A Proposal for Relaxed Low Pass Filtering in DDS Used as a Local Oscillator in Heterodyne Receivers

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Abstract – **This paper defines the conditions allowing the implementation of relaxed low pass Filtering in DDS Used As a Local Oscillator in Heterodyne Receivers, and a specific practical example is presented. As a result, considerable reduction of the order of the low-pass filter, with an almost full exploitation of the frequency capability of DDS, were achieved.**

Keywords – **heterodyne receiver, DDS, local oscillator, spurious responses**

I. Introduction

Low-cost high-performance DDS became available during the last several years, allowing their broader use as local oscillators (LO) in heterodyne receivers. From technical point of view, the fine frequency resolution and the low phase noise are perhaps the main features that make DDS very suitable for the role of LO.

Due to the digital nature of DDS, its output signal contains not only the desired oscillation but also unwanted components with frequencies $nf_{CLK} \pm f_{OUT}$, where f_{OUT} is synthesized frequency, f_{CLK} is DDS clock frequency and *n* is an arbitrary integer [1, 2]. DDS output spectrum is shown in Fig. 1.

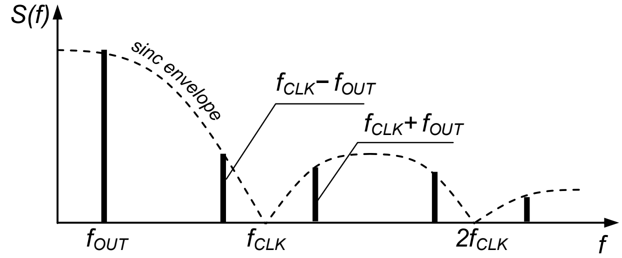

Figure 1.DDS output spectrum

When DDS is used as a LO in a heterodyne receiver, the unwanted components of its output spectrum lead to additional spurious responses. A straightforward approach for avoiding these spurious responses is the use of a low-pass filter that suppresses all unwanted components of the DDS output spectrum. Unfortunately, making this filter requires the use of off-chip components. This is a drawback for the use of DDS as local oscillators. Additionally, the closer to the Nyqiust limit is the synthesized frequency, the higher the order of the necessary filter. This results in a higher number of off-chip components in the receiver. On the contrary, the simplification of the filter leads to a DDS clock frequency, which is considerably higher than the theoretical lower bound, resulting in incomplete use of the frequency capabilities of DDS and increased power consumption.

Here it is proposed to tolerate some of the unwanted components of DDS output spectrum, and using appropriate DDS clock frequency, receiver's intermediate frequency (IF), etc., the corresponding spurious responses would be limited to the stop-band of the receiver input circuitry. In particular, this paper defines the conditions that allow tolerating the component with frequency $f_{CLK} - f_{OUT}$. The frequency of the latter is the closest to the synthesized frequency, therefore the suppression of this component creates the greatest difficulties in the low-pass filtering.

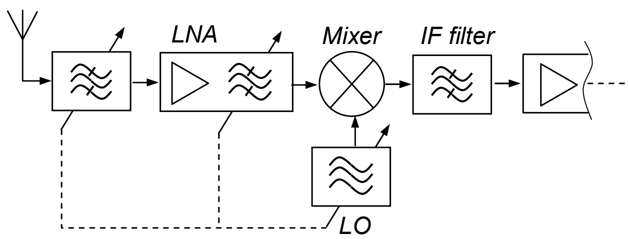

Figure 2. Fragment of heterodyne receiver block diagram.

II. Receiver's spurious responses, created by the proposed relaxed low-pass filtering

Part of the block diagram of a heterodyne receiver is shown in Fig. 2. The basic concepts of heterodyne receivers have been described in details in numerous textbooks, e.g., [3,4], and will not be discussed here. In the proposal presented in this paper, a sum of two oscillations, instead of only one, is fed to the LO input terminal of the mixer. Their frequencies are f_{LO} and $f_{CLK} - f_{LO}$, where f_{LO} is LO frequency, needed for the reception of the desired channel. The frequencies of the receiver's spurious responses, created by the presence of the second oscillation, are derived below.

