted" delta function to be done

(1) $$\delta(\theta) = \begin{cases} 1, & \theta = \pi/2 \\ 0, & \theta \neq \pi/2 \end{cases}; \theta \in [0, \pi]$$

in the interval $[0, \pi]$ with polynomial, containing basis function

(2) $$y(\theta) = \cos\left[nC \arctan(kd\cos\theta) - \frac{n\pi}{2} \right].$$

In the above equation $k = 2\pi/\lambda$, θ is elevation angle between axis of antenna array and the direction of transmitted/received signal. The factor C is defined by

(3) $$C = \frac{\pi}{2}\arctan(kd).$$

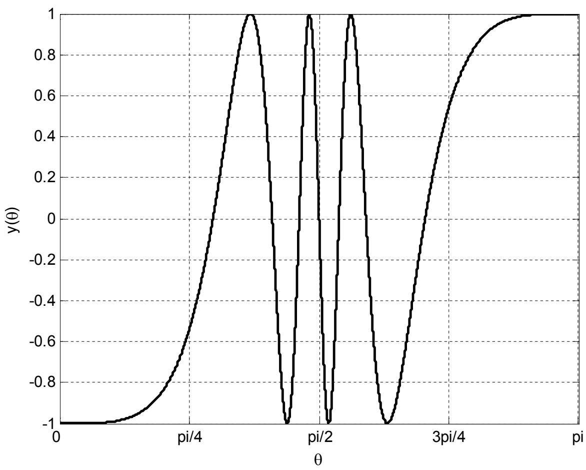

Fig.1. Basis function $n = 7$, $d = \lambda/2$

On Fig.1 is shown the function graphics by $n = 7$ and $d = \lambda/2$. From the figure it is seen that the function thickens its oscillations in the middle of the definition area, where the mainlobe lies. This property makes it appropriate for obtaining a narrow beam patterns. The polynomial is obtained by Remes' algorithm. It comprises an iterative computing of a system of $n+2$ linear equations. The obtained $n+2$ solutions are $n+1$ coefficients of the desired polynomial of power *n*, and the approximation error ε. The polynomial looks like this

(4) $P_n(\theta)=\sum_{m=1}^{n+1} b_m \cos\left[mC\arctan(kd\cos\theta)-\frac{m\pi}{2}\right].$

On Fig. 2 the approximation of "shifted" delta function with polynomial of power 7 is shown.

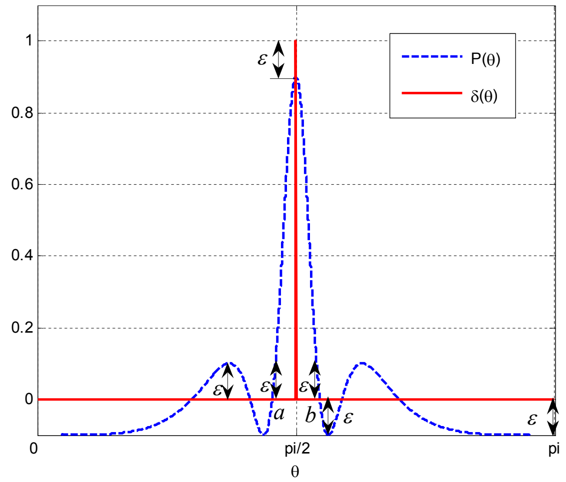

Fig.2. Approximation of "shifted" delta function with $P_7(\theta)$

The approximation is optimal, because is equiripple outside the mainlobe.

Excitation amplitude currents of the antenna array are obtained by nonzero coefficients of polynomial in following sequence

(5) $I_n=\frac{|b_{n+1}|}{2},\frac{|b_n|}{2},...,\frac{|b_2|}{2},|b_1|,\frac{|b_2|}{2},...,\frac{|b_n|}{2},\frac{|b_{n+1}|}{2}.$

The final form of array factor has complex function and looks like this

(6) $A_n(\theta)=\sum_{m=1}^{n} I_m \exp\left[j(n-m)C\arctan(kd\cos\theta)-j(n-m)\frac{\pi}{2}\right].$

Its module defines the antenna array's pattern and the argument – its phase response.

