Application of Genetic Algorithms in MATLAB to Parameter Extraction Errors Minimization of Wide-Band On-Chip Spiral Inductor Model

Vladislav Durev[1]

Abstract – **An automated procedure for minimizing the parameter extraction errors of the on-chip wide-band spiral inductor model is developed in this paper. A MATLAB program based on the Genetic Algorithm toolbox minimizes the errors between the measured and the extracted Y-parameters of the spiral inductor model using symbolic analysis to construct the expressions needed. The proposed algorithm shows excellent agreement with the measured data over the whole frequency range.**

Keywords – **On-Chip Spiral Inductor, Parameter Extraction, MATLAB, Genetic Algorithms (GA), Modified Nodal Analysis (MNA)**

I. INTRODUCTION

The on-chip spiral inductor is a very important constructive component of the contemporary CMOS microelectronics. In the CMOS SoC RFs the use of the spiral inductors in designs like VCOs, mixers, RF amplifiers, impedance-matching circuits is widespread. Many papers, devoted on the on-chip spiral inductors modeling were published in the recent years. The wide-band spiral inductor model, presented in [1] is simple and has an excellent accuracy compared to the measured results. The main advantage of the model is that its model parameters are not frequency dependent.

The on-chip spiral inductor accurate systematic model parameter extraction [2] gives very accurate results when applied. Despite that, every parameter extraction procedure generates errors in the extracted parameters. Even the accurate extraction procedure in [2] has small errors comparing the obtained Y-parameters to the measured ones. It is often that a post-extraction optimization procedure is needed to refine the obtained parameters to be closer to the measured ones.

The MATLAB environment is chosen as an industry standard for the optimization procedure development, based on the MATLAB Genetic Algorithm (GA) toolbox [3] that minimizes the errors between the measured and the extracted Y-parameters of the spiral inductor model from [1] using symbolic analysis with respect to the model parameters.

The main advantage of the GA is that they do not need preliminary information or other knowledge for the investigated problem. Only the definition of the purpose

[1]Vladislav Durev is with the Department of Electronics at Technical University of Sofia, 8 Kl. Ohridski Blvd, Sofia 1000, Bulgaria, E-mail: v_p_durev@yahoo.com.

function is important for realizing the effective global search. There is a variety of examples for multiple variable optimizations in microelectronics based on the use of GA [4]. The geometric technology parameters for a given technology can be optimized to achieve given electrical characteristics [5] and the parameter extraction itself can be realized in some cases using GA [6].

In the present paper, the input data variables are the values of the model parameters from the enhanced model of the spiral inductor, presented in [1]. The ranges of the input variables are defined as a percentage of the values, obtained after the application of the extraction procedure, described in [2]. The optimization procedure makes the difference between the measured and the obtained Y-parameters minimal and thus minimizes the corresponding full range average errors.

II. SPIRAL INDUCTOR MODEL ANALYSIS IN MATLAB

A. Basic circuit analysis

The enhanced model of spiral inductor [1] can be treated as a parallel combination of an upper and a lower subcircuits (Fig. 1). Because of the DC blocking property of the oxide capacitors C_{ox1} and C_{ox2}, the spiral inductor model can be separated into two parts: upper subcircuit and lower subcircuit.

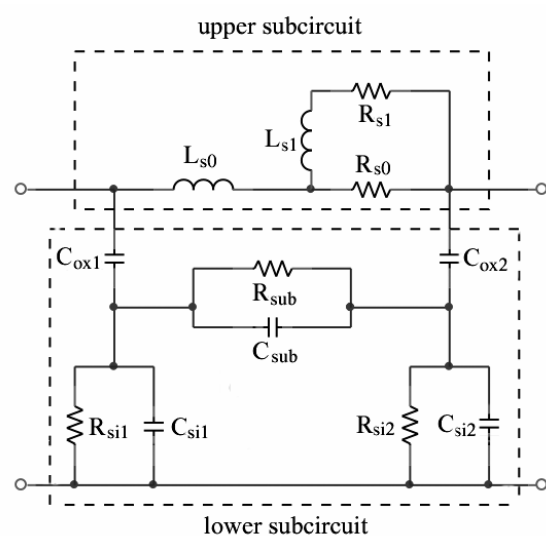The inductor can be approximately characterized by the

Fig. 1. The enhanced model of spiral inductor from [1]

upper subcircuit for lower frequencies and by the lower subcircuit for high frequencies. The model can be simplified from the analysis point of view, if presented using the equivalent schematic, shown in Fig. 2.

