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Non-Minimum Phase Filter Synthesis using Genetic Algorithms in MATLAB

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Abstract – **A novel approach for non-minimum phase filter synthesis is discussed. A MATLAB program, based on the Genetic Algorithm toolbox, minimizes the errors between the required magnitude and phase responses and the obtained ones.**

Keywords – **Non-Minimum Phase Filter, Frequency Detector (FD), MATLAB, Genetic Algorithms (GA), Modified Nodal Analysis (MNA).**

I. INTRODUCTION

The non-minimum phase systems are systems with zeros in the right half *s* plane. Their magnitude and phase response are not uniquely connected. This makes them suitable where the minimum phase filters can not ensure the desired magnitude and phase responses. Though, analytical methods for non-minimum phase filters synthesis are still missing. There are methods developed for the synthesis of a specific type of filters only – mainly all-pass [1]. In these cases, when a specific magnitude and phase response are required, the use of iterations is recommended [2].

There is a variety of examples for multiple variable optimizations in microelectronics based on the use of GA [3]. Passive circuit synthesis procedures were developed to generate passive filters for preliminary defined frequency responses [4-5]. This kind of developments gave us the idea to synthesize a non-minimum phase filter, using computer and GA.

The MATLAB environment is chosen as an industry standard for the passive filter circuit synthesis procedure development, based on the MATLAB Genetic Algorithm (GA) toolbox [6] that minimizes the errors between the required magnitude and phase responses and the obtained ones.

The main advantage of the GA is that they do not need preliminary information for to find a solution for a certain problem. Only the definition of the purpose function is important for realizing the effective global search.

In the present paper, the input data variables are the values of the three types of passive components – resistors, inductors and capacitors along with their corresponding circuit nodes' numbers.

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II. NON-MINIMUM PHASE FILTERS AND FREQUENCY DETECTORS

A. Frequency Detector and Frequency Locked Loops

Phase noise is of critical concern in communications, radar and other timing applications. Noise reduction techniques, based on feedback topologies, have been employed over the past few decades to improve phase noise of oscillators [7]. These topologies, called Frequency Locked Loop (FLL), consist of noisy Voltage Controlled Oscillator (VCO), Frequency detector (FD) and integrator, connected in a feedback loop Fig. 1. The sensitivity of the FD determines the phase noise reduction in FLL. The higher the sensitivity, the higher the phase noise reduction is. The FD used in [8] is based on a resonant cavity and has the characteristic, shown in Fig. 2. The tuning range of the VCO is ΔF and depends on the sensitivity of FD. The higher the sensitivity, the lower the ΔF is.

