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Legendre Orthogonal Functional Network Applied in Modeling of Dynamical Systems

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Abstract – **Functional networks represent extension of neural networks, which deal with general functional models. In this paper, we present Legendre-type orthogonal functional network and their application in the modeling of dynamical systems. As an example of the proposed modeling method, we considered the modeling of the hydraulic multi tank system.**

Keywords – **Legendre polynomials, Orthogonal functional networks, Genetic algorithm, Modeling.**

I. Introduction

Functional networks were first introduced in [1, 2]. They represent extension of the standard neural networks and unlike neural networks, they deal with general functional models. Functional network have many advantages, so the problem that can be solved by the neural network, also can be formulated by functional network [1]. There are also many examples that cannot be solved by the neural network but can [1-3] be naturally formulated using the functional network. Neuron functions may be multivariate, multi-argument, and different. Functional network is a very useful general framework for solving a wide range of problems: the solving of differential functional and difference equation [2], nonlinear time series and prediction modeling [3], factorization model of multivariate polynomials [4], the identification of nonlinear system [5], linear and nonlinear regression [4], approximation functions [6] etc. Some of these applications have been developed in [5-7]. In this paper, an orthogonal functional network model is presented whose neuron functions are approximated by Legendre orthogonal basis functions.

The Legendre polynomials and their orthogonal properties were established during eighteenth century. One of the most important applications of Legendre orthogonal functions is designing orthogonal filters [8-11] which are useful for forming orthogonal signal generators, least square approximations and practical realizations of optimal and adaptive systems. This paper describes a method for obtaining classical orthogonal Legendre functions using a new transform. The obtained functions have been used to design orthogonal neurons that enable generating the known orthogonal functions in the form of real physical signals. The parameters and weights of the Legendre orthogonal functional network are determined by genetic algorithm as the learning algorithm and known optimization technique. As a case study, an experimental three tank hydraulic system was considered. Experiments were performed to approve theoretical results and demonstrate that the method described in the paper is very suitable for modeling dynamical systems in the sense of accuracy and algorithm speed.

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II. Legendre orthogonal functions

Consider the orthogonal, shifted Legendre polynomials in their explicit form [11-12]:

$$P_n(x) = \frac{1}{n!}\sum_{j=0}^{n}(-1)^{n-j}\binom{n}{j}\frac{(n+j)!}{j!}x^j \tag{1}$$

These polynomials are orthogonal over interval (0, 1). From Eq. (1) we obtain the sequence of Legendre type orthogonal polynomials:

$$\begin{aligned}
&P_0(x) = 1,\\
&P_1(x) = 2x-1,\\
&P_2(x) = 6x^2-6x+1,\\
&P_3(x) = 20x^3-30x^2+12x-1,....
\end{aligned} \tag{2}$$

After applying the substitution $x = e^{-t}$ into Eq. (1) and Laplace transform, the following rational function can be obtained:

$$W_n(s) = \frac{1}{s}\frac{\prod_{i=1}^{n}(s-i)}{\prod_{i=1}^{n}(s+i)} \tag{3}$$

Denote for $\varphi_n(t) = L^{-1}\left[W_n(s)\right]$ series of Legendre exponential functions orthogonal over interval $(0,\infty)$ with weight $w(t) = e^{-t}$ i.e.:

$$\int_0^\infty \varphi_i(t)\varphi_j(t)e^{-t}dt = \begin{cases} 0, & i \neq j \\ N_i, & i = j \end{cases} \quad (4)$$

Rational functions given with (3) can be factorized and realized in the form of the filter. Each filter section with transfer function (s-i)/(s+i) can be easily practically realized (Fig. 1).

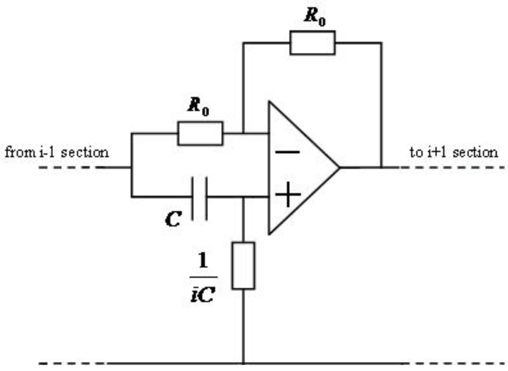

Fig. 1. Practical realization of the single filter section

III. Legendre orthogonal functional networks

Typical architecture of a functional network [6] is given in Fig. 2.

