Properties of an Algorithm for Automatic Modulation Classification Based on Sixth-Order Cumulants

Vladimir Orlic[1] and Miroslav L. Dukic[2]

Abstract – **In this paper one algorithm for automatic modulation classification based on sixth-order cumulants is presented. Expressions for standard deviations of sixth-order cumulant sample estimates are derived, and performance of proposed algorithm and algorithm based on fourth-order cumulants is compared. Sixth-order cumulants method shows much better accuracy in distinguishing BPSK from complex valued modulation techniques, and this conclusion is confirmed via Monte-Carlo simulations.**

Keywords – **Automatic modulation classification, cumulants, higher-order statistics.**

I. Introduction

Automatic modulation classification (AMC) represents one of most up-to-date topics in telecommunications, with natural application in cognitive radio, electronic warfare and surveillance systems. It considers methods for identification of modulation techniques used at transmitter by observing and processing received data samples, commonly distorted during propagation. Because of their simplicity, pattern-recognition methods of AMC, based on extraction of key features of received signal, are very popular. As key features in pattern-recognition, higher-order statistics – cumulants and moments, are usually considered. While fourth-order cumulants in AMC applications are analyzed in detail ([1], [2]), and their variance of the sample estimates are described with expressions derived in [2], sixth-order cumulants have not been evaluated in such manner yet, to the best of our knowledge.

In this paper we present one algorithm for AMC based on sixth-order cumulant features, then we derive expressions for its variance sample estimates, and suggest a criterion for comparison of performance of different cumulant-based algorithms. We have analyzed algorithm's performance in cases with BPSK, QPSK, 16-QAM and 64-QAM constellations, with goal to compare it with algorithm based on forth-order cumulants. Although it is commonly believed that sixth and higher-order cumulants provide no additional benefit due to their increased measurement-error variance [3], analysis presented in this paper shows that in some scenarios sixth-order cumulants are characterised with better classification selectivity.

[1]Vladimir Orlic is with IMTEL Komunikacije A.D., Mihaila Pupina Blvd 165B, 11070 Belgrade, Serbia, E-mail: cheggy@insimtel.com.

[2]Miroslav L. Dukic is with the Faculty of Electrical engineering at University of Belgrade, Kralja Aleksandra Blvd 73, 11000 Belgrade, Serbia.

II. AMC Algorithm

The received signal sequence $y(n)$ in any communication system can be represented by:

$$y(n) = \sum_{k=0}^{L-1} h(k)x(n-k) + g(n) \tag{1}$$

where $x(n)$ stands for transmitted modulated symbols, $h(k), k = 0...L-1$ are coefficients of channel of length L, and $g(n)$ is additive white Gaussian noise with a zero mean and a variance of σ_g^2. Cumulants of random variable x can be expressed using the joint cumulant formula [4]:

$$cum(x_1,...,x_n) = \sum_{\pi} (|\pi|-1)!(-1)^{|\pi|-1} \prod_{B\in\pi} E(\prod_{i\in B} x_i) \tag{2}$$

where the π runs through the list of all partitions of $\{1,\ldots,n\}$, and B runs through the list of all blocks of the partition π. For zero-mean random variable x, associated with transmitted data sequence $x(n)$, the second-order cumulant $C_{21,x} = cum(x, x^*)$ is given by:

$$C_{21,x} = E(|x|^2) \tag{3}$$

The sixth-order cumulant $C_{63,x} = cum(x,x,x,x^*,x^*,x^*)$ can be expressed as:

$$C_{63,x} = E(|x|^6) - 9E(|x|^4)E(|x|^2) + 12|E(x^2)|^2 E(|x|^2) + 12E^3(|x|^2) \tag{4}$$

We will assume, without loss of generality, that the constellations are normalized to have unit energy ($C_{21,x} = 1$). In practice, the self-normalized cumulants are estimated:

$$\hat{C}_{63,x} = C_{63,x} / (C_{21,x})^3 \tag{5}$$

In practice, cumulants can be estimated only from the samples of the received signal $y(n)$; if channel is considered to be non-dispersive, i.e. received data is corrupted only by noise, we get the following expression for sixth-order cumulants of transmitted sequence:

$$\hat{C}_{63,x} = \frac{C_{63,y}}{(C_{21,y} - \sigma_g^2)^3} \tag{6}$$

An estimate of the variance of additive noise σ_g^2 is usually available in practice.