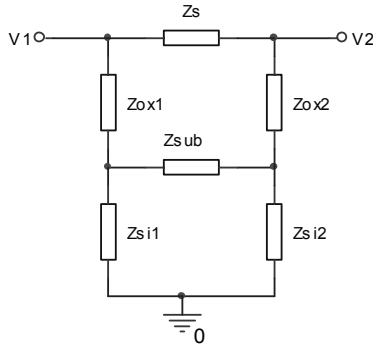The following expressions can be written for the model, shown in Fig. 2:

Fig. 2. Equivalent schematic of the enhanced spiral inductor model

$$Z_{Ls0} = sL_{s0}\,;\ Z_{s1} = R_{s1} + sL_{s1}\,;\ Z_{Rs0} = R_{s0} \qquad (1)$$

$$Z_s = Z_{Ls0} + \frac{Z_{s1}.Z_{Rs0}}{Z_{s1} + Z_{Rs0}} \qquad (2)$$

$$Z_{ox1,2} = 1/sC_{ox1,2} \qquad (3)$$

$$Z_{Rsi1,2} = R_{si1,2}\,;\ Z_{Csi1,2} = 1/sC_{si1,2} \qquad (4)$$

$$Z_{si1,2} = \frac{Z_{Rsi1,2}.Z_{Csi1,2}}{Z_{Rsi1,2} + Z_{Csi1,2}} \qquad (5)$$

$$Z_{Rsub} = R_{sub}\,;\ Z_{Csub} = 1/sC_{sub} \qquad (6)$$

$$Z_{sub} = \frac{Z_{Rsub}.Z_{Csub}}{Z_{Rsub} + Z_{Csub}} \qquad (7)$$

*B. Circuit analysis with respect to Y-parameters*

The *Y*-parameters can be found using equations (2), (3), (5) and (7). The corresponding schematics are shown in Fig. 3 for both cases – $V_2 = 0$ (Fig. 3a) and $V_1 = 0$ (Fig. 3b). According to the two-port definition for the *Y*-parameters, Y_{11} and Y_{21} are defined for the case in Fig. 3a, and Y_{22} and Y_{12} – for the case, shown in Fig. 3b.

The equivalent impedance Zeq_{Y11} for the schematic, shown in Fig. 3a can be found, using the following equations:

$$Z_{ox2si2} = \frac{Z_{ox2}.Z_{si2}}{Z_{ox2} + Z_{si2}}\,;\ Z_{subox2si2} = Z_{ox2si2} + Z_{sub} \quad (8)$$
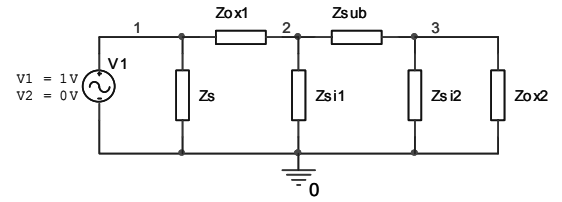
(a)
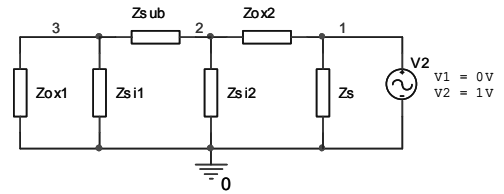
(b)

Fig. 3 Representation of the Y-parameter analysis:

