Identification of One Class of Distributed Parameter Systems Based on Orthogonal Functions

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Abstract – **In this paper the identification issue of one class of distributed parameter systems (DPS) by means of two-dimensional orthogonal polynomials and functions is studied. Suitable m-functions are proposed and numerical examples are conducted to demonstrate the validity and accuracy of the method.**

Keywords – **Identification, Distributed parameter system, Orthogonal functions**

I. INTRODUCTION

Knowledge of the DPS mathematical model is a crucial point in its investigation of dynamic behavior and design of control systems. Two approaches to find models are usually applied: analytical and experimental.

Mathematical modeling in the first approach, which is associated with the design phase of the facility, shall be made on certain assumptions and assumptions that largely determine the accuracy of the model.

The second approach uses identification methods, including questions about the structure of the system model and its relevant parameters. Typically, the type of equations describing the model structure is chosen in advance based on a priori information about the ongoing physical processes and the task of the identification is reduced to parameter estimation. Identification of DPS is far more complex than the case of lumped parameter systems, because the mathematical model of DPS is often partial differential equations (PDE) or integral equations, and the solutions of DPS are concerned with not only initial conditions but also boundary conditions, which increases the complexity.

Two methods of identification of DPS are usually used. The first class of methods uses some optimization approaches, such as Kalman filter algorithm, gradient method, genetic algorithm optimization [10], etc. The second class of methods translates the mathematical model of DPS into algebraic, transcendental or differential equations in advance and then identificates the translated system. Some of the approximation methods are: differential quadrature method (Bellman and Roth, 1979), Walsh functions (Sinha et al. 1980), finite element method (Brahmanandam and Chatterji, 1982), Laguerre polynomials [1, 2, 5], block – pulse functions [4, 6, 7, 8], Haar wavelets [7, 9], etc.

In this paper the attention is focus on the second class of methods. The identification of one class of DPS based on two-dimensional orthogonal polynomials and functions is considered.

The rest of the paper is organized as follows. In section II some mathematical preparations about often used orthogonal functions and polynomials are given. The properties of two-dimensional orthogonal functions are considered. The algorithm of parameter identification for one class of DPS based on two-dimensional orthogonal polynomials and functions is presented in section III. In section IV, numerical simulations to demonstrate the validity and efficiency of the methods are given. Some conclusion in section V is made.

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II. MATHEMATICAL PREPARATIONS

Orthogonal polynomials have very useful properties in the solution of mathematical and physical problems.

The Laguerre polynomials $\lambda(z)$ are defined over a range [0; ∞). They can be defined recursively, defining the first two polynomials as $\lambda_0(z)=1; \lambda_1(z)=1-z$ and then using the following recurrence relation

$$(n+1).\lambda_{n+1}(z)-(2.n+1-z).\lambda_n(z)+n.\lambda_{n-1}(z)=0\,, \qquad (1)$$

for any $n \geq 1$.

The first class of Chebyshev polynomials *T(z)* is defined over a range [-1, 1]. In this paper we briefly call it Chebyshev polynomials. They are orthogonal polynomials with respect to the weighting function $\rho(z)=\dfrac{1}{\sqrt{1-z^2}}$ and can be defined recursively by using the following recurrence relation

$$T_{n+1}(z)-2.z.T_n(z)+T_{n-1}(z)=0\,, \qquad (2)$$

where: $T_0(z)=1; T_1(z)=z$.

A block – pulse function (BPF) *B(t)* is defined over a time interval $t\in[0,T]$ as

$$\{B_i(t)\}, \quad i=1,2,...,m\,, \qquad (3)$$

where:

$$\left.\begin{array}{l} B_i(t)=1, \qquad for\ t\in[0;T/m] \\ B_i(t)=\begin{cases}1, & for\ t\in[(i-1)T/m;\, iT/m] \\ 0, & elsewhere\end{cases} \\ \qquad\qquad for\ i=2,3,...,m \end{array}\right\}$$

The orthogonal set of Haar functions *H(t)* is a group of square waves with magnitude of $\pm 2^{m/2}$ in some intervals and zeros elsewhere. The Haar functions are defined as

$$H_m(t)=H_1(t).(2^j.t-\frac{k}{2^j})\,, \qquad (4)$$

where: $j\geq 0; \quad m=2^j+k; \quad 0<k\leq 2^j$;

$H_1(t)$ - scaling function, pleased during the whole observed interval [0, T].

