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Adaptive Semilogarithmic Quantizer for Quantization of Laplacean Source

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Abstract – **In this paper, we have analyzed a novel companding quantizer for nonuniform scalar quantizion of Laplacean source. We have compared it with standard semilogorithmic A-low companding and we have shown better performances of proposed model. An adaptive scalar nonuniform quantization of Laplacean source based on proposed model has been analyzed. In this case quantizers have been adapted to the maximal amplitudes of input signals. This scalar quantization model is used in order to achieve higher quality of signal-to-quantization noise ratio (SNRQ) in a wide range of signal volumes (variances) with respect to it's necessary robustness over a broad range of input variances. The main contribution of this model is achieving higher quality of transmission and posibilty of his applying for digitaliztion of countinious signals in wide range.**

Keywords – **Semilogorithmic companding, Laplacean source, Signal-to-quantization noise ratio.**

I. Introduction

In various papers, analysis of Laplacean source quantization is given, because for larger number of speech samples, the probability density function of speech signal is better represented with Laplacean then with Gaussian distribution [1], [2]. In paper [3], robust and switched non-uniform scalar quantization model is analyzed for the case when the power of an input signal varies in a wide range. In this paper a new compounding characteristic for non-uniform scalar quantization of Lapacean source is analyzed. Main goal of our research is to find a simply way for realization non-uniform system quantization with high quality of performance, with maintaining robustness in wide range of input signals. A general analysis of non-uniform scalar quantization as well as analysis of A-low companding is given in selection 2. Comparation of A-low companding characteristic with newely proposed compounding function is also given this section. Expressions for granular and overload distortion by using Bennett's integral are derived. We have performed an optimization of number of quantization levels. In section 3, we have presented adaptive quantizer model and also have given numerical result and discussed derived solutions. Here we can see how increase of number of quantization levels in adaptive system effects the dependence of SQNR value in function of the input power. Optimization is done for one or more quantizer, in observed range value, taking in consideration total distortion. Finally, we have discussed derived result and on the basis of this discussion we have driven conclusions which are refered on possibility implementation this model on speech processiong. The main contribution of this model is increase quality of transmission and possibility of his applying for digitalization of continiuos signals in wide range.

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II. A novel-proposed semilogarithmic compandor

By choosing a set of N real-valued quantization points $\{y_1^{(N)}, y_2^{(N)},..., y_N^{(N)}\}$, and N decision thresholds $\{t_0^{(N)}, t_1^{(N)},..., t_N^{(N)}\}$, the N-point scalar quantizer $Q^{(N)}$, could be characterized. By relating decision thresholds as $-\infty=t_0^{(N)}< t_1^{(N)}<...< t_N^{(N)}=\infty$, we define quantization rule as many-to-one mapping, $Q^{(N)}(x)=y_i$ if $t_{i-1}^{(N)}<x\leq t_i^{(N)}$ for i=1,2,…,N. In other words, a quantized signal has the value $y_i^{(N)}$ when the original signal belongs to the quantization cell $S_i^{(N)}=(t_{i-1}^{(N)}, t_i^{(N)}]$ for i=1,2,…,N. Nonuniform quantization can be achieved by compressing the signal x using nonuniform compressor characteristic $c(\cdot)$ (also called compounding low), then, by quantizing the compressed signal $c(x)$ employing a uniform quantizer, and at the end by expanding the quantized version of the compressed signal using a nonuniform transfer characteristic $c^{-1}(\cdot)$, which is inverse to the characteristic of the compressor. The overall structure, which consists of the compressor, the uniform quantizer, and the expandor in casscade is called compandor. A general model for any nonuniform quantizer based on compounding technique can illustrated in Figure 1. where $c(x)$ and $c^{-1}(x)$ are compressor and expandor functions respectively.