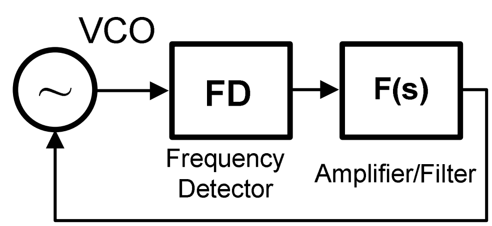

Fig. 1. Frequency Locked Loop

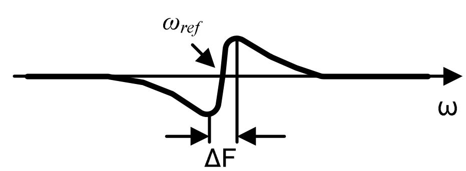

Fig. 2. FD characteristic

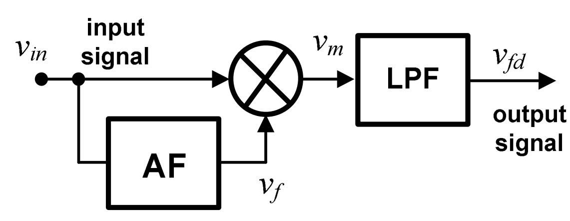

Fig. 3. Wideband Frequency Detector with an All Pass Filter

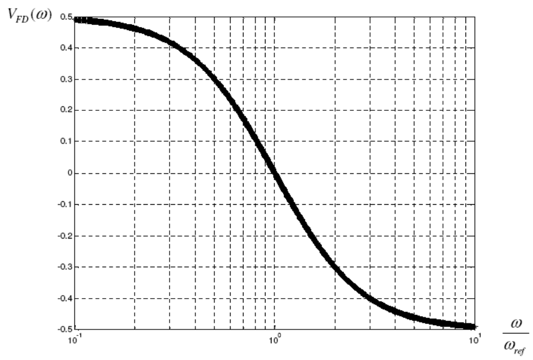

Fig. 4. Output characteristic of the FD

The frequency, where the FD output signal is zero, is called reference frequency ω_{ref} [8]. In the vicinity of this frequency, the sensitivity of the FD is greater. The operating frequency of the FLL is equal to ω_{ref}. If a variable operating frequency, greater than ΔF is needed ω_{ref} has to be changed, but the polarity of the FD output voltage trends to zero on both sides of the ΔF limit. This involves acquisition problems, because during the start-up, the VCO frequency could be outside ΔF.

B. Wideband Frequency Detector

To solve the problem with frequency acquisition we propose a Wideband FD (WFD) [8] with characteristic shown in Fig. 4. The output signal of the WFD is positive, when the input frequency is below ω_{ref}, and negative, when the input frequency is above ω_{ref}. In this way the VCO locks on to the ω_{ref} independently on its initial frequency.

The WFD consists of a mixer, an All Pass Filer (AF) and a

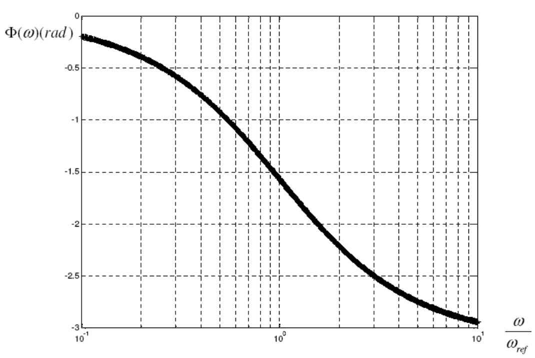

Fig. 5. Phase response of the AF

Low Pass filter (LPF) – Fig. 3. The AF has the following non-minimum phase transfer function:

$$H_{AF} = \frac{\omega_{ref} - s}{\omega_{ref} + s} \quad (1)$$

The signal at the output of the AF is described by Eq. 2:

$$\begin{aligned} v_m(t) &= v_{in} * v_f = a\sin(\omega t) * b\sin(\omega t + \phi) \\ &= 0,5ab\left[\cos(-\phi) + \cos(2\omega t + \phi)\right] \end{aligned} \quad (2)$$

The LPF at the output of the mixer removes the high frequency components. At the output of the WFD remains only the DC value which depends on the input frequency. The phase response of the AF determines the output voltage of the WFD

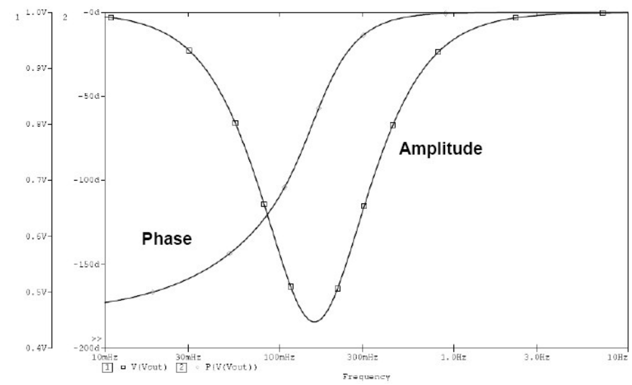

Fig. 6. Magnitude and phase responses of the proposed non-minimum phase filter

$$\begin{aligned} &V_{FD}(\omega) = 0,5AB\sin(-\Phi(\omega)) \\ &\Phi(\omega) = -2arctg\left(\frac{\omega}{\omega_{ref}}\right) \end{aligned} \quad (3)$$

From Eq. 3 follows:

$$V_{FD}(\omega) = 0,5AB\cos\left[-2arctg\left(\frac{\omega}{\omega_{ref}}\right)\right] \quad (4)$$

The AF does not change the amplitude of the signal. Hence, A=B. In Fig. 5 the phase response of AF is shown and in Fig. 4 - the frequency characteristic of FD.

