Critical Speed-Capacitance Requirements for Self-Excited Induction Generator

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Abstract – **A novel method for solving nonlinear algebraic equations describing behaviour of self-excited induction generator, based on the MATLAB Optimization Toolbox routines, is presented in paper. Using this approach, critical rotor speed and excitation capacitance requirements under different loading conditions have been considered. Accuracy of prediction has been improved by including core losses into the calculations.**

Keywords – **Induction generator, Self-excitation, MATLAB, fsolve.**

I. INTRODUCTION

The concept of self-excited induction generator (further: SEIG) has been known since 30's years of last century, when Basset and Potter in [1] reported that an induction machine, whose rotor was externally driven by some prime mover, could act as autonomous generator if capacitor bank of appropriate capacitance was connected across stator terminals. Except a few sporadic responses, there was no serious research of SEIG untill the 80's years of the 20st century. During that long period it has been usual to consider SEIG as something that is possible, but not practically acceptable, in first line due to it's poor regulation characteristics.

After the first signs of the global energetic crisis, scientists and researchers from all over the world increased their interest in SEIG, recognizing it's huge potentials for stand-alone power generation. Recently published research overviews [2], [3] indicate that serious work is still to be done in order to achieve acceptable level of performance to price ratio.

Difficulties related to solving of equations describing the SEIG behaviour were perhaps the greatest problem that researches had faced in earlier years. The main obstacle emerges from the fact that when machine operates as a SEIG, both voltage and frequency are independent variables. Different methods and formulas for approximate analysis have been proposed, but common point in the most of approaches has been to simplify equivalent circuit of the machine, neglecting core losses for example. Having in mind the physics of self-excitation process [4], [5] which implies that machine's operating point has to be in the saturated region of the magnetizing curve, it is clear that such simplifications affect accuracy of prediction. On the other hand, they appeared to be necessary, due to great mathematical difficulties that emerged even in trying to solve nonlinear equations obtained from simplified circuits.

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There are numerous phenomena that can be analysed when research of SEIG is performed. Determination of critical speed or capacitance necessary for self-excitation is of great importance, and there was significant work in the past considering this type of problems [6], [7]. Besides mentioned simplifications in analysis, the usual approach in literature is to consider critical conditions for self-excitation of an unloaded machine, which is operating state with no practical significance. Also, the analysis is usually performed across one dimension, meaning that critical speed for exact capacitance or critical capacitance for exact speed is calculated.

In this paper, an unified approach is used to calculate contours defining critical speed-capacitance combinations under different values of symmetrical three-phase load connected to the stator terminals. Calculation has been simplified using a method for solving nonlinear algebraic equations based on the MATLAB Optimization Toolbox routines [8]. This method also allowed core losses to be taken into account easily.

II. MATHEMATICAL MODEL

Equivalent circuit of an SEIG with R-L load connected to stator terminals is shown on Fig. 1. Core losses are modeled by adding resistor R_c in parallel with magnetizing inductance X_m, as it was proposed in [8].

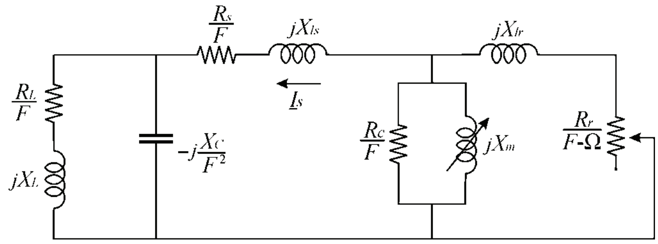

Fig.1. Per-phase equivalent circuit of a three-phase SEIG

The meaning of used symbols is:

R_s, R_r, R_L	-	stator, rotor and load resistance;
R_c	-	magnetizing resistance;
X_m	-	magnetizing reactance at rated frequency ;
X_{ls}, X_{lr}, X_L	-	stator leakage reactance, rotor leakage reactance and load reactance at rated frequency;
X_C	-	capacitor reactance at rated frequency;
$F = f_s / f_{sn}$	-	actual to rated stator frequency ratio [p.u.];
$\Omega = n/n_{sn}$	-	actual rotor speed to rated synchronous speed ratio [p.u.].

Values of all machine parameters are expressed in ohms. Also, parameters describing the rotor part of the equivalent circuit are refered to stator, although there is no explicit notification neither in Fig. 1, nor in the text.

In general, both R_c and X_m are variables, depending on the actual conditions of operation, but in this analysis they will be considered as constants. Assumption that R_c has a constant value is questionable, because it neglects the fact that core losses depend on frequency. On the contrary, assumption that magnetizing reactance is constant, when critical conditions for self-excitation are considered, absolutely matches the truth.

