An Application of Brand-Level Diffusion Model in the Mobile Communication Market

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Abstract – **We present a brand-level diffusion model and apply it in the mobile communication market. We analyzed the effects of a new brand entry on the diffusion dynamic of the existing brands. The model is tested using data from the Serbian mobile communication market.**

Keywords - **brand, diffusion, market.**

I. INTRODUCTION

Marketing researchers have been developing many different types of diffusion models to address various issues surrounding the sales growth of new services. A careful analysis of the diffusion literature reveals that most of the diffusion models focus only on service-level diffusion and that there are only a few models that research diffusion at the brand level. In this paper we want to analyze the impact of a later entrant on the sales growth of the existing brands services.

According to diffusion theory, a new service's sales growth at any time largely depends on the strength of word of mouth from its previous adopters. Similarly, a brand's sales growth should then depend on the extent to which it receives good word of mouth from its own previous adopters. In this article, we presented an expression for brand-level sales as a function of the time variable alone. Subsequently, we extend the proposed model to capture the impact of a late entrant on the diffusion of the service.

The model is tested using data from the Serbian mobile communication market characterized by two long-standing market leaders (Telekom and Telenor), which has recently been outpaced by a new mobile operator (VIP). We conclude the article, giving managerial implications and directions for further research.

II. A BRAND-LEVEL DIFFUSION MODEL

There is an extensive amount of research in marketing on the development and applications of new service/product adoption models. The most important model in this stream of research, that, in fact, pioneered a long subsequent line of research inquiry over four decades, is the Bass Model [1]. The attractive feature of this is that it allows the hazard function characterizing new service/product adoption at a point of time to be a linear function of the cumulative distribution function of the adoption process. Such a formulation is based on the following behavioral premise: Some consumers, called innovators, adopt the new service/product for reasons that represent their independent decision-making ability. Such reasons may arise, for example, due to their being better informed about the new service/product from reading technology magazines, their adventurous needs for novelty etc. Other consumers, called imitators, adopt the new service/product on account of social learning effect, influenced by previous adopters, either by observation or through explicit word of mouth.

The mathematical structure of the Bass model is derived from a hazard function corresponding to the conditional probability that an adoption will occur at time t given that it has not occurred yet. If $f(t)$ is the density function of time to adoption and $F(t)$ is the cumulative fraction of adopters at t, the basic hazard function underlying the Bass model is:

$$\frac{f(t)}{1-F(t)} = p + q \cdot F(t) \qquad (1)$$

Parameter q reflects the influence of those consumers who have already adopted the product (i.e. word-of-mouth communication from previous adopters), while p captures the influence that is independent from the number of adopters (i.e. external communication). The cumulative number of adoptions at time t is $N(t) = M \cdot F(t)$, where M refers to the market potential for the new product.

The Bass model assumes that word-of-mouth effect (collective force) is the same for all the brands. However, to ensure that its influence is different on different brands, the brand-specific coefficients are introduced as follows, for example, for Brand 1[5]:

$$\frac{f_1(t)}{1-F(t)} = p_1 + q_1 \cdot F(t) \qquad (2)$$

To derive the closed-form expression for the model expressed in Equation 2, the model (3) is derived by summing both sides of Equation 2 over all the brands as follows:

$$\frac{\sum f_i(t)}{1-F(t)} = \sum p_i + \sum q_i \cdot F(t) \qquad (3)$$

Denoting $\Sigma f_i(t)$ by $f_c(t)$, Σp_j by p, and Σq_i, by q equation (3) reduces to (4).

$$\frac{f_c(t)}{1-F(t)} = p + q \cdot F(t) \qquad (4)$$

It is easy to see that the model is simply the classic Bass (1969) model. The solution of equation (1) is given by equation (5):

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$$F(t)=\frac{1-e^{-(p+q)\cdot t}}{1+\frac{q}{p}\cdot e^{-(p+q)\cdot t}} \tag{5}$$

Substituting expression 5 in the brand-level Equation 2 and solving the resulting differential equation, it is obtained:

$$F_i(t)=\frac{q_i}{q}\cdot\frac{1-e^{-(p+q)\cdot t}}{1+\frac{q}{p}\cdot e^{-(p+q)\cdot t}}+\frac{p}{q}\cdot\left(\frac{p_i}{p}-\frac{q_i}{q}\right)\cdot ln\frac{1+\frac{q}{p}}{1+\frac{q}{p}\cdot e^{-(p+q)\cdot t}} \tag{6}$$

Equation 6 expresses the cumulative sales function of Brand i as a function of the time variable alone.