(a) V2 = 0; (b) V1 = 0

$$Z_{subox2si2si1} = \frac{Z_{si1}.Z_{subox2si2}}{Z_{si1} + Z_{subox2si2}} \qquad (9)$$

$$Z_{subox2si2si1ox1} = Z_{ox1} + Z_{subox2si2si1} \qquad (10)$$

$$Z_{eqY11} = \frac{Z_s.Z_{subox2si2si1ox1}}{Z_s + Z_{subox2si2si1ox1}} \qquad (11)$$

Then, using (11), the following expression for Y_{11} can be written:

$$Y_{11} = V_1 / Z_{eqY11} \qquad (12)$$

Because of the symmetry of the model from Fig. 2, for the equivalent impedance Zeq_{Y11} and Y_{22} in the case, shown in Fig. 3b, when $V_1 = 0$, the following expressions are valid, using equations (11) and (12):

$$Z_{eqY22} = \frac{Z_s.Z_{subox1si1si2ox2}}{Z_s + Z_{subox1si1si2ox2}} \qquad (13)$$

$$Y_{22} = V_2 / Z_{eqY22} \qquad (14)$$

Y_{21} can be easily expressed if the voltage U_{Zox2} over Z_{ox2} in Fig. 3a is known. For this case this is the voltage between node number 3 and the ground. It can be symbolically expressed using the symbolic analysis possibilities in MATLAB. If the input current, node voltages and admittances in Fig. 3a are declared as symbols using *syms*, the *Modified Nodal Analysis* (MNA) circuit matrix equation can be solved and for node number 3 the following expression can be written:

$$U_{Z_{ox2}} =$$

$$\frac{Y_{ox1}.Y_{sub}}{\left[(Y_{ox1} + Y_{si1}).(Y_{sub} + Y_{ox2} + Y_{si2}) + Y_{sub}.(Y_{ox2} + Y_{si2})\right]} \quad (15)$$

In order to obtain Y_{21}, the currents I_{Zox2} and I_s are expressed, using the following equations:

$$I_{Z_{ox2}} = U_{Z_{ox2}}/Z_{ox2} \; ; \; I_s = V_1/Z_s \quad (16)$$

Using the expressions from (16) and the Y_{21} two-port definition, Y_{21} is expressed in the form:

$$Y_{21} = -\left(I_s + I_{Z_{ox2}}\right)/V_1 \quad (17)$$

Because of the symmetry of the model from Fig. 2, for Y_{12} in the case, shown in Fig. 3b when $V_1 = 0$, the following expression is valid, using equation (17):

$$Y_{12} = -\left(I_s + I_{Z_{ox1}}\right)/V_2 \quad (18)$$

III. Minimization the Parameter Extraction Errors Using GA

A. Purpose function and general structure

Once the basic circuit analysis and *Y*-parameter expressions are present, a GA approach can be applied to minimize the errors, coming from the parameter extraction procedure. The idea is to compare the *Y*-parameters, obtained for the model parameter values from the parameter extraction and the *Y*-parameters, obtained when the model parameter values are varied in a certain range. The actual comparison is done in the purpose function, which compares the absolute value for every frequency point of every two-port *Y*-parameter in one expression, using the sum of the least squares values:

$$G_{fun} = \sum_{i=1}^{n} \left| \left(Y_{jk}(f_i) - Y^{(m)}{}_{jk}(f_i) \right) \right|^2 \quad (19)$$

where j,k = 1,2;

$Y_{jk}^{(m)}$ – measured *Y*-parameters;

Y_{jk} - simulated *Y*-parameters of the model;

n – number of the measured/simulated frequency points

The values of the model parameters are ranged in a certain predefined range. The GA optimization is done using 1000 iterations, generation gap of 0.7 for population of 200 individuals. The function body contains two *for* cycles. The first cycle runs the calculations for every frequency point until the end (*DATA_ROWS*), and the second cycle runs the calculations for every individual for a given frequency point until the end (*Nind*).