From the theory of self-excitation it is known that machine's operating point have to be in saturated zone of the magnetizing curve, what can be expressed as $X_m < X_{muns}$, where X_{muns} is the value of magnetizing reactance in unsaturated state. In other words, limit of stability is determined by relation $X_m = X_{muns}$, which explains previous statement.

From the equivalent circuit shown on Fig. 1, using impedance loop method, can be written:

$$\underline{I}_s(\underline{Z}_s + \underline{Z}_1 + \underline{Z}_2) = \underline{I}_s \underline{Z}_{tot} = 0 \quad (1)$$

where

$$\underline{Z}_s = \frac{R_s}{F} + jX_{ls} \quad (2)$$

$$\underline{Z}_1 = (\frac{R_L}{F} + jX_L) \,||\, \frac{-jX_C}{F^2} \quad (3)$$

$$\underline{Z}_2 = \frac{R_c}{F} \,||\, jX_m \,||\, (\frac{R_r}{F-\Omega} + jX_{lr}) \quad (4)$$

Since in steady-state stator current I_s differs from zero, it follows that $\underline{Z}_{tot}$ must be zero, and that can be written as:

$$\mathrm{Re}\{\underline{Z}_{tot}\} = 0 \quad (5)$$

$$\mathrm{Im}\{\underline{Z}_{tot}\} = 0 \quad (6)$$

Eqs. (5) and (6) are basic equations of an SEIG, and they have to be satisfied in any stable operating condition.

After some mathematical operations, Eqs. (5) and (6) can be written as:

$$\mathrm{Re}\{\underline{Z}_{tot}\} = \frac{\frac{R_L X_C^2}{F^3}}{R_L^2 + \left(FX_L - \frac{X_C}{F}\right)^2} + \frac{R_s}{F} + \frac{\frac{R_c X_m^2}{F}\left[\frac{R_r^2}{(F-\Omega)^2} + \frac{R_c R_r}{F(F-\Omega)} + X_{lr}^2\right]}{\frac{R_c^2}{F^2}\left[\frac{R_r^2}{(F-\Omega)^2} + (X_m + X_{lr})^2\right] + X_m^2\left[\frac{R_r^2}{(F-\Omega)^2} + \frac{2R_c R_r}{F(F-\Omega)} + X_{lr}^2\right]} = 0 \quad (7)$$

and

$$\mathrm{Im}\{\underline{Z}_{tot}\} = \frac{\frac{(X_L X_C - R_L^2)X_C}{F^2} + X_L^2 X_C}{R_L^2 + \left(FX_L - \frac{X_C}{F}\right)^2} + X_{ls} + \frac{\frac{R_c^2 X_m}{F^2}\left[\frac{R_r^2}{(F-\Omega)^2} + X_m X_{lr} + X_{lr}^2\right]}{\frac{R_c^2}{F^2}\left[\frac{R_r^2}{(F-\Omega)^2} + (X_m + X_{lr})^2\right] + X_m^2\left[\frac{R_r^2}{(F-\Omega)^2} + \frac{2R_c R_r}{F(F-\Omega)} + X_{lr}^2\right]} = 0 \quad (8)$$

If all parameters of the equivalent circuit, including reactance of the capacitor X_C are known, it is possible to determine relative rotor speed Ω and relative frequency F by solving Eqs. (7) and (8). However, it is necessary to use some numerical method, since Eqs. (7) and (8) are high-degree nonlinear equations. If any of standard methods is selected for this purpose (e.g. Newton-Raphson iterative method), it demands long-lasting mathematical transformations in order to get appropriate form of equations. Even if this work is properly done, serious problems related to convergency of the process will occur in the next phase, which makes such approach almost useless.

In this paper, system of two nonlinear equations is solved using user-friendly numerical routine from MATLAB Optimization Toolbox, named *fsolve*. This routine uses nonlinear least-squares algorithm that employs Levenberg-Marquardt method.

The first important feature of the *fsolve* routine is that further mathematical derivations are not needed, because it can easily operate with Eqs. (7) and (8). Also, it has extremely good convergence, and if no serious mistake is made during definition of starting vector values, zeros of the system are easily calculated.