If m denotes the market potential, it can be shown that the market potential and the peak sales time of brand i are:

$$m_i=m\cdot F_i(t)=m\cdot\frac{q_i}{q}\cdot\left\{1+\frac{p}{q}\left(\frac{p_i}{q}\cdot\frac{q}{q_i}-1\right)\cdot ln\left(1+\frac{q}{p}\right)\right\} \tag{7}$$

and the peak sales time of brand i,

$$t_i^*=\frac{1}{p+q}\cdot ln\left(\frac{2q_i p+q(q_i-p_i)}{p(q_i+p_i)}\right) \tag{8}$$

we show that brand j will reach its peak before the category does if and only if $q_i/p_i < q/p$. Similarly, we show that brand i will reach its peak earlier than brand j if and only if $q_i/p_i < q_j/p_j$. A smaller ratio of internal to external force suggests either a poorer word-of-mouth effect and/or a stronger brand equity, both of which lead to an earlier peak sales time but not necessarily a higher peak.

The model formulated thus far assumes that the brands enter the market simultaneously. But in many markets, there are successful late entrants. In the event of a successful late entrant, the managers of incumbent brands will be curious to know exactly bow the new entrant will affect the category as a whole, their own brands in particular, and the competing brands. In the next section, we extend the proposed model to answer these questions.

III. Modeling of a New Brand Entrance

Suppose that two brands are present in the market from the time of introduction of the service; that is, t = 0. Suppose at time t_n, a third brand enters the market, where $t_n>0$. One of three things can be expected to happen to the market at the category level. The adoption of service will expand (i.e., m will become larger), the service will start diffusing faster (i.e., q will be higher), or both market expansion and faster diffusion will happen simultaneously. At the brand level, the existing brands may get affected in their diffusion speed either positively or negatively, and the exact effect depends on the parameters m, q and q_i.

Generally, it is possible that the new entry affects both the market potential and the diffusion speed of the service. We assume that Brands 1 and 2 entered the market at $t = 0$ and Brand 3 entered the market at $t = t_n$.

The following differential equations represent the sales growth of the service and the three brands before the third brand entry and after the third brand entry [1].

$$\frac{f(t)}{1-F(t)}=p+q\cdot F(t),\ \ \forall t\le t_n \tag{7}$$

$$\frac{f(t)}{k-F(t)}=p+Q\cdot F(t),\ \ \forall t\ge t_n \tag{8}$$

$$\frac{f_i(t)}{1-F(t)}=p_i+q_i\cdot F(t),\ \ \forall t\le t_n,\ \ i=1,2 \tag{9}$$

$$\frac{f_i(t)}{k-F(t)}=p_i+Q_i\cdot F(t),\ \ \forall t\ge t_n,\ \ i=1,2 \tag{10}$$

$$f_3(t)=0,\ \ \forall t\le t_n \tag{11}$$

$$\frac{f_3(t)}{k-F(t)}=Q_3\cdot F(t),\ \ \forall t\ge t_n \tag{12}$$

Q represents the factor of imitation for service, Q_i *represents the factor of imitation for brand i,* after the third brand entry. The estimated value of k will tell us by what percentage the market has expanded or contracted because of the third brand entry.

Equations 7 and 8 represent the diffusion of the service before and after the third brand entry, respectively; Equations 9 and 10 represent the diffusion of Brand i (i = 1,2) before and after the third brand entry; and Equations 11 and 12 represent the diffusion of Brand 3. Table I, presents all the notations that are used in the model.

TABLE I
NOTATIONS OF ALL PARAMETERS

	Before t_n			After t_n		
	Innovation	Imitation	Market Potential	Innovation	Imitation	Market Potential
service	p	q	m	p	Q	M
Brand 1	p_1	q_1	/	p_1	Q_1	/
Brand 2	p_2	q_2	/	p_2	Q_2	/
Brand 3	/	/	/	/	Q_3	/

In Equations 8, 10, and 12, the parameter k implies that eventually more or less market potential will be realized because of the third brand entry. In other words, $F(\infty)$ may have a value different from 1 (to be specific, it will be k, as is shown subsequently), and it is important to realize that though statistically speaking $F(\infty)$ should be 1, what we do here is a simple scaling to ensure that the model takes care of the possible market contraction or expansion. Note that we focus our attention on modeling the changes that may occur in the coefficients of imitation (of the category and of the incumbent brands) because of the third brand entry, and thus we let the coefficients of innovation remain unchanged in the process. This helps us study the change in the diffusion speed much more clearly, because in this case we only need to examine the estimates of coefficients of imitation before and after the third brand entry to understand whether the diffusion has speeded up or not. For the same reason, we do not model the coefficient of innovation for the late entrant.

Before we proceed further, a caveat is in order. A new entrant in a market changes the market dynamics not only by its entry but also by the marketing actions and reactions that follow the entry. These actions include price cutting, more advertising and promotional efforts, wider distribution, and so forth, which cause the changes observed in the diffusion dynamics. In this article, we study those changes and not their causes."