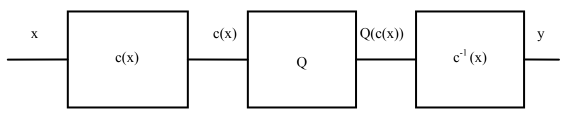

Fig. 1. Block diagram of non-uniform companding technique

In situations such as speech coding, the exact value of the input variance changes with time and is not known in advance. In cases like that, by using logorithmic companding law we can obtaine a signal-to-quantization ratio which is constant over a broad range of input variance.

The A-law companding is used for PCM telephone systems in the Europe, with the standard value of A = 87.6, and A-law compression characteristic is characterized by:

$$c_1(x) = \begin{cases} \dfrac{Ax}{1+\ln A}\operatorname{sgn} x & za\ |x| < x_{\min} \\ \dfrac{x_{\max}(1+\ln \dfrac{Ax}{x_{\max}})}{1+\ln A}\operatorname{sgn} x & za\ x_{\min} < |x| < x_{\max} \end{cases} \quad (1)$$

For a source, that is characterized as a continuous random variable with probability density $p(x)$, the N-point nonuniform scalar quantizer distortion could be defined as the expected mean square error between original and quantized signal. So the total distortion consists of two components, the granular and the overload distortion:

$$D_t = D_g + D_o, \quad (2)$$

with:

$$D_g = D_{g1} + D_{g2}$$

$$D_g = \frac{\Delta^2}{6}\int_{x_{\min}}^{x_{\max}} \frac{p(x)}{[c'(x)]^2}dx + \frac{\Delta_1^2}{12}P_1 \quad (3)$$

$$D_o = 2\int_{x_{\max}}^{\infty}(x - y_L(N))^2 p(x)dx \quad (4)$$

considering:

$$P_1 = 1 - e^{-\frac{\sqrt{2}}{A}\frac{x_{\max}}{\sigma}};\ \Delta_1 = \frac{2x_{\min}}{N_1};\ \Delta = \frac{2x_{\max}}{N}$$

Here, Laplacean source in wide broad of range is analyzed. Probability density function of Laplacean original random variable x with unit variance is given with:

$$p(x,\sigma) = \frac{1}{\sqrt{2}\sigma}e^{-\frac{\sqrt{2}}{\sigma}x}. \quad (5)$$

After substituting (5) into (3) and (4), and by aproximating y_N with x_{max}, and taking in consider that $x_{\min} = \dfrac{x_{\max}}{A}$ after some straightforward mathematical manipulations, we obtain :

$$D_{g2} = \frac{2}{3}\frac{(1+\ln A)^2}{N^2\sqrt{2}\sigma}[e^{-\frac{\sqrt{2}}{\sigma}\frac{x_{\max}}{A}} (\frac{\sigma}{\sqrt{2}}(\frac{x_{\max}}{A})^2 + \sigma^2\frac{x_{\max}}{A} + \frac{\sigma^3}{\sqrt{2}}) - e^{-\frac{\sqrt{2}}{\sigma}x_{\max}}(\frac{\sigma}{\sqrt{2}}x^2_{\max} + \sigma^2 x_{\max} + \frac{\sigma^3}{\sqrt{2}})] \quad (6)$$

$$D_o = \frac{1}{\sigma}e^{-\frac{\sqrt{2}}{\sigma}x_{\max}}(\sigma x_{\max}^2 - 2\sigma(x_{\max} + \frac{\sigma}{\sqrt{2}})(y_L(N) - \frac{\sigma}{\sqrt{2}}) + \sigma y_L^2(N)) \quad (7)$$

$$D_{g1} = \frac{x^2_{\max}}{3A^2N_1^2}(1 - e^{-\frac{\sqrt{2}}{A}\frac{x_{\max}}{\sigma}}) \quad (8)$$

where N_1 is given with:

$$N_1 = \frac{N}{1+\ln A} \quad (9)$$

Our novel proposed characteristic is given with:

$$c_2(x) = \begin{cases} x & za\ |x| < x_{\min} \\ \ln\dfrac{x}{x_{\min}} + x_{\min} & za\ x_{\min} < |x| < x_{\max} \end{cases} \quad (10)$$