The two-dimensional orthogonal functions $F_{ij}(x,t)$ are defined as the set of orthogonal functions over the intervals $t \in [0,T],\ x \in [0,X]$ as

$$\{F_{ij}(x,t)\} = F_i(t).F_j(x) \qquad (5)$$

A function y(x,t), absolutely integrable in the region $t \in [0,T],\ x \in [0,X]$, may be approximated to

$$y(x,t) \cong \sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij}F_{ij}(x,t) = F_M^T(t).Y.F_N(x)\ , \qquad (6)$$

where: Y - two-dimensional orthogonal function coefficient matrix of the function $y(x,t)$ $Y = [y_{ij}]_{m\times n}$;

$$y_{ij} = \frac{T.X}{m.n}\int_{(j-1)X/n}^{j.X/n}\int_{(i-1).T/m}^{i.T/m} y(x,t).\,dt.dx\ ;$$

$F_M(t) = [F_1(t)\ \ F_2(t)\ \ ...\ \ F_m(t)]^T$;

$F_N(x) = [F_1(x)\ \ F_2(x)\ \ ...\ \ F_n(x)]^T$

The orthogonal function $F(t)$ has the property

$$\underbrace{\int_0^t ... \int_0^t}_{r-times} F(t).dt^r = P_M^r.F(t) \qquad (7)$$

If $y(x,t) \cong F_M^T(t).Y.F_N(x)$, applying the property (7) to $y(x,t)$ yields

$$\underbrace{\int_0^x ... \int_0^x}_{b-times}\underbrace{\int_0^t ... \int_0^t}_{a-times} y(x,t).dt^a.dx^b \cong F_M^T(t).(P_M^T)^a.Y.P_N^b.F_N(x)\ , \quad (8)$$

where P_M and P_N are correspondingly $(m\times m)$ and $(n\times n)$ integral operational matrices.

III. ALGORITHM OF IDENTIFICATION

A DPS described by the following first order PDE is considered

$$a_1\frac{\partial y(x,t)}{\partial x} + a_2\frac{\partial y(x,t)}{\partial t} + a_3 y(x,t) - u(x,t) = 0\ , \qquad (9)$$

where:
u(x,t) and y(x,t) - are the input and the output of the DPS respectively;
x – the variable of location ($x \in [0;L]$);
t – time ($t \in [0;T]$);
a_1, a_2 and a_3 – the unknown system parameters.

The initial condition is $y(x,0) = f(x)$ and the boundary conditions are $y(0,t) = g(x)$.

Orthogonal polynomials and functions implementation reduces the problem of parameter identification of DPS to a computationally convenient form. The identification process includes the following fundamental steps.

- Expansion of the input u(x, t) and output y(x,t) functions of the PDE into two-dimensional orthogonal polynomials or functions;
- Rewriting of the PDE in the matrix form using the orthogonal functions properties and after some well known manipulations, i.e.

$$A.Q = H\ , \qquad (10)$$

where: Q – vector of the estimation values of the parameters $\hat{a}_i,\ i = \overline{1,3}$.

- Solving of the obtained matrix equation for the vector of unknown parameters using least – squares technique

$$Q = (A^T.A)^{-1}.A^T.H \qquad (11)$$

IV. NUMERICAL SIMULATION

Take a DPS, described by the PDE (9), where the real values of parameters are: $a_1 = 2, a_2 = 4, a_3 = 1$, the input is *u(x,t)=4x+2t+xt, y(x,0)=0, y(0,t)=0,* $x,t \in [0;10]$*)*.