During analysis whose results are presented here, reactance of capacitor has been varied across wide range, using program loop and considering that all other parameters in the equivalent circuit are constant. For each different value of capacitor reactance (i.e. capacitance), two different values of relative speed Ω and frequency F have been calculated. The first solution can be obtained if starting vector $[F \quad \Omega]^+$ is defined as $[0.98\Omega \quad \Omega]^+$, and the second one if starting assumption is approximatelly $[0.6\Omega \quad \Omega]^+$. As a result, this calculations finally give pairs of values (C, Ω) that define closed contour in the $C-\Omega$ plane. This contour presents critical speed-capacitance conditions that have to be fulfilled in order to sustain self-excitation under specific loading conditions.

III. Numerical Results

Presented approach has been used to calculate critical speed-capacitance contours for real three-phase squirrel-cage induction machine operated as an SEIG. Stator of the machine is star-connected, with rated values for motor mode of operation $380\,V$, $50\,Hz$, $3.2\,A$, 2860 rpm. Parameters of the machine are given in Table I.

TABLE I
MACHINE PARAMETERS

R_s	$4.05\ \Omega$
R_r	$2.75\ \Omega$
X_{ls}	$4.34\ \Omega$
X_{lr}	$2.77\ \Omega$
X_{muns}	$226\ \Omega$
R_c	$1200\ \Omega$

Load impedance connected across stator terminals can be expressed as $\underline{Z}_L = Z_L e^{j\varphi_L}$, where φ_L is angle of impedance at rated frequency. Created mathematical model allows investigation of performance of an SEIG under any type of symmetrical R-L load. In order to identify influence of load impedance variation to the shape of critical $C-\Omega$ contours, in this analysis pure resistive load has been assumed. Results can be seen in Fig. 2. Log scale is used for the sake of better visibility. The actual load resistance used in calculation is assigned to it's resulting contour. Values of load resistance per phase are expressed in p.u., where $1\,\text{p.u.} = 68.5\,\Omega$.

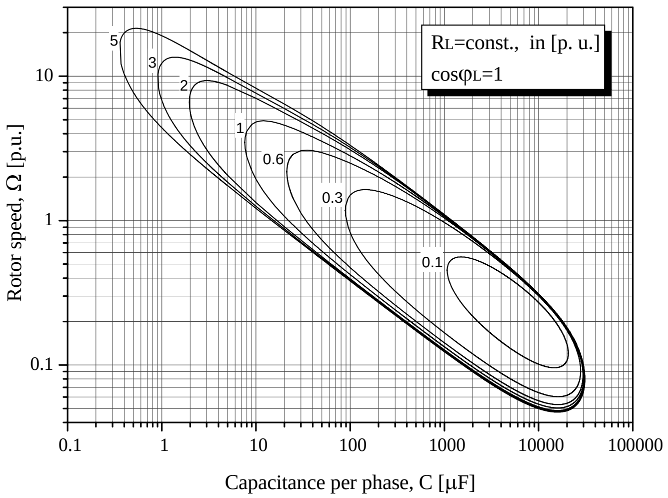

Fig. 2. Critical $C-\Omega$ contours for different load resistances R_L

From Fig. 2 it is obvious that if load resistance is set to the constant value, self-excitation can be achieved for many different combinations of rotor speed and excitation capacitors. Pairs of values (C,Ω) lie on the closed contour of irregular shape. Identification of such contours is of great importance because they represent critical conditions that have to be fulfilled in order to sustain self-excitation of symetrically loaded SEIG. For a specific value of load resistance R_L, point whose coordinates are (C,Ω) must be inside relevant contour. Only in this case SEIG will be able to generate power in steady state. Otherwise, if the point defined by (C,Ω) lies outside the critical contour, self-excitation will fail.

There is the point defined by minimum possible capacitance $C_{\min}$, and also the point defined by maximum possible capacitance $C_{\max}$, on each contour. At this points, self excitation can exist only if relative rotor speed has the exact value. For any other value of capacitance that lies between $C_{\min}$ and $C_{\max}$, relative rotor speed can take random value between boundry values defined by contour. Similiar analysis could be done if relative rotor speed is observed instead of capacitance.

Important conclusion that issues from the Fig. 2 is that dimensions of critical $C-\Omega$ contours expand as load resistance R_L takes higher values. The contour will be largest when generator is not loaded, since $R_L \to \infty$, but this operating state is not of great practical importance. This is the reason why critical contour of SEIG under no load has not been calculated. If Fig. 2 is carefully studied, it could be noticed that dimensions of contours rapidly shrink when load resistance falls under 1 [p.u.]. Intuitive conclusion can be made that there is certain minimum of load resistance $R_{L\min}$, at which critical contour does not exist anymore. Instead of contour, there is only point defined by the exact pair of values (C,Ω). It is obvious that load resistance must not take value lower than $R_{L\min}$, because SEIG will not be able to sustain self-excitation under such circumstances. This critical value of load resistance is function of machine's equivalent circuit parameters, and can not be changed.