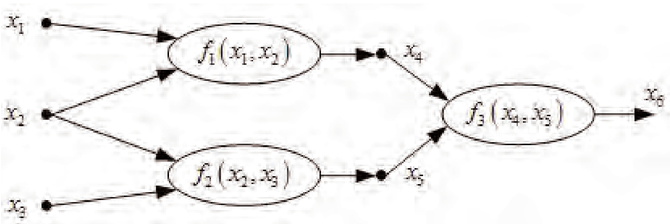

Fig. 2. A functional network model

Besides of the input (x_1, x_2, x_3) and output (x_6) layers, a functional network consists of one or more layers of intermediate storing units (x_4, x_5), which store information produced by neuron units, and one or more layers of processing units (f_1, f_2, f_3). A neuron unit evaluates a set of input values, coming from the previous layer (or input units) and delivers a set of output values to the next layer (or output units). Each neuron has associated neuron function that can be multivariate and can have as many arguments as inputs. Once the input values are given, the output is determined by the neuron type and its function.

If we use orthogonal basis functions for neurons, we can obtain general orthogonal functional network [13] (Fig. 3) with the following output:

$$y = \sum_{i=1}^{n} w_i f_i(x) \quad (5)$$

If we use Legendre orthogonal series described in Section 2 as orthogonal basis, we can design Legendre orthogonal functional networks. Function f_i in this case will represent the first i sections connected in series $f_i(s) = \prod_{j=1}^{i} \frac{s-j}{s+j}$. Output signals of these sections are orthogonal in time domain. These networks can approximate any given function and model any system by adjusting variable weights w_i. These weights are adjusted in such a way that we minimize the error $e = y - f(x)$ where $f(x)$ represents either the function to be approximated or the response to the applied input signal x of the system to be modeled. In our experiments, optimal weights will be determined by genetic algorithm [14-15].

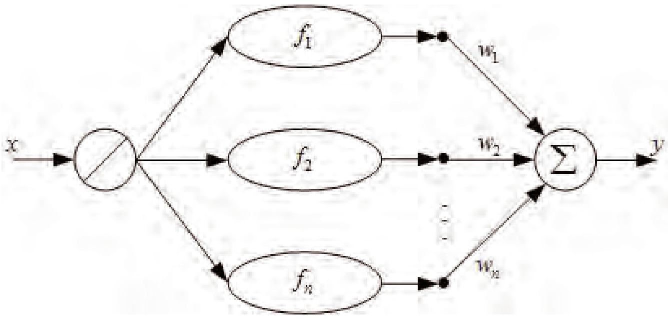

Fig. 3. A general orthogonal function network

IV. Experimental results

For the purpose of verification of our modeling method, we consider a multi tank hydraulic system manufactured by "Inteco", Poland [16] (Fig. 4). The multitank system relates to liquid level control problems commonly occurring in industrial storage tanks. It comprises three separate tanks fitted with drain valves. The separate tank mounted in the base of the set-up acts as a water reservoir for the system. Some of the tanks have a constant cross section, while others are spherical or conical, so having variable cross section (this creates main nonlinearities of the system). Several issues have been recognized as potential impediments to high accuracy modeling and control of level or flow in the tanks: nonlinearities caused by shapes of tanks, saturation-type nonlinearities (introduced by maximum or minimum allowed level in tanks), valve geometry and flow dynamics, pump and valves input/output characteristic curve.

Fig. 4. Multitank hydraulic system

In order to obtain a model for the given hydraulic system, method described in previous section is applied. The same step input signal (x=1) is applied to both the unknown system and Legendre orthogonal functional network with four neurons in the middle layer (four function in orthogonal basis). The only known data about the system is the measured output - tank liquid level H_1 ($f(x)$), given in Fig. 5.