The output current of a mixer is $i_{out}(t) = u_S(t)g(u_{LO})$, where $u_S(t)$ is mixer's input signal, $g(u_{LO})$ is the conductance or transconductance of the nonlinear component(s) used in the mixer, and $u_{LO}(t)$ is local oscillator voltage. The function $g(u_{LO})$ is nonlinear. Under the assumption that memoryless nonlinear components are used, $g(u_{LO})$ can be expanded in Taylor series [5]. Then

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$$i_{out}(t) = u_S(t) g(u_{LO}) = = \sum_{k=0}^{\infty} g_k u_S(t) u_{LO}^{\ k}(t) . \quad (1)$$

Let

$$u_{LO}(t) = \sin(2\pi f_{LO1} t + \varphi_{0LO1}) + + \sin(2\pi f_{LO2} t + \varphi_{0LO2}) \quad (2)$$

and

$$u_S(t) = \sin(2\pi f_S t + \varphi_{0S}). \quad (3)$$

Substituting (2) and (3) in (1) results in the following form of the components of the mixer output spectrum:

$$f = \left| f_S \pm \left| m f_{LO1} + n f_{LO2} \right| \right|, \quad (4)$$

where *m* and *n* are arbitrary integer numbers.

Assuming that a balanced mixer is used in the receiver, the terms of this power series having even exponents are suppressed [6] and the current is:

$$i_{out}(t) = g_1 u_S(t) u_{LO}(t) + g_3 u_S(t) u_{LO}^{\ 3}(t) + ...$$

The magnitudes of the terms rapidly decrease with the increase of *k* if $u_{LO}(t)$ is not too large. Here it is assumed that terms with exponent $k \ge 5$ are with negligible levels. This is achievable at the expense of a slightly reduced conversion gain, and the application of techniques for linearization of $g(u_{LO})$. In general, this linearization is also desirable.

Under these assumptions, only spectral components with $|m| + |n| = 1$ and $|m| + |n| = 3$ will be present in the output spectrum. In the specific case discussed in this paper, $f_{LO1} = f_{LO}$ and $f_{LO2} = f_{CLK} - f_{LO}$.

From (4) it follows that the frequencies of the spurious responses of interest will be of the form:

$f_{SP} = \left\| m f_{LO} + n(f_{CLK} - f_{LO}) \right| \pm f_i \right|$, where f_i is the IF of the receiver.

The spurious responses that will be taken into consideration are shown in Table 1.

III. CONDITIONS FOR THE PROPOSED RELAXED LOW-PASS FILTERING

With $B_{stop} = 2\Delta f_{stop}$ we denote the stop-band of the receiver input circuitry and with $f_{R\max}$ we denote the upper bound for the received frequency corresponding to $f_{LO} = f_{CLK}/2$. In case of low side injection, $f_{R\max} = f_{CLK}/2 + f_i$, and in case of high side injection, $f_{R\max} = f_{CLK}/2 - f_i$.

In this paper we will discuss the scenario when IF is not greater than $f_{CLK}/2$. Investigations were also made for $f_i > f_{CLK}/2$, but the results are not presented here due to their low practical value. When IF is too large, the ratio $f_{R\max}/f_{R\min}$ becomes too small, and in addition, one of the main advantages of heterodyne receivers is lost. With the limitation described above, even the infradyne receiver is well covered (it can be easily shown that when $f_i > f_{CLK}/4$, then $f_{R\max} < f_i$).

TABLE 1
RECEIVER'S SPURIOUS RESPONSES

m, n	$\left\| m f_{LO} + n(f_{CLK} - f_{LO}) \right\| \pm f_i \right\|$	*Spurious response*
1, 0	*Desired channel/Image channel*	
0, 1	$f_{CLK} - f_{LO} + f_i$	f_{SP1}
	$\left\| f_{CLK} - f_{LO} - f_i \right\|$	f_{SP2}
3, 0	*Not only in DDS LO therefore not discussed here*	
0, 3	$3f_{CLK} - 3f_{LO} + f_i$	f_{SP3}
	$\left\| 3f_{CLK} - 3f_{LO} - f_i \right\|$	f_{SP4}
1, 2	$2f_{CLK} - f_{LO} + f_i$	f_{SP5}
	$\left\| 2f_{CLK} - f_{LO} - f_i \right\|$	f_{SP6}
-1, 2	$2f_{CLK} - 3f_{LO} + f_i$	f_{SP7}
	$\left\| 2f_{CLK} - 3f_{LO} - f_i \right\|$	f_{SP8}
2, 1	$f_{CLK} + f_{LO} + f_i$	f_{SP9}
	$\left\| f_{CLK} + f_{LO} - f_i \right\|$	f_{SP10}
-2, 1	$\left\| f_{CLK} - 3f_{LO} \right\| + f_i$	f_{SP11}
	$\left\| \left\| f_{CLK} - 3f_{LO} \right\| - f_i \right\|$	f_{SP12}