Similary to the previous procedure for A-low compounding , we obtain following expressions:

$$D_{g2} = \frac{2(\ln\frac{x_{\max}}{x_{\min}})^2}{3N_2^2\sqrt{2}\sigma}P \quad (11)$$

$$P = e^{-\frac{\sqrt{2}}{\sigma}x_{\min}}(\frac{\sigma}{\sqrt{2}}x_{\min} + \sigma^2 x_{\min} + \frac{\sigma^3}{\sqrt{2}}) - e^{-\frac{\sqrt{2}}{\sigma}x_{\max}}(\frac{\sigma}{\sqrt{2}}x^2_{\max} + \sigma^2 x_{\max} + \frac{\sigma^3}{\sqrt{2}}) \quad (12)$$

$$D_o = \frac{1}{\sigma}e^{-\frac{\sqrt{2}}{\sigma}x_{\max}}(\sigma x_{\max}^2 - 2\sigma(x_{\max} + \frac{\sigma}{\sqrt{2}})(x_{\max} - \frac{\sigma}{\sqrt{2}}) + \sigma(x_{\max})^2) \quad (13)$$

$$D_{g1} = \frac{x^2_{\min}}{3N_1^2}(1 - e^{-\frac{\sqrt{2}}{\sigma}x_{\min}}) \quad (14)$$

where N_1 and N_2 are given with:

$$N_1 = \frac{x_{\min}N}{\ln(x_{\max}/x_{\min}) + x_{\min}} \quad (15)$$

$$N_2 = N - N_1 \quad (16)$$

Now using well famous relationship between signal power and total distortion we can easily calculate signal-to-quantization noise ratio (SQNR):

$$\text{SQNR} = 10\lg\frac{\sigma^2}{D_t} \quad (17)$$

For the case of N=256 and A=87.6, we obtain optimal value for A-low compounding SQNR of 38.16 dB, whereby N_1=46.77 and N_2=209.23.On the other hand observing the novel proposed characteristic and by considering that $x_{\min} = x_{\max}/A$, for the same case of N=256 and A=87.6, we obtain SQNR value of 39.11 dB, whereby N_1=6.2 and N_2=249.8. The comparation is shown in Fig.2. Here it can be easily observed that novel proposed characteristic shows better performances of SQNR quality over 0.95 dB.

We can see that our charachteristic has better performances in range with high value of variances. If we are using one quantizer, A-law companding characteristic show better performances. But in our case, when we are using adaptive quantization our proposed characteristic has an advance.

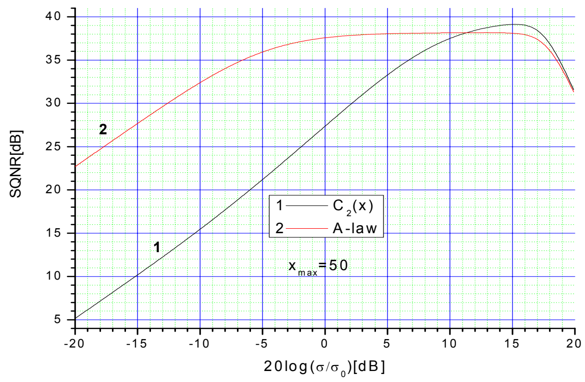

Fig.2. Comparation of transmition quality (SQNR), between A-law companding characteristic and novel-proposed characteristic in a wide dynamic input range of power

Now we will derived expression for granular and overload distortion which is reffered for the case when the number of quantic levels is optimize and independent related each other. From derived result we can see that expression for A-law characteristic and our proposed characteristic is the same:

$$D_{g2} = \frac{2(\ln\frac{x_{max}}{x_{min}})^2}{3N_2^2\sqrt{2}\sigma}P \tag{18}$$

$$D_o = \frac{1}{\sigma}e^{-\frac{\sqrt{2}}{\sigma}x_{max}}(\sigma x_{max}{}^2 - 2\sigma(x_{max}+\frac{\sigma}{\sqrt{2}})(x_{max}-\frac{\sigma}{\sqrt{2}})+\sigma(x_{max})^2) \tag{19}$$

$$D_{g1} = \frac{x_{min}^2}{3N_1^2}(1-e^{-\frac{\sqrt{2}}{\sigma}x_{min}}) \tag{20}$$