III. Analisis Antenna Array's Parameters. Comparison with Dolph-Chebyshev's and Riblet's Antenna Arrays

On Fig.3 antenna array's pattern with following initial data: number of elements $n=5$, element spacing $d=\lambda/2$; sidelobe attenuation $R=-20\text{dB}$ is shown. Normalized excitation amplitude currents are: $I_1=1$; $I_2=3.0087$; $I_3=4.1402$; $I_4=3.0087$; $I_5=1$. The circle with dashed line defines the angle $\Delta\theta_{3\text{dB}}$ of the pattern at the level -3dB. In the particular case $\Delta\theta_{3\text{dB}}=16.431^0$. In the Remes' algorithm, for every iteration are assigned $n+2$ points from the definition area of the argument θ, for which the graphics of approximation function must be at ε distance from approximated. This means, that $\Delta\theta_{3\text{dB}}$ can be varied by appropriate selection of the two closest points near the middle of the definition area $\pi/2$. On Fig.2 they are signed by a and b. The shrinking of the angle leads to increasing of the sidelobe level.

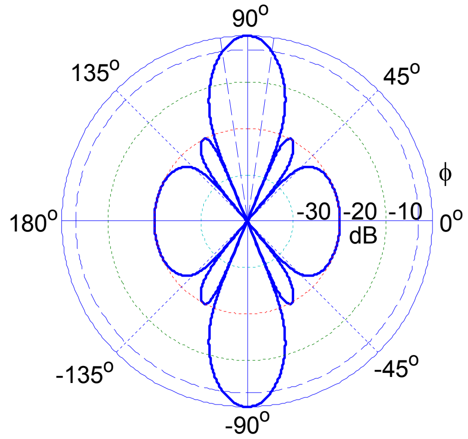

Fig.3. Offered method: $n=5$; $d=\lambda/2$; $R=-20\text{dB}$

Increasing the array element spacing d leads to significant improvement of its selectivity – shrinks the mainlobe and decreases the level of the sidelobes. On Fig.4 the pattern of array when $d=\lambda$ is shown.

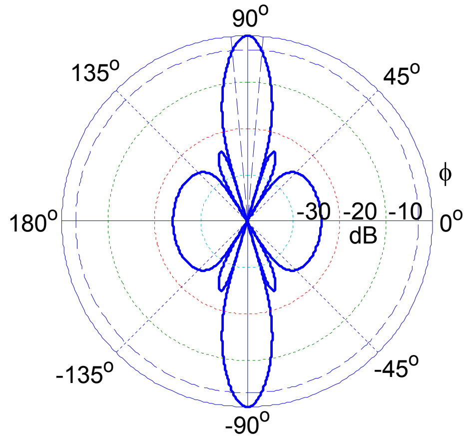

Fig.4. Offered method: $n=5$; $d=\lambda$; $R=-23.98\text{dB}$

In this case $R=-23.98\text{dB}$, and $\Delta\theta_{3\text{dB}}=9.648^0$. Normalized excitation amplitude currents are: $I_1=1$; $I_2=3.3463$; $I_3=4.746$; $I_4=3.3463$; $I_5=1$. The decreasing *d* leads to opposite effect.

On Fig.5 the pattern of Dolph-Chebyshev's antenna array with the same initial data like this on Fig.3 is shown.

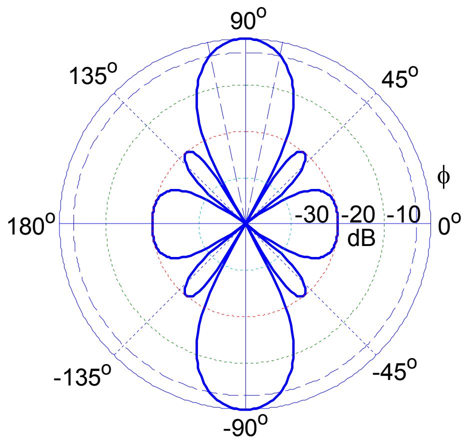

Fig.5. Dolph-Chebyshev: $n = 5$; $d = \lambda/2$; $R = -20\text{dB}$

The angle $\Delta\theta_{3\text{dB}} = 23.538^0$ is with 7.1^0 wider from this on the pattern from Fig.3. In this case with the offered method a narrower beam with 30.2% is obtained.

The Riblet's antenna arrays are with odd number of elements and have improved selectivity for $d < \lambda/2$. For higher values of d they have Dolph-Chebyshev's pattern. On the following two figures the patterns of antenna arrays, designed by the offered method and the Riblet's with initial data: $n = 5$; $d = \lambda/4$; $R = -20\text{dB}$ are shown.