Table I shows the values of $\hat{C}_{63,x}$ for various modulation constellations, along with values of the fourth-order cumulants $\hat{C}_{42,x}$, self-normalized values of:

$$C_{42,x} = E(|x|^4) - |E(x^2)|^2 - 2E^2(|x|^2) \quad (7)$$

$\hat{C}_{63,x}$ and $\hat{C}_{42,x}$ values can be used as key features in AMC. Although the theoretical feature-vector (i.e. mutual distance between nearby values of cumulants for considered constellation types) is increased with sixth-order cumulants, it is of interest to derive expressions for the variance of the $\hat{C}_{63,x}$ sample estimates, in order to evaluate the algorithm in proper manner.

III. Variance of sample estimates of sixth-order cumulants

It is usually assumed that variance of the estimate $C_{21,x}$ is small enough to be ignored; however, in [2] it is demonstrated that this is not good assumption in general. Swami derived expressions for variance of sample estimates of fourth-order cumulants $\hat{C}_{42,x}$, in both cases when value of $C_{21,x}$ is known exactly and when it needs to be estimated from the data. We will focus on the second case, since it describes the situation in practice in more realistic manner, i.e. when AMC is performed at the receiver without any a priori knowledge about energy of transmitted signal. According to [2], variance of the sample estimates of $\hat{C}_{42,x}$ for complex constellations, with N samples, is given by:

$$N\,\mathrm{var}(\hat{C}_{42,x}) \approx [m_{8,4} - m_{4,2}^2] + 4m_{2,1}[3m_{4,2}m_{2,1} - 2m_{6,3} + 2m_{2,1}^3] \quad (8)$$

where $m_{k,m} = E[y^{k-m}(y^*)^m]$ represents mixed moment of order k with m conjugations. For real constellations variance is given by:

$$N\,\mathrm{var}(\hat{C}_{42,x}) \approx [m_{8,4} - m_{4,2}^2] + 6m_{2,1}[5m_{4,2}m_{2,1} - 2m_{6,3} + 3m_{2,1}^3] \quad (9)$$

We derive the variance of the estimates of the cumulants in (4), under the same assumptions as those used in Swami's work: sample estimates $m_{k,m}$ are unbiased and asymptotically Gaussian with variance $(m_{2k,k} - |m_{k,m}|^2)/N$, and similar results hold asymptotically for the sample estimates of the cumulants.

A. Complex signals

For complex signals $E(x^2) = 0$ and equation (4) is reduced:

$$C_{63,x} = m_{6,3} - 9m_{4,2}m_{2,1} + 12m_{2,1}^3 \quad (10)$$

$$E[m_{6,3}] = \frac{1}{N}\sum_{m=1}^{N} E(|x(m)|^6) = m_{6,3} \quad (11)$$

$$\begin{aligned} E[m_{4,2}m_{2,1}] &= \frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N} E(|x(m)^2 x(n)|^2) \\ &= \frac{1}{N^2}[N\cdot m_{6,3} + N(N-1)\cdot m_{4,2}m_{2,1}] \\ &= m_{4,2}m_{2,1} + \frac{1}{N}[m_{6,3} - m_{4,2}m_{2,1}] \end{aligned} \quad (12)$$

$$\begin{aligned} E[m_{2,1}^3] &= \frac{1}{N^3}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{l=1}^{N} E(|x(m)x(n)x(l)|^2) \\ &= \frac{1}{N^3}[Nm_{6,3} + 3N(N-1)m_{4,2}m_{2,1} + N(N-1)(N-2)m_{2,1}^3] \\ &\approx m_{2,1}^3 + \frac{3}{N}[m_{4,2}m_{2,1} - m_{2,1}^3] \end{aligned} \quad (13)$$

$$E[C_{63,x}] \approx C_{63,x} - \frac{9}{N}[m_{6,3} - 5m_{4,2}m_{2,1} + 4m_{2,1}^3] \quad (14)$$