Let us try to optimize total distortion by using Lagrange multiplicators in order to obtain even better performances. Optimization can be done by optimizing function J considering limiting condition of total number of real-valued quantization points with:

$$J = D_t + \lambda(N_1 + N_2) \tag{21}$$

Limitation is presented as:

$$N_1 + N_2 = N \tag{22}$$

After solving following expressions:

$$\frac{\partial J}{\partial N_1} = 0 \tag{23}$$

$$\frac{\partial J}{\partial N_2} = 0 \tag{24}$$

and considering (22), we obtain following expressions for N_1 and N_2

$$N_1 = \frac{N}{h}\sqrt[3]{\frac{2}{3}x_{min}^2(1-e^{-\frac{\sqrt{2}}{\sigma}x_{min}})} \tag{25}$$

$$N_2 = \frac{N}{h}\sqrt[3]{\frac{4}{3}\frac{(\ln\frac{x_{max}}{x_{min}})^2}{\sqrt{2}\sigma}P} \tag{26}$$

$$h = \sqrt[3]{\frac{2}{3}x_{min}^2(1-e^{-\frac{\sqrt{2}}{\sigma}x_{min}})} + \sqrt[3]{\frac{4}{3}\frac{(\ln\frac{x_{max}}{x_{min}})^2}{\sqrt{2}\sigma}P} \tag{27}$$

where P is given with (12)

$$\frac{1}{\lambda} = (\frac{N}{h})^3 \tag{28}$$

Now, when we apply obtained optimized values for codebook sizes of $N_{1opt} = 10.93$ i $N_{2opt} = 245.07$ for the case of A=87.6, we reach SQNR value of 39.317 dB. Because of it's better performances and easier practical realization, we will now focus on novel proposed characteristic.

III. Adaptive Quantization Based on Novel Semilogarithmic Compandor

Analysis that we will be performed is simillar to analysis performed in [5].

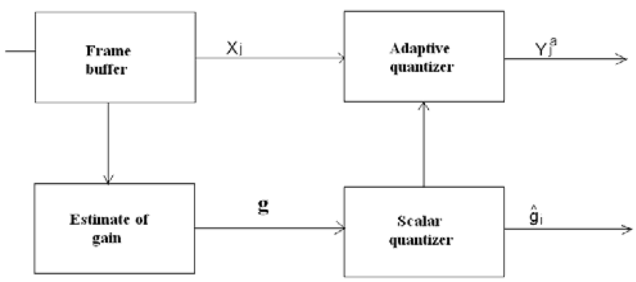

Fig.4 Semilogarithmic adaptive scalar quantizer

Decision thresholds and representation levels are:

$$t_j^a = \hat{g}_i t_j^f$$

$$y_j^a = \hat{g}_i y_j^f$$

$$g = \frac{\sigma}{\sigma_{ref}}$$

t_j^f - decision thresholds for given quantizer ($\sigma = 1$)

y_j^f - representation levels for given quantizer ($\sigma = 1$)

$\hat{g}_i$ - representation levels of gain.

For transmitting the encoded information about signal power we use log-uniform quantizer:

$$(\hat{G}_i)^2\ [dB] = -20 + \frac{2i-1}{2}\Delta\quad [dB]$$

$$(\hat{G}_i)^2\ [dB] = 20\log\hat{g}_i$$

from the last expression we obtain value for $\hat{g}_i$.