The output voltage of the WFD depends on the phase response only. In some articles [9-10], in order to increase the sensitivity, the magnitude response of the frequency dependent component (resonant cavity) is used in addition to the phase response. We used this approach in order to improve the sensitivity of the WFD.

C. Wideband FD with improved sensitivity

If somehow we manage to change the characteristic of the AP so that the amplitude response trends to zero at ω_{ref}, the sensitivity will increase. The magnitude and the phase responses of the proposed filter are shown in Fig. 5. After a few iterations [8] we found a function with amplitude and a phase response very close to the desired ones, where ζ is the damping factor:

$$H_{NF}=\frac{s^2+2\zeta\omega_{ref}s+\omega_{ref}^2}{s^2+2\zeta\omega_{ref}s-\omega_{ref}^2} \qquad (5)$$

This function (5) has poles in the right half ***s*** plane. Hence, it is unrealizable. Our attempts to find a realizable function with a similar response have failed. Thus we decided to use genetic algorithms. By using them, we expected to find a circuit with a magnitude and a phase response close to these, shown in Fig. 6.

III. GA Passive Circuit Synthesis Procedure Description and Parameters

A. Purpose function

A GA approach can be applied to minimize the difference between the preliminary defined magnitude and phase responses and the obtained ones. The idea is to compare the magnitude and the phase responses, obtained using the transfer function equation and the magnitude and phase responses, obtained when the model parameter values are varied in a certain range. The actual comparison is done in the purpose function, where the absolute values for every frequency point of the magnitude and the phase responses are compared in one expression, using the sum of the least squares values:

$$G_{fun}=W_M\sum_{i=1}^{n}\left[\left|\left(M(f_i)-M^{(m)}(f_i)\right)\right|^2\right]+ W_P\sum_{i=1}^{n}\left[\left|\left(P(f_i)-P^{(m)}(f_i)\right)\right|^2\right] \qquad (6)$$

, where: $M^{(m)}$ – required magnitude response values;

$M_{.}$ - obtained magnitude response values during synthesis;

W_M - magnitude response weight coefficient;

$P^{(m)}$ – required phase response values;

$P_{.}$ - obtained phase response values during synthesis;

W_P - phase response weight coefficient;

f_i - current frequency point;

n – number of the frequency points.

B. General structure

The values of the components (resistors, inductors or capacitors) and the number of nodes, where each component is connected in the circuit, are the input data for the GA synthesis procedure. The procedure body contains two *for* cycles. The first cycle runs the calculations for every frequency point until the end, and the second cycle runs the calculations for every individual for a given frequency point until the end. The MNA circuit matrix equations are formulated (represented in the MATLAB program) automatically from the input data on condition that the type and the number of the components are fixed to optimize the search space of the GA. Once formulated, the MNA circuit equations are solved for every individual.

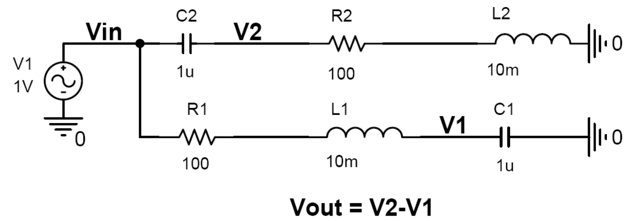

Fig. 7. The circuit described with Eq. 7.

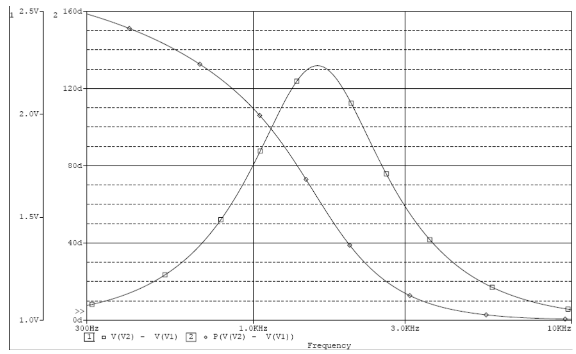

Fig. 8. Magnitude and phase responses (Eq. 7).

IV. Results

The GA circuit synthesis procedure is done using 1000 to 3000 iterations, a generation gap of 0.7 for a population of 200 individuals and 200 frequency points. Two experiments were done – GA passive circuit synthesis, using realizable transfer function (7) and the transfer function (5).