B. Optimization procedure realized in MATLAB

The representation of equations (1 - 19) and the construction of the optimization procedure in MATLAB are shown below:

```
    for i = 1:DATA_ROWS %Calculate the frequency response of the circuit
        s = j*2*pi*frequency(i);

    for ix = 1:Nind

        ZLs0 = s.*Ls0(ix);
        Zs1 = Rs1(ix) + s.*Ls1(ix);
        ZRs0 = Rs0(ix);
        Zs = ZLs0 + ((Zs1.*ZRs0)./(Zs1 + ZRs0));

        Zox1 = 1./(s.*Cox1(ix));
        Zox2 = 1./(s.*Cox2(ix));

        ZRsi1 = Rsi1(ix);
        ZCsi1 = 1./(s.*Csi1(ix));
        Zsi1 = (ZRsi1.*ZCsi1)./(ZRsi1 + ZCsi1);

        ZRsi2 = Rsi2(ix);
        ZCsi2 = 1./(s.*Csi2(ix));
        Zsi2 = (ZRsi2.*ZCsi2)./(ZRsi2 + ZCsi2);

        ZRsub = Rsub(ix);
        ZCsub = 1./(s.*Csub(ix));
        Zsub = (ZRsub.*ZCsub)./(ZRsub + ZCsub);

        %Calculate Y11, U1 = 1, U2 = 0
        Zox2si2 = (Zox2.*Zsi2)./(Zox2 + Zsi2);
        Zsubox2si2 = Zox2si2 + Zsub;
        Zsubox2si2si1 = (Zsi1.*Zsubox2si2)./(Zsi1 + Zsubox2si2);
        Zsubox2si2si1ox1 = Zox1 + Zsubox2si2si1;
        ZeqY11 = (Zs.*Zsubox2si2si1ox1)./(Zs + Zsubox2si2si1ox1);
        Y11(ix, 1) = U1./ZeqY11;

        %Calculate Y22, U1 = 0, U2 = 1
        Zox1si1 = (Zox1.*Zsi1)./(Zox1 + Zsi1);
        Zsubox1si1 = Zox1si1 + Zsub;
        Zsubox1si1si2 = (Zsi2.*Zsubox1si1)./(Zsi2 + Zsubox1si1);
        Zsubox1si1si2ox2 = Zox2 + Zsubox1si1si2;
        ZeqY22 = (Zs.*Zsubox1si1si2ox2)./(Zs + Zsubox1si1si2ox2);
        Y22(ix, 1) = U2./ZeqY22;

        Ysub = 1./Zsub;
        Yox1 = 1./Zox1;
        Yox2 = 1./Zox2;
        Ysi1 = 1./Zsi1;
        Ysi2 = 1./Zsi2;

        %Calculate Y12, U1 = 0, U2 = 1
        %Expressions taken from symmetry considerations with Y21
        UZox1 = (Yox2.*Ysub)./((Yox2 + Ysi2).*(Ysub + Yox1 + Ysi1) + Ysub.*(Yox1 + Ysi1));
        IZox1 = UZox1./Zox1;
        Is = U2./Zs;
        I1 = Is + IZox1;
```

```
        Y12(ix, 1) = -I1./U2;

        %Calculate Y21, U1 = 1, U2 = 0
        %Expressions taken from the symbolic extraction of
Y21
        UZox2 = (Yox1.*Ysub)./((Yox1 + Ysi1).*(Ysub +
Yox2 + Ysi2) + Ysub.*(Yox2 + Ysi2));
        IZox2 = UZox2./Zox2;
        Is = U1./Zs;
        I2 = Is + IZox2;
        Y21(ix, 1) = -I2./U1;
    end

    %Least squares sum of the difference between the
modules of the required and the current Y-parameters
    g_fun = g_fun + ((abs(Y11) - abs(Y11_req(i)))).^2 +
((abs(Y12) - abs(Y12_req(i)))).^2 + ((abs(Y21) -
abs(Y21_req(i)))).^2 + ((abs(Y22) - abs(Y22_req(i)))).^2;

    end
```