The bit rate per sample for proposed adaptive quantizer is given by:

$$R = \log_2 N + \frac{K}{M}$$

M-frame lenght
K-number of quantization levels for gain

Here the optimization of proposed characteristic is performed, considering maximal and minimal values of input signal amplitudes (considering A and *xmax*), for constant input power value. We have presented numerical results for the case of standard value for parameter A and side information of 1,2 and 3 bits (k=2,4,8). For the 8-level case, we can see that SQNR varies from it's peak value, for 0.317 [dB] in case of codebook size of 256 and 16 codebooks. Also we can observe that SQNR never falls under 39 dB for the complete wide range of input power. Also we have presented the performances of the adaptive quantizer based on our novel proposed compounding characteristic and for optimal value of parameter A (optimal value of $x_{\min}^{opt}$).

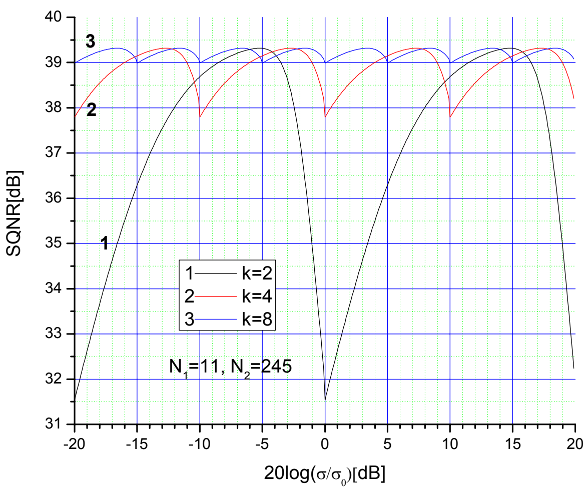

Fig.5 Comparation of quality (SQNR) reffered to A-law companding characteristic, for two, four and eight quantizers and nonoptimized values of x_{min} ($x_{min}=x_{max}/A$, A=87.6) in a wide dynamic input range of power

The optimization of parametres *xmin* and *xmax*, and the 16-level side information implementation is coNsidered in Fig.6. Here, we can see that SQNR varies from it's peak value, for 0.280 dB. By aplying an adaptive quantization with the novel quantization characteristic and *xmin* optimal value (optimal *A*), we reach maximal SQNR value gain of 2.7 dB over standard quantization which is discussed in [1,6]. Also, we we reach maximal SQNR value gain of 1.56 dB over case of adaptive quantization with the novel quantization characteristic and nonoptimized *A*.

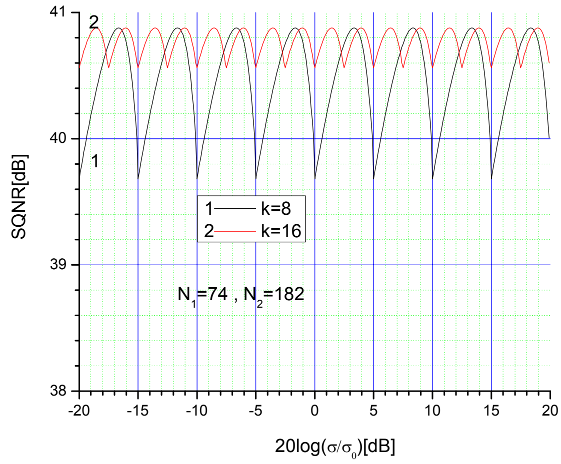

Fig.6 Comparation of quality (SQNR) reffered to novel proposed characteristic, for eight and sixteen quantizers and optimized values values of x_{min} ($x_{min}=x_{max}/A$, A=87.6) in a wide dynamic input range of power

IV. Conclusion

In this paper, we have presented and analyzed the novel semilogarithmic compounding characteristic for nonuniform scalar quantization of Laplacean source in a wide range of input signal's power. From obtained results, we can see that proposed characteristic has better performances over standard A-low compounding characteristic. By comparing we observe gain of 2.7 dB over A-law(A=87.6)**.** Compounding with using proposed characteristic produce accomplishing high quality of signal-to-quantization noise ratio (SNRQ), for digitalized signal in a wide range of signal volumes (variances), with respect to it's necessary robustness over a broad range of input variances, so it can be applied for coding of speech signals.

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