However, previous considerations have mostly theoretical meaning. Practical application imposes limitations in rotor speed and capacitance of the capacitors connected to stator. It is not safe to run rotor of a conventional induction machine at speeds that are several times higher than rated synchronous speed, since mechanical stress and vibrations may cause serious damage. Also, from the aspect of economy, it is not reasonable to use large capacitor units, whose prices are exceeding price of induction machine. Having this in mind, it is enough to define only those parts of critical $C-\Omega$ contours that are situated in the region of practically acceptable speeds and capacitances, as shown on Fig. 3.

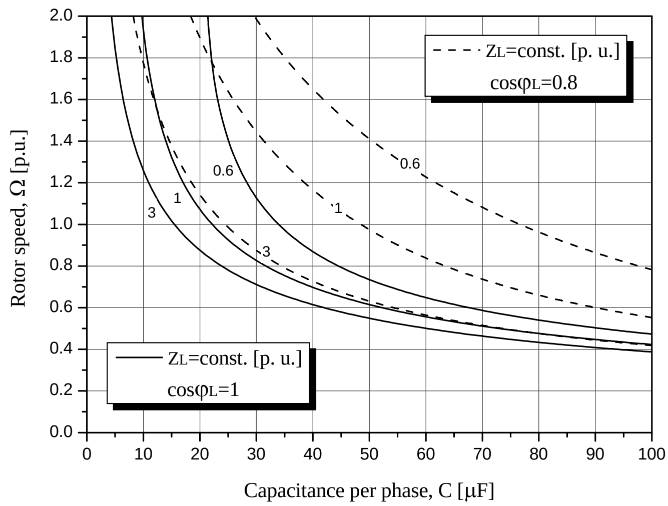

Fig. 3. Practically significant segments of $C-\Omega$ contours

Influence of combined resistive-inductive load can be observed from the Fig. 3. Curves calculated with assumption that load is purely resistive ($\cos\varphi_L = 1$) have been drawn with solid lines. Dashed lines represent critical $C-\Omega$ curves calculated for $\cos\varphi_L = 0.8$. It is obvious that presence of inductivity in load impedance leads to increased demands in applied capacitance, if rotor speed remains constant. In the case that capacitance remains constant, rotor has to run at higher speed to enable self-excitation, if combined resistive-inductive load is connected to the stator.

Curves presented in the Fig. 3 can also be used for quick estimation of possible consequences of desired loading, according to the actual operating state. For example, if machine whose parameters are given in Table I operates as an SEIG with excitation capacitance $C = 20\,\mu F$ and at rotor speed $\Omega = 1.2\,\text{p.u.}$, it will be possible to decrease load resistance from $R_{L1} = 3\,\text{p.u.}$ to $R_{L2} = 1\,\text{p.u.}$, without loss of self-excitation. The previous statement is clear if we notice that the point with coordinates $(C,\Omega) = (20\,\mu F, 1.2\,\text{p.u.})$ is placed above both critical $C-\Omega$ curves. On the contrary, if rotor speed is constant, but has the value $\Omega = 1\,\text{p.u.}$, described manipulation would lead to loss of self-excitation because the point $(C,\Omega) = (20\,\mu F, 1\,\text{p.u.})$ lies below critical curve for $R_{L1} = 1\,\text{p.u.}$

IV. CONCLUSIONS

Critical rotor speeds and excitation capacitances required for sustainable operation of an SEIG loaded with symmetrical three-phase load of arbitrary value and type, have been investigated in this paper. Mathematical difficulties that had been following any intention to solve nonlinear equations describing an SEIG in the past, have been succesfuly eliminated by using *fsolve* routine from MATLAB Optimization Toolbox. Based on this approach, full calculations have been performed, without need to simplify exact equivalent circuit of an SEIG by neglecting core losses.

It has been shown that, for arbitrary load impedance that is greater than minimum allowed value, exact closed contour in $C-\Omega$ plane can be defined. This contour can be considered as a set of critical rotor speed and capacitance combinations that allow operation of an SEIG at the limit of stability. Due to it's nature, it could be named as „critical $C-\Omega$ contour“. For a given load impedance $\underline{Z}_L$, sustainable self-excitation will exist only if the point whose coordinates are (C,Ω) is placed inside the critical contour. Dimensions of critical contour expand as load impedance tends to get higher values. Shape of the contour is not uniform, and for the exact value of $|\underline{Z}_L|$ it will be affected by actual value of load power factor at rated frequency.

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