IV. RESULTS

The presented procedure for optimization the parameter extraction results of wide-band on-chip spiral inductor model in MATLAB was verified according to the published data in [1, 2]. The average error over the frequency range for the modules of the extracted and the measured Y-parameters is used to estimate the accuracy of the optimization procedure for various geometry RF spiral inductors. The maximal full range average error is calculated over the modules of the Y-parameters, using the formula (20):

$$AvgErr_Y = \max_i \left\{ 100 \left| \left| 1 - \left| \frac{Y_{jk}(f_i)}{Y_{jk}^{(m)}(f_i)} \right| \right| \right| \right\} \qquad (20)$$

where j,k = 1,2;

$Y_{jk}^{(m)}$ - measured Y-parameters;

Y_{jk} - simulated Y-parameters of the model.

The obtained results for the full range average errors from the extraction procedure and after the optimization procedure are compared and the improvements for every Y-parameter are shown in Table 1. For example, the extracted values for the case of *4.5 x 30 x 14.5 x 2* geometry (Table 1) are: $R_{s0} = 6.680\ \Omega$, $R_{s1} = 7.588\ \Omega$, $L_{s0} = 3.02$ nH, $L_{s1} = 1.183$ nH, $C_{ox1} = 119.57$ fF, $C_{ox2} = 112.745$ fF, $R_{si1} = 291.376\ \Omega$, $R_{si2} = 286.831\ \Omega$, $C_{si1} = 34.133$ fF, $C_{si2} = 32.993$ fF, $R_{sub} = 946.544\ \Omega$ and $C_{sub} = 72.409$ fF.

The corresponding average errors are: $AvgErr_{Y11} = 0.254$ %, $AvgErr_{Y12} = 0.062$ %, $AvgErr_{Y21} = 0.062$ %, $AvgErr_{Y22} = 0.239$ %. After the GA optimization, the following model parameter values are obtained:
$R_{s0} = 6.697\ \Omega$, $R_{s1} = 7.558\ \Omega$, $L_{s0} = 3.02$ nH, $L_{s1} = 1.184$ nH, $C_{ox1} = 119.776$ fF, $C_{ox2} = 112.479$ fF, $R_{si1} = 292.638\ \Omega$, $R_{si2} = 285.733\ \Omega$, $C_{si1} = 33.997$ fF, $C_{si2} = 32.958$ fF, $R_{sub} = 944.4\ \Omega$ and $C_{sub} = 72.516$ fF.

The corresponding average errors after the GA optimization are: $AvgErr_{Y11} = 0.099$ %, $AvgErr_{Y12} = 0.041$ %,

$AvgErr_{Y21} = 0.041$ %, $AvgErr_{Y22} = 0.154$ %. Thus the errors' improvement is 61%, 34%, 34% and 36% respectively. The improvement is achieved with 0.5% variation of the model parameter values.

TABLE I
ERROR IMPROVEMENTS AFTER THE GA OPTIMIZATION PROCEDURE

Dimension (N x R x W x S)	Average error minimization, %			
	Y_{11}	Y_{12}	Y_{21}	Y_{22}
2.5 x 60 x 14.5 x 2	15	19	19	16
4.5 x 60 x 14.5 x 2	32	33	33	43
6.5 x 60 x 14.5 x 2	14	9	9	7
4.5 x 30 x 14.5 x 2	61	34	34	36
3.5 x 60 x 9 x 7.5	29	37	37	14

* N: number of turns, R: inner radius (μm), W: metal width (μm), S: spacing (μm)

V. CONCLUSION

An automated procedure for minimizing the parameter extraction errors of the on-chip wide-band spiral inductor model is developed in this paper. Using symbolic analysis and the GA toolbox in the environment of MATLAB the program reduces even the small values for the maximal Y-parameter full range average error with up to 61%, which has its impact, when a system with multiple spiral inductors and other components has to be simulated. The proposed algorithm shows excellent agreement with the measured data over the whole frequency range.

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