The Laguerre and first class of Chebyshev polynomials, BPF and Haar orthogonal functions are used and m – files are created in Matlab based on the algorithm above. The estimation values of the parameters for a few number *m* of two-dimensional orthogonal polynomials and functions are calculated. The obtained parameters and relative parameter errors E are given in Table 1 and Table 2, where:

$$E = \sqrt{\frac{\sum_{i=1}^{3}(a_i - \hat{a}_i)^2}{\sum_{i=1}^{3}a_i^2}}.100\% \qquad (12)$$

Table 1
Estimation values of the parameters $\hat{a}_i$
and relative parameter errors E by using *m=4*

orthogonal polynomials and functions	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	E [%]
Laguerre	2,0043	3,9965	0,9998	0,12
Chebyshev	1,9979	3,9903	1,0079	0,28
BPF	2,0000	4,0000	1,0000	0
Haar	2,1200	4,2500	0,7000	8,9

Table 2
Estimation values of the parameters $\hat{a}_i$
and relative parameter errors E by using *m=8*

orthogonal polynomials and functions	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	E [%]
Laguerre	2,0004	3,9864	1,0087	0,35
Chebyshev	2,0001	3,9954	0,9994	0,10
BPF	2,0000	4,0000	1,0000	0
Haar	2,0918	4,1836	0,8469	5,59

Obviously, the estimation precision is bigger when block – pulse function is implemented. By using a few number of two-dimensional Haar orthogonal functions the estimation precision is unsatisfactory.

The noise immunity of the identification algorithms is investigated. The output signal y(x,t) is simulated and independent zero – mean white Gaussian noise is applied for this purpose. The ratio nose-to-signal is q = 10% and q=14%. The relative parameter errors E, obtained by applying considered orthogonal polynomials and functions are shown in Figs.1-2.

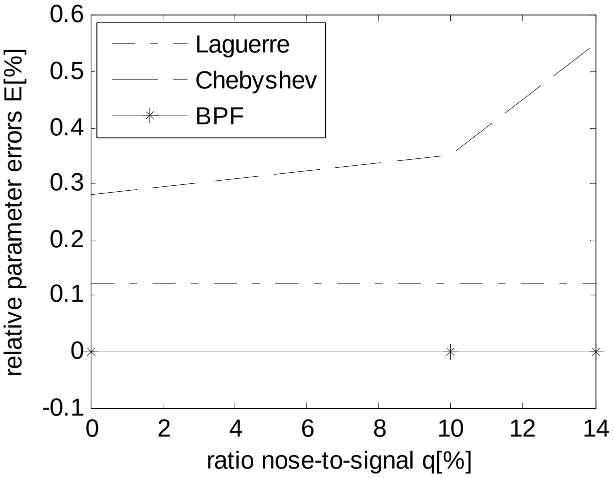

Fig.1 The relative parameter errors, obtained by applying *m=4*

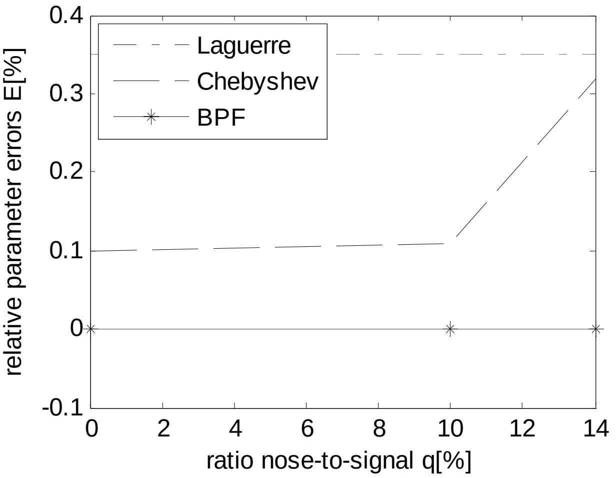

Fig.2The relative parameter errors, obtained by applying *m=8*

As shown in Figs.1-2 the estimates of the parameters via BPF-approach are very accurate.

V. CONCLUSION

The Laguerre and first class of Chebyshev orthogonal polynomials, BPF and Haar orthogonal functions are used for identification of one class of distributed parameter systems. Suitable m – files are created in Matlab based on the considered algorithms. Numerical examples are conducted to demonstrate the validity and accuracy of the method. The estimation values of the parameters for different number of two - dimensional orthogonal polynomials and functions are calculated. The noise immunity of algorithms for different ratio nose-to-signal is investigated.

The following conclusions can be made.

- The algorithms are comparatively simple in form and have low computer memory requirement.
- The obtained identification precision is bigger when BPF are implemented. Results from numerical simulation show that just a few number of BPF are need for high accuracy.
- The parameter estimations can be obtained comparatively accurate when bigger number of Haar orthogonal functions is applied.

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