From above expression we can notice that $C_{63,x}$ is only asymptotically unbiased; if N is large enough, the bias can be considered for negligible. In eq. (13) and (14) we have omitted the $O(1/N^2)$ and $O(1/N^3)$ terms in asymptotic analysis – the same will be done in following considerations. The exact error variance can be expressed as:

$$\begin{aligned} \mathrm{var}(C_{63,x}) &= \mathrm{var}(m_{6,3}) + 81\,\mathrm{var}(m_{4,2}m_{2,1}) + 144\,\mathrm{var}(m_{2,1}^3) \\ &\quad - 18\,\mathrm{cov}(m_{6,3}, m_{4,2}m_{2,1}) + 24\,\mathrm{cov}(m_{6,3}, m_{2,1}^3) \\ &\quad - 216\,\mathrm{cov}(m_{4,2}m_{2,1}, m_{2,1}^3) \end{aligned} \quad (15)$$

$$\mathrm{var}(m_{6,3}) = \frac{1}{N}(m_{12,6} - m_{6,3}^2) \quad (16)$$

$$\begin{aligned} E[(m_{4,2}m_{2,1})^2] &= \frac{1}{N^4}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{l=1}^{N}\sum_{k=1}^{N} E(|x(n)^2 x(m) x(l)^2 x(k)|^2) \\ &\approx \frac{(N-1)(N-2)(N-3)}{N^3} m_{4,2}^2 m_{2,1}^2 \\ &\quad + \frac{(N-1)(N-2)}{N^3}[m_{8,4}m_{2,1}^2 + m_{4,2}^3 + 4m_{6,3}m_{4,2}m_{2,1}] \end{aligned} \quad (17)$$

From eq. (12) and (17) we get:

$$\begin{aligned} \mathrm{var}(m_{4,2}m_{2,1}) &= E[(m_{4,2}m_{2,1})^2] - [E(m_{4,2}m_{2,1})]^2 \\ &\approx \frac{1}{N}(m_{8,4}m_{2,1}^2 + m_{4,2}^3 + 2m_{4,2}^2 m_{2,1}^2 + 2m_{6,3}m_{4,2}m_{2,1}) \end{aligned} \quad (18)$$

Similarly, we have:

$$\begin{aligned} \mathrm{var}(m_{2,1}^3) &= \frac{1}{N^6}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{l=1}^{N}\sum_{k=1}^{N}\sum_{p=1}^{N}\sum_{q=1}^{N} E(|x(n)x(m)x(l)x(k)x(p)x(q)|^2) \\ &\quad - [E(m_{2,1}^3)]^2 \\ &\approx \frac{(N-1)(N-2)(N-3)(N-4)(N-5)}{N^5} m_{2,1}^6 \\ &\quad + \frac{15(N-1)(N-2)(N-3)(N-4)}{N^5} m_{4,2}m_{2,1}^4 \\ &\quad - [m_{2,1}^3 + \frac{3}{N}(m_{4,2}m_{2,1} - m_{2,1}^3)]^2 \approx \frac{3}{N}(3m_{4,2}m_{2,1}^4 + 2m_{2,1}^6) \end{aligned} \quad (19)$$