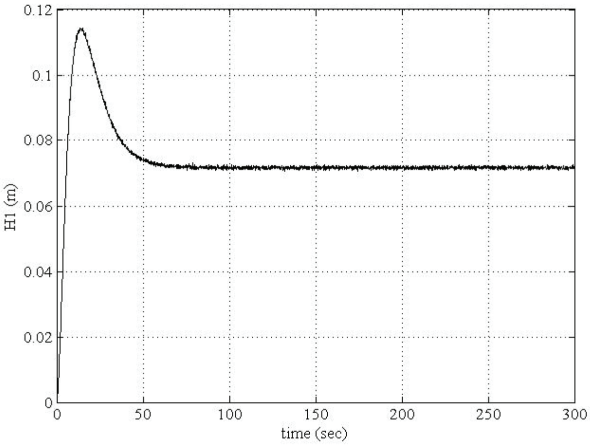

Fig. 5. Step response of unknown hydraulic system ($f(x)$)

The next step is to form difference of measured system output and functional network output as well as to calculate the error. Optimal parameters values for the best model of unknown system are determined by using genetic algorithm. Genetic algorithm used in experiment was with the following parameters: initial population of 200, number of generations 100, stochastic uniform selection, reproduction with 10 elite individuals, Gaussian mutation with shrinking and scattered crossover. The goal of the experiment was to make a error (difference between the outputs from the system and from the functional network) as small as possible for a choosen input, i.e., to obtain the best model of the unknown system. So, we used error as the fitness function for the genetic algorithm. Experimental time was 300 seconds.

After the experiment, following weights for the Legendre orthogonal functional network were obtained: w_1=0.0691, w_2=-0.0602, w_3=-0.0332 and w_4=0.0243. With these weights, final functional network output (y) is given in Fig. 6. We can see from the Figs. 5 and 6 that our functional network now represents accurate model of considered three tank hydraulic system.

V. CONCLUSION

In this paper we present a new method for obtaining models of dynamical systems based on Legendre orthogonal functional networks. Theoretical background on this method is given with full description of Legendre functions. Using these functions, we designed Legendre orthogonal functional network. Filters are described in details and illustrated. The idea for a new method for systems modeling is based on the fact that functions generated by Legendre network are orthogonal in the left semi plane of the complex plane, so this network is convenient for the modeling of continuous systems.

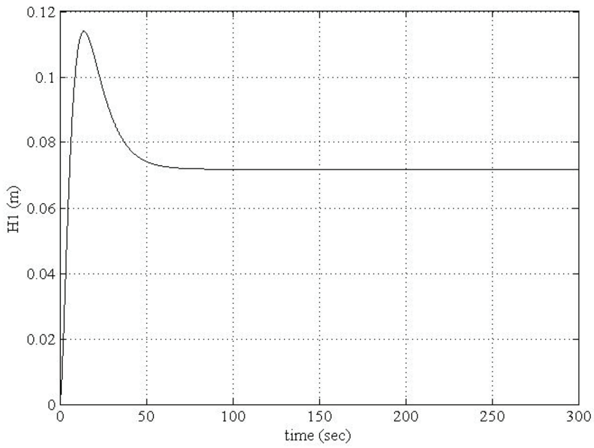

Fig. 6. Step response of Legendre orthogonal functional network (y)

Adjustable network which can be used for modeling of arbitrary systems can be designed on the basis of these functions. Specific models are obtained using adjustable weights. During the modeling of concrete unknown system, these weights are being adjusted in order to obtain the best model of unknown system in the sense of mean square error. Optimal adjustment of the weights is accomplished using genetic algorithms, which have demonstrated very good performances as global optimizers in many types of applications.

In order to verify the obtained theoretical results, a modeling of a multitank hydraulic system is considered. Model of this system is obtained using a proposed modeling method. Experiments have demonstrated that the method described in the paper is very suitable for systems modeling and it achieves excellent results in the sense of modeling algorithm simplicity and speed as well as model accuracy.

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