Solutions to Equations 7 and 9 are given by Equations 5 and 6, respectively. To solve the differential Equations 8 and 10, we make use of the fact that the cumulative sales functions are continuous at time t_n. In other words, we first evaluate $F(t_n)$, $F_i(t_n)$ $(i = 1, 2)$ by replacing t by t_n, in Equations 5 and 6 and then use those as the initial conditions for solving the corresponding differential Equations 8 and 10 to obtain:

$$F(t)=\frac{k\left[1+\frac{Q}{p}F_n\right]-(k-F_n)e^{-(p+kQ)(t-t_n)}}{\left(1+\frac{Q}{p}F_n\right)-\frac{Q}{p}(k-F_n)e^{-(p+kQ)(t-t_n)}} \qquad (13)$$

$$F_i(t)=F_{in}+\frac{Q_i(p+qF_n)}{Q^2}\cdot\left[\frac{p+kQ}{p+QF_n+Q(k-F_n)e^{-(p+kQ)(t-t_n)}}-1\right]-\frac{Q_ip-p_iQ}{Q^2}\cdot ln\left[\frac{p+kQ}{p+QF_n+Q(k-F_n)e^{(p+kQ)(t-t_n)}}\right] \qquad (14)$$

Note that we denote $F_i(t_n)$ and $F_i(t_n)$ by F_n and F_{in}, respectively. Equations 13 and 14 describe the cumulative sales growth of the category and of brand i, respectively, after the third brand entry. By letting $t\to\infty$ in Equation 13, we find that $F(\infty) = k$, and because the market potential of the category before the third brand entry is scaled to 1, the estimated value of k will tell us by what percentage the market has expanded or contracted because of the third brand entry. Similarly, by comparing the estimated value of $F_i(\infty)$ before and after the third brand entry, we can compute what happened to the market potential of Brand i because of the late entrant. To obtain the cumulative sales function for Brand 3, we recognize that its cumulative sales at $t = t_n$ are 0. Then, similar to Equation 14, which describes the cumulative sales function of Brands 1 and 2 after t_n, the cumulative sales function for Brand 3 can be shown as:

$$F_3(t)=\frac{Q_3(p+qF_n)}{Q^2}\left[\frac{p+kQ}{p+QF_n+Q(k-F_n)e^{-(p+kQ)(t-t_n)}}-1\right]\cdot-\frac{Q_3p}{Q^2}\cdot ln\left[\frac{p+kQ}{p+QF_n+Q(k-F_n)e^{-(p+kQ)(t-t_n)}}\right] \qquad (15)$$

The set of equations given by 13, 14, and 15, along with the set of Equations 5, 6, and 11, provides us with the cumulative sales growth of the category, of Brand i $(i = 1,2)$, and of Brand 3 for the entire duration, that is, from $t = 0$ to well after $t = t_n$.

The proposed model can be used for several purposes. For example, by applying these equations to an empirical data set and by comparing the estimates of equivalent parameters before and after the third brand entry, we can infer the changes in the speed of diffusion of the category and of each brand and the changes in the market potential of the category and of each brand.

IV. Empirical demonstration

The data for the study were provided by the wireless service providers that have national presence. The competition in wireless mobile service in Serbia was happened in 1997, when the second operator started in the market. In 2007 the license was granted for the third operator to enter the market. We collected the data set on the number of new subscribers during the last decade.

The estimated values of relevant parameters for aplication of presented model are given by Table II.

TABLE II
ESTIMATION PARAMETERS OF MARKET

Market potential	7.350.000			
	Before t_n		***After t_n***	
	p	***q***	***p***	***Q***
Service	0.02	0.35	0.02	0.65
Telenor	0.005	0.17	0.005	0.22
Telekom	0.025	0.18	0.025	0.28
VIP	-	-	-	0.15

From the estimates reported on the coefficients of imitation in all three markets, the sum of the brand's imitation coefficients equals the corresponding category imitation coefficient, both before and after the third brand's entry. With respect to the coefficients of innovation, however, the sum of the two brands' innovation coefficients does not equal the corresponding category innovation coefficient. This is because we have forced the innovation coefficient of the third brand to be zero.

We tested the performance of the proposed model in a case of only the diffusion speed is changed. The market potential seems to be constant before and after the third operator entrance. It could be noticed that the coefficient of imitation is increased because of the marketing actions and reactions that follow the new entry. These actions include price cutting, more advertising and promotional efforts, wider distribution, and so forth, which cause the changes observed in the diffusion dynamics.

It is possible to consider the scenario in which the market potential is increased and diffusion speed is not affected by the third operator entry. The scenario in which both, the market potential and the diffusion speed, are changed should be considered, too.

The normalized cumulative sales functions before and after the third mobile operator entrance are shown in Figure 1 and Figure 2, respectively.