$$
\begin{aligned}
\operatorname{cov}(m_{6,3}, m_{4,2}m_{2,1}) &= \frac{1}{N^3}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{l=1}^{N} E(\left|x(n)^3 x(m)^2 x(l)\right|^2) \\
&\quad - E(m_{6,3})E(m_{4,2}m_{2,1}) \\
&\approx \frac{(N-1)(N-2)}{N^2} m_{6,3}m_{4,2}m_{2,1} \\
&\quad + \frac{(N-1)}{N^2}[m_{10,5}m_{2,1} + m_{8,4}m_{4,2} + m_{6,3}^2] \\
&\quad - m_{6,3}[\, m_{4,2}m_{2,1} + \frac{1}{N}(m_{6,3} - m_{4,2}m_{2,1})] \\
&\approx \frac{1}{N}(m_{10,5}m_{2,1} + m_{8,4}m_{4,2} + m_{6,3}m_{4,2}m_{2,1})
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\operatorname{cov}(m_{6,3}, m_{2,1}^3) &= \frac{1}{N^4}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{l=1}^{N}\sum_{k=1}^{N} E(\left|x(n)^3 x(m)x(l)x(k)\right|^2) \\
&\quad - E(m_{6,3})E(m_{2,1}^3) \\
&\approx \frac{(N-1)(N-2)(N-3)}{N^3} m_{6,3}m_{2,1}^3 \\
&\quad + \frac{3(N-1)(N-2)}{N^3}[m_{8,4}m_{2,1}^2 + m_{6,3}m_{4,2}m_{2,1}] \\
&\quad - m_{6,3}[m_{2,1}^3 + \frac{3}{N}(m_{4,2}m_{2,1} - m_{2,1}^3)] \\
&\approx \frac{3}{N}(m_{8,4}m_{2,1}^2 + m_{6,3}m_{2,1}^3)
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\operatorname{cov}(m_{4,2}m_{2,1}, m_{2,1}^3) &= \frac{1}{N^5}\sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{l=1}^{N}\sum_{k=1}^{N}\sum_{p=1}^{N} E(\left|x(n)^2 x(m)x(l)x(k)x(p)\right|^2) \\
&\quad - E(m_{4,2}m_{2,1})E(m_{2,1}^3) \\
&\approx \frac{(N-1)(N-2)(N-3)(N-4)}{N^4} m_{4,2}m_{2,1}^4 \\
&\quad + \frac{2(N-1)(N-2)(N-3)}{N^4}[2m_{6,3}m_{2,1}^3 + 3m_{4,2}^2 m_{2,1}^2] \\
&\quad - [m_{4,2}m_{2,1} + \frac{1}{N}(m_{6,3} - m_{4,2}m_{2,1})][m_{2,1}^3 + \frac{3}{N}(m_{4,2}m_{2,1} - m_{2,1}^3)] \\
&\approx \frac{1}{N}(3m_{6,3}m_{2,1}^3 + 4m_{4,2}m_{2,1}^4 + 3m_{4,2}^2 m_{2,1}^2)
\end{aligned} \tag{22}
$$

By combining the various sub-expressions at eq. (15), we finally obtain:

$$
\begin{aligned}
N\operatorname{var}(C_{63,x}) &= [m_{12,6} - m_{6,3}^2] + 9[m_{2,1}^2(48m_{4,2}m_{2,1}^2 - 54m_{4,2}^2 + 96m_{2,1}^4 \\
&\quad - 64m_{6,3}m_{2,1}) + m_{4,2}(9m_{4,2}^2 + 16m_{6,3}m_{2,1} - 2m_{8,4}) \\
&\quad + m_{2,1}(17m_{8,4}m_{2,1} - 2m_{10,5})]
\end{aligned} \tag{23}
$$