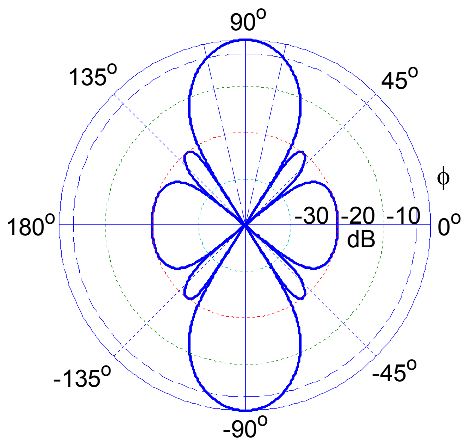

Fig.6. Offered method: $n = 5$; $d = \lambda/4$; $\Delta\theta_{3\text{dB}} = 25.632^0$

From the comparison can be seen that the pattern of Fig.6 is with better selectivity. The difference between the angles $\Delta\theta_{3\text{dB}}$ is 7.362^0. In this case with the offered method a narrower beam with 22.3% is obtained.

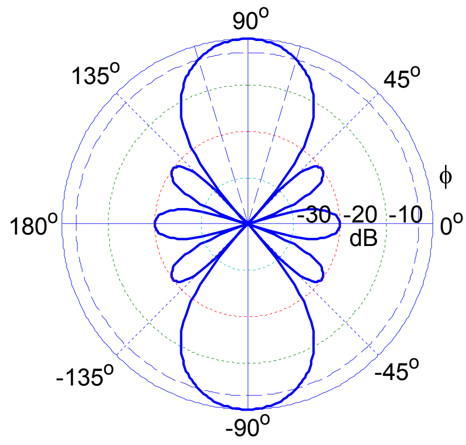

Fig.7. Riblet's method: $n = 5$; $d = \lambda/4$; $\Delta\theta_{3\text{dB}} = 32.994^0$

IV. Practical Realisation

On Fig. 8 an equivalent configuration for the determining of array factor is shown.

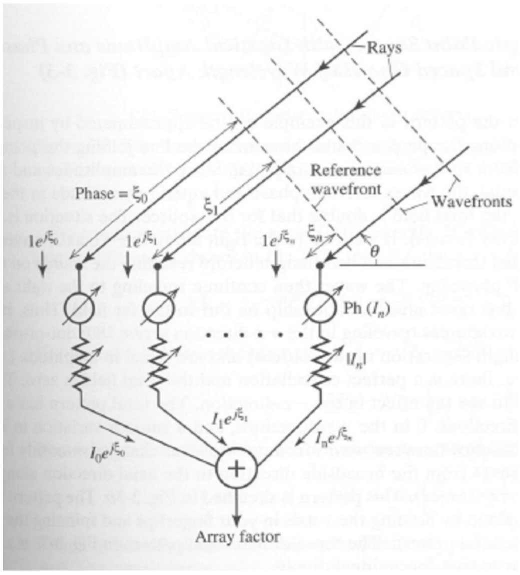

Fig.8

In previous part it was shown that the excitation amplitude currents $|I_n|$ are given by Remes' algorithm. The currents distribution realization is obtained with variable attenuators. Their signal attenuation will be inversely proportional to the excitation currents relation. From the equation (6) can be seen, that the function presenting phase's change takes the form

$$(7)\quad \varphi(\theta) = C\arctan(kd\cos\theta) - \frac{\pi}{2} = C\arctan(\psi) - \frac{\pi}{2},$$

where $\psi = kd\cos\theta$. On Fig. 9 a function graphic's is shown.

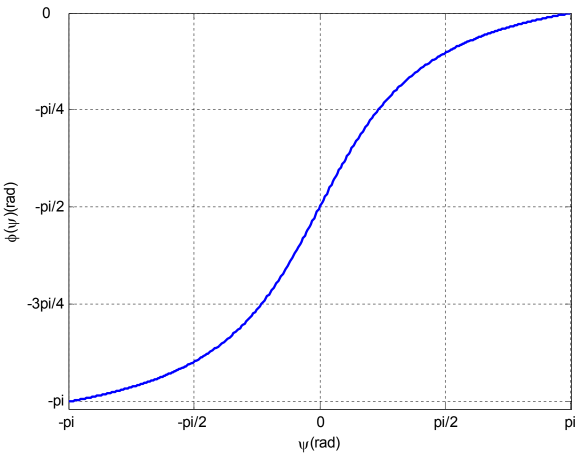

Fig. 9. Phase response

The implementation of the phase function can be done with phase modulators. It must be applied to modulation inputs simultaneously, with scanned frequency, a signal proportional to the change of angle $\psi = kd\cos\theta$.

V. Conclusion

With the offered theory and method for linear array antenna design optimal patterns with high selectivity are obtained. That is due to the specific properties of the offered basis function and the use of Remes' algorithm. Methods, with better properties than those based on Dolph-Chebyshev and Riblet can not be found in literature and that is why the offered theory is original. The theory and the method enrich the knowledge in the domain of antenna arrays synthesis and can find practical application.

References

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