Table 1 shows that f_{SP1} to f_{SP10} are relatively high, therefore they can be avoided by sacrificing a small part of the possible received range in the area close to $f_{R\max}$. It can be proved that f_{SP2} is with the lowest frequency. Therefore, a sufficient condition for avoiding all spurious responses from f_{SP1} to f_{SP10} is:

$$f_R + \Delta f_{stop} < f_{SP2} . \quad (5)$$

Avoiding f_{SP11} and f_{SP12} is more difficult. Regardless of the choice of f_i and f_{CLK}, these spurious responses create 3 or 4 affected intervals in the possible received frequency range. In the design of receivers for relatively narrow received frequency ranges it is not a problem to select a combination of f_{CLK} and f_i, where the whole received range could fit between two affected intervals. There are two possible approaches when the received range is broader:

- Finding a combination of f_{CLK} and f_i, where a given affected interval can fit between two subbands of interest (e.g., in SW DRM broadcasting).
- Changing from low side to high side injection or vice versa could help in some cases.

The limits of the affected intervals in the possible received frequency range are found by solving equations in the form $f_R \pm \Delta f_{stop} = f_{SPn}$. The results are shown in Fig. 3. In addition, $f_i < f_{CLK}/2 - \Delta f_{stop}/2$ should always be satisfied. Otherwise, the whole received frequency range will be affected by f_{SP10}.

The results show that tolerating the first unwanted spectral component can considerably facilitate the low-pass filtration or enable a more complete use of the capabilities of DDS in the following cases:

- when IF is low (to approx. 1/20 от $f_{R\max}$);
- when B_{stop} is narrow;
- when for some reasons it has been decided in advance to use high side injection in the receiver.

If there are no other considerations, it is preferable to use low side injection. It allows achieving a high received frequency with a lower f_{CLK}. However, when low side injection is used and when f_i and B_{stop} are relatively large, satisfying the condition (5) leads to an allowable received frequency that is significantly lower than $f_{R\max}$. Therefore, the corresponding f_{LO} becomes considerably lower than $f_{CLK}/2$. Then the advantage of the proposed relaxed filtering is significantly reduced. However, when the values of IF are comparable to $f_{R\max}$ ($f_i > f_{CLK}/5$), two new spurious-free frequency windows become broad enough to be considered. These are the intervals from $f_{R\max} - f_i + \Delta f_{stop}/2$ to $f_{R\max} - f_i/2 - \Delta f_{stop}/4$ and from $f_{R\max} - f_i/2 + \Delta f_{stop}/4$ to $f_{R\max} - \Delta f_{stop}/2$. One of them closely approaches $f_{R\max}$.

The benefits of the proposed relaxed low–pass filtering become problematic when B_{stop} is broad. In most cases, however, the narrowing of B_{stop} is favorable. The proposed relaxed low-pass filtering is even less applicable when the receiver input circuitry is not tunable, except when the received range is too narrow.

It is also clear that the proposed relaxed low-pass filtering is particularly suitable for low IF receivers and homodyne receivers.

Further relaxation of low-pass filtering is possible when tolerating also the DDS output spectral component with frequency $f_{LO} + f_{CLK}$. In this case, the stop-band of the low-pass filter can begin from approx. $3f_{CLK}/2$. It turned out that only two of the additional spurious responses, created by