B. Real signals

In case of real signals, $E(x^2) = E(|x|^2)$, so eq. (4) becomes:

$$C_{63,x} = m_{6,3} - 9m_{4,2}m_{2,1} + 24m_{2,1}^3 \tag{24}$$

$$E[C_{63,x}] \approx C_{63,x} - \frac{9}{N}[m_{6,3} - 9m_{4,2}m_{2,1} + 8m_{2,1}^3] \tag{25}$$

and $C_{63,x}$ can be considered for asymptotically unbiased, under the same assumptions as were made in previous section. The error variance can now be expressed as:

$$
\begin{aligned}
\operatorname{var}(C_{63,x}) &= \operatorname{var}(m_{6,3}) + 81\operatorname{var}(m_{4,2}m_{2,1}) + 576\operatorname{var}(m_{2,1}^3) \\
&\quad - 18\operatorname{cov}(m_{6,3}, m_{4,2}m_{2,1}) + 48\operatorname{cov}(m_{6,3}, m_{2,1}^3) \\
&\quad - 432\operatorname{cov}(m_{4,2}m_{2,1}, m_{2,1}^3)
\end{aligned} \tag{26}
$$

Using the sub-expressions (16)-(22), with N samples, we get the variance expression:

$$
\begin{aligned}
N\operatorname{var}(C_{63,x}) &= [m_{12,6} - m_{6,3}^2] + 9[m_{2,1}^2(384m_{4,2}m_{2,1}^2 - 126m_{4,2}^2 + 384m_{2,1}^4 \\
&\quad - 128m_{6,3}m_{2,1}) + m_{4,2}(9m_{4,2}^2 + 16m_{6,3}m_{2,1} - 2m_{8,4}) \\
&\quad + m_{2,1}(25m_{8,4}m_{2,1} - 2m_{10,5})]
\end{aligned} \tag{27}
$$

IV. PERFORMANCE ANLYSIS AND SIMULATION

Variance of the sample estimates for the $C_{42,x}$ and $C_{63,x}$, calculated for BPSK, QPSK, 16-QAM and 64-QAM signals from eq. (8), (9), (23) and (27), are presented in Table I. It can be noticed that sixth-order cumulants, as expected, are characterized with increased variance of sample estimates, along with larger distances between nearby values for considered modulations. In order to compare performance of fourth and sixth-order cumulants-based algorithms, we use a ratio of standard deviation and mutual distance of nearby values (ρ), for both $C_{42,x}$ and $C_{63,x}$, as a measure of algorithm's selectivity.

TABLE I
STATISTICAL PARAMETERS OF INTEREST FOR VARIOUS CONSTELLATION TYPES

Constellation:	BPSK		QPSK		16-QAM		64-QAM	
$\hat{C}_{42,X}$	-2		-1		-0.68		-0.6191	
distance (Δ)		1		0.32		0.0609		
variance (σ^2)	36		12		9.54		8.82	
std. dev. (σ)	6		3.464		3.089		2.97	
$\rho=\sigma/\Delta$		6	3.464	11.375	9.653	50.722	48.768	
$\hat{C}_{63,X}$	16		4		2.08		1.797	
distance (Δ)		12		1.92		0.283		
variance (σ^2)	5040		576		332.306		289.38	
std. dev. (σ)	70.993		24		18.229		17.011	
$\rho=\sigma/\Delta$		5.916	2	12.5	9.494	64.413	60.11	

By observing values of ρ from Table I, we can confirm that distinguishing BPSK from QPSK is better with sixth-order cumulants criteria (because of 3.464/2=1.732 times lower ratio of standard deviation and distance from BPSK value for QPSK signals), while classification of QAM-modulated signals shows to be slightly better with fourth-order cumulants. Considering complex-valued noise, we find that its ratio ρ is also 1.732 times lower for BPSK-QPSK values of $C_{63,x}$, meaning that for low values of SNR, where impact of noise on classification algorithm is significant,

$C_{63,x}$ features give the same performance with approximately 10log10(1.7322)=4.77dB lower SNR in comparison with case where $C_{42,x}$ is used for classification criteria. If only QAM-modulated signals in AWGN are considered, $C_{42,x}$ shows better performance for approximately 1.7dB lower SNR in comparison with $C_{63,x}$.