A. Experiment to obtain realizable transfer function (7)

At first we tested the ability of GA to synthesize non-minimum phase filters. As a purpose function we used (7).

$$H_{NF}=\frac{s^2-2\zeta\omega_{ref}s-\omega_{ref}^2}{s^2+2\zeta\omega_{ref}s+\omega_{ref}^2} \qquad (7)$$

We chose this function because it is a second order function – like Eq. 5. Eq. 7 has the circuit, shown in Fig. 7. The values of the components are estimated for ζ=0,5 and ω_{ref}=10krad/s. The magnitude and the phase responses are shown in Fig. 8. The GA passive circuit synthesis procedure run, using the following constraints:

- maximum generations: 1000
- number of nodes: 6
- number of components: 6 (2-resistors, 2-inductors, 2-capacitors)
- components' values ranges – resistors (1Ω - 500 Ω), inductors (1nH – 100mH), capacitors (1nF – 1mF)
- W_M = 10000; W_P = 1.

The obtained circuit (Fig. 9) is fully compatible with the requirements and shows excellent performance (Fig. 10). It is obvious from the purpose function value optimization in Fig. 11 that the circuit can be further optimized, but this is not necessary for this case.

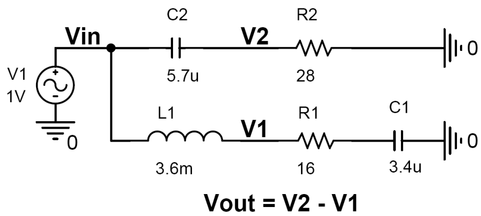

Fig. 9. Synthesized realizable transfer function

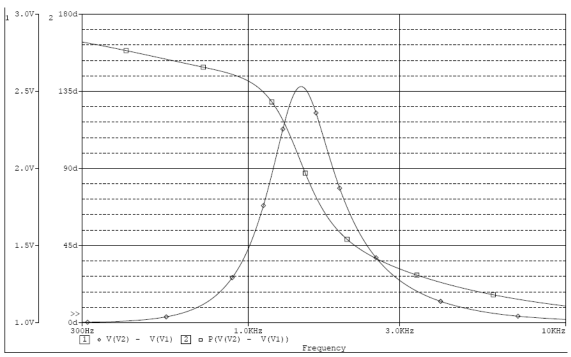

Fig. 10. Magnitude and phase response of the Synthesized Circuit

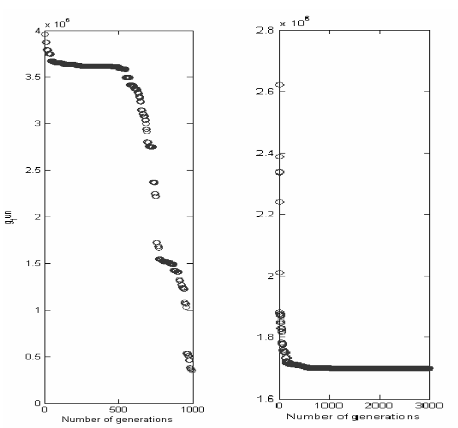

Fig. 11. Purpose function value during the GA synthesis procedure: experiment to obtain (7) (left) and experiment to obtain (5) (right)

B. Experiment to obtain the transfer function (5)

The GA constrains are enlarged for this experiment, to cover more search space:

- maximum generations: 3000
- number of nodes: 15
- number of components: 15 (5-resistors, 5-inductors, 5-capacitors)
- components' values ranges – resistors (1Ω - 5000 Ω), inductors (1μH – 300mH), capacitors (10pF – 300μF)
- $W_M = 100000$; $W_P = 1$.

The obtained circuit is not compatible with the requirements. It is obvious from the purpose function value optimization in Fig. 11, that the circuit can not be further optimized although the components' values are still in the preliminary defined ranges.

V. Conclusion

In this paper we report a wideband frequency detector with improved sensitivity. The key element in the detector is a non-minimum phase filter with unique magnitude and phase response. A novel method based on GA, for synthesis of non-minimum phase filters was proposed. Two different functions were synthesized. The algorithm successfully found solution for realizable functions. In the case of unrealizable functions, finding compatible circuit has failed.

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