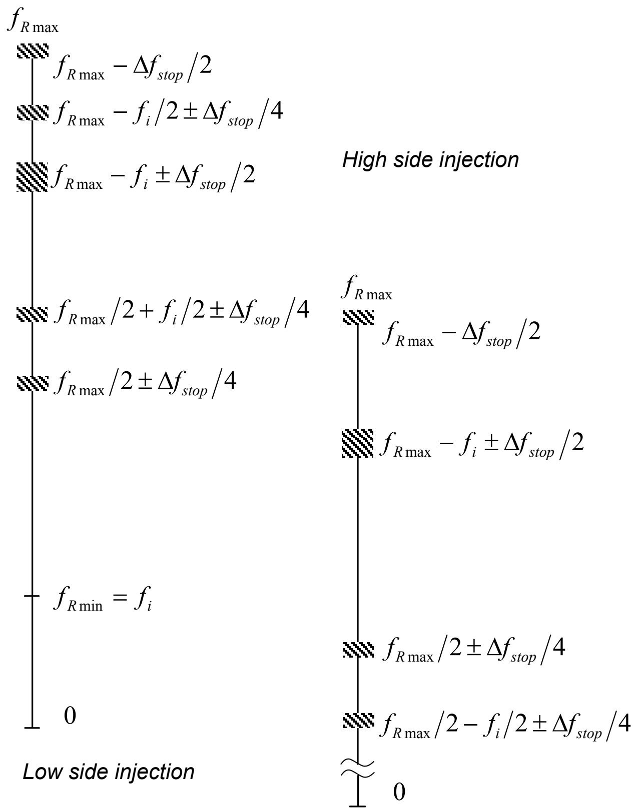

Figure 3. Affected intervals in the possible received frequency range.

the presence of the above DDS output spectral component, need to be considered. These are $f_{SP13} = 3f_{LO} + f_i$ and $f_{SP14} = 3f_{LO} - f_i$. It is easy to determine that avoiding them requires slight narrowing of the possible received range from below.

IV. EXAMPLE

One potential application of the proposed approach is in a DRM receiver. In this case the use of DDS as LO is valuable due to the low phase noise (highly desirable because of the OFDM used in DRM). In addition, two quadrature oscillations with almost perfectly maintained $\pi/2$ phase difference can be generated using DDS technique. This makes DDS suitable for LO in an image reject mixer. The use of the latter is particularly appropriate in portable DRM receivers. It allows the use of low IF, and therefore reduced power consumption.

Let's consider a DRM receiver for LW, MW and SW including 11 m band (25.6 - 26.1 MHz) with IF=455 kHz. An easily achievable stop-band $B_{stop} = 2\Delta f_{stop} = f_R/5$ was assumed. Obviously, these values dictate the use of an image reject mixer in the receiver. It was determined that the maximum value of f_{LO} would be $0.45 f_{CLK}$. Under these conditions, it was calculated that $f_{CLK} = 56.99$ MHz. The intervals of the received frequency range affected by spurious responses were found to be above the 11 m band, and in the gap between the 22 m and 19 m bands.

A reasonable value of 55 dB was assumed for the attenuation of the unwanted spectral components of the DDS output.

Using Matlab, elliptic filters were designed and analyzed when all unwanted spectral components of the DDS output were suppressed, as well as when the spectral component with frequency $f_{LO} - f_{CLK}$ was tolerated. In the proposed relaxed low-pass filtering, a filter of third order was sufficient to achieve an attenuation of 58 dB for the spectral components that had to be suppressed. Using the traditional approach, a filter of sixth order was needed to achieve the same attenuation.

It was also determined that using the traditional approach, a third order elliptic filter would achieve the desired attenuation when $f_{CLK} = 78.6$ MHz. This is approx. 40% more than f_{CLK} in the case of the proposed relaxed filtration. The essential results from this example are presented in Table 2.

TABLE 2.
ESSENTIAL RESULTS FROM THE EXAMPLE, ALLOWING A COMPARISON BETWEEN THE TRADITIONAL AND THE PROPOSED APPROACHES FOR LOW-PASS FILTERING IN DDS LO.

Approach	Achieved attenuation* of all rejected spectral components	Filter order	f_{CLK}
Proposed	**≥58 dB**	**3**	**56.99 MHz**
Traditional, version 1	**≥58 dB**	**6**	**56.99 MHz**
Traditional, version 2	**≥58 dB**	**3**	**78.6 MHz**

* *Attenuation caused by the sinc response of the DDS DAC is included in the presented values.*

V. CONCLUSIONS

It was shown that relaxed low-pass filtering in DDS used as LO can be successfully applied under certain conditions. This leads to a considerable reduction of the order of the low-pass filter, with an almost full exploitation of the frequency capability of DDS.

Further investigations could examine the possibilities to tolerate even more of the unwanted spectral components. Another direction would be to investigate the relaxed band-pass filtering in frequency conversion, which would use some of the higher order spectral components (traditionally considered as unwanted) of DDS output, with the goal of overcoming the frequency limitations of DDS.

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