With goal to test both algorithms in modulation classification problems, we carry out the simulations through 2,000 Monte-Carlo trials and N received data samples are collected for AMC in each trial. Algorithm with $C_{63,x}$ features is simulated along with algorithm based on $C_{42,x}$ features in non-dispersive channel conditions (no multipath, AWGN only), and with noise power σ_g^2 considered to be known. Correct classification probability P_{CC} was calculated versus SNR, with two scenarios of different sets of modulation candidates considered: (i) {BPSK, QPSK} and (ii) {QPSK, 16-QAM, 64-QAM}. Simulation results for scenario (i) are presented in Fig. 1, while results for scenario (ii) are shown in Fig. 2. The simulation results are in good agreement with theoretical arguments: while the algorithm based on $C_{63,x}$ features shows significantly better performance than the one based on $C_{42,x}$ features in case of {BPSK, QPSK} scenario, in case of {QPSK,16-QAM,64-QAM} scenario, difference in performance is negligible. For the best performance, hybrid classification criteria can probably be used, for example: some that's using sixth-order cumulants for classification of BPSK signals, while for complex-valued signals fourth-order cumulants are used. However, achieved results are promising that sixth-order cumulants could show good performance even in the channel context that goes beyond the scope of influence of AWGN only; such are channel models that we have considered within our previous work at the field of AMC [5].

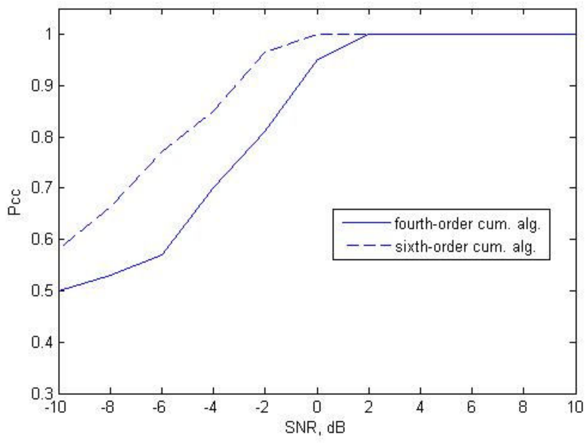

Fig. 1. Correct classification probability in {BPSK, QPSK} scenario, N=250

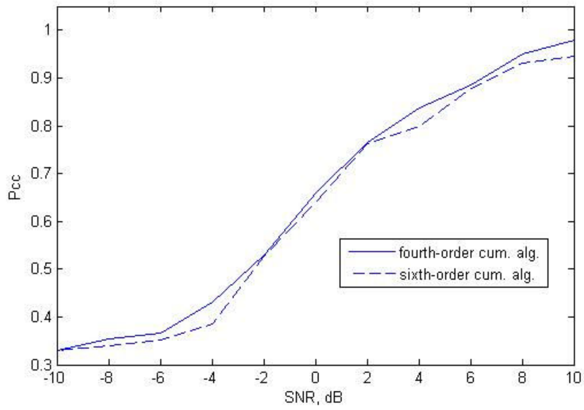

Fig. 2. Correct classification probability in {QPSK, 16-QAM, 64-QAM} scenario, N=2000

V. CONCLUSION

In this paper algorithm for automatic modulation classification based on sixth-order cumulants is presented. Expressions for variance of the sample estimates of sixth-order cumulants are derived, and proposed solution is compared with fourth-order cumulants. As a criterion for comparison of algorithms, the ratio of standard deviation and feature vector is used, and theoretical conclusions are tested via simulations. Achieved results show that proposed algorithm shows very good performance in distinguishing BPSK from complex-valued modulations, and slightly lower performance when used for classification of QAM-signals. In this paper only channel with AWGN was considered; realistic picture of algorithm's performance can be formed if multipath channels are considered as well, and that topic represents the scope of our future work.

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