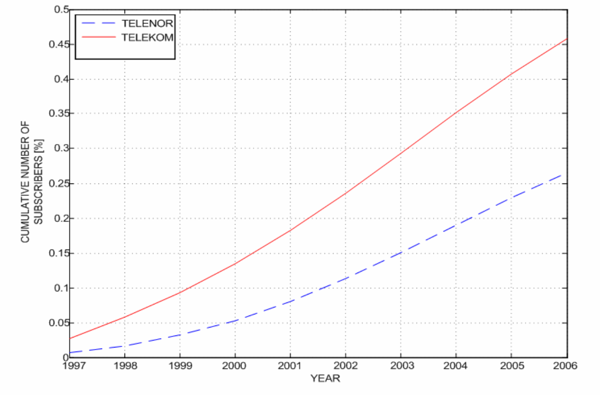

Fig. 1. The cumulative sales functions before t_n

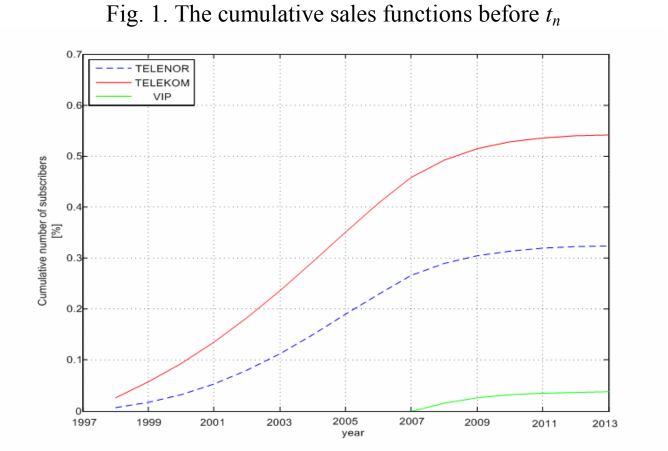

Fig. 2. The cumulative sales functions after t_n

From the Figures 3 and 4, it could be noticed that the number of new adopters is significantly influenced by the earlier entrant of the third operator in the telecommunication market. In the considered scenario, the third operator entrance was occured when the total number of subscribers is reached nearly 70 % of the total market.

V. Conclusion

In this paper we presented a brand level diffusion model and analyzed the effects of the third operator entry on the diffusion dynamics of the existing brand. We applyied this model in Serbian mobile telecommunication market. Presented model will be usefull to product managers, because the simple graph of the saled data does not explain the complex effect the third brand entry has on the market.

Managers can use the proposed model to compare their operators' performance before and after a third operator entry in the mobile market, evaluate their operators' performance with respect to the competing opearators, and assess the changes take in place in the diffusion process because of the new operator entry.

Although our focus was the third entrant effect, the proposed model can be used to study the effects of any discrete changes in the market, such as reposition of an existing operator, a major price reduction, and a grand promotional campaign by an existing operator.

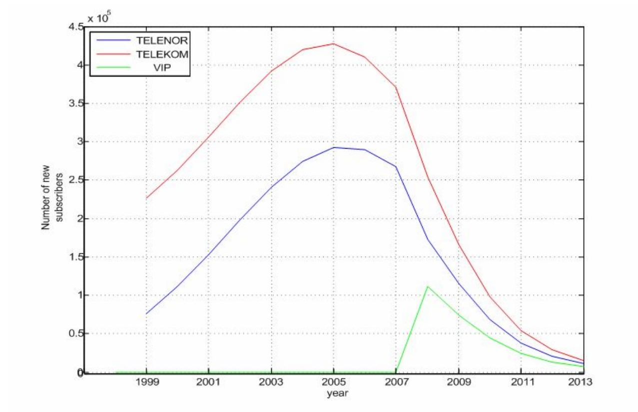

Fig. 3. The number of new users-later entrance

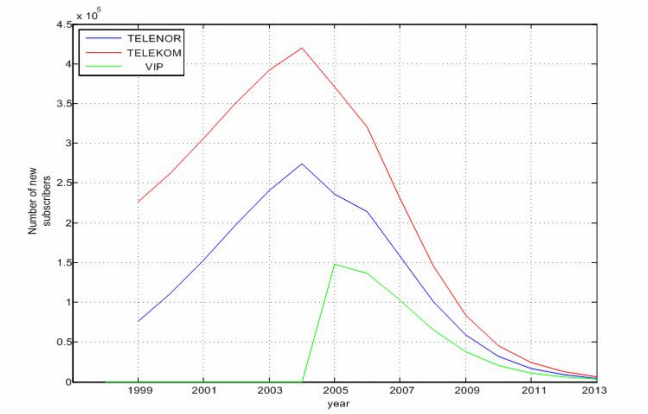

Fig. 4. The number of new users-earlier entrance

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