Block-based Wave Digital Network of an Elliptic Filter in MATLAB/Simulink

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Abstract – **Microstrip structures with two cascaded transmission lines in parallel branches can be modeled and analyzed by use of one-dimensional wave digital approach. A complex structure, divided into uniform segments, is efficiently modeled here by wave digital network. Frequency response is obtained by direct analysis of the block-based network formed in Simulink toolbox of MATLAB environment. An Elliptic filter, proving the response accuracy of the new technique, is given.**

Keywords – **Wave digital approach, wave digital networks, microstrip circuits, Elliptic filter.**

I. INTRODUCTION

Modeling of the planar microstrip structures by wave digital elements, based on well known theory of wave digital filters [1-4], can be efficiently used for analysis in both the time and the frequency domains.

Wave digital network (WDN) represents a wave digital model of a complex microstrip structure. A structure has to be divided into several uniform transmission lines (uniform segments) where each segment is modeled by unit wave digital elements [1-2]. A lossless uniform transmission line is modeled by a two-port digital element with a delay occurring in the forward path – called the unit element (UE) [2]. The port resistances of UE are equal and correspond to the characteristic impedance of uniform segment.

There does not essentially exist just one type of microstrip structures, but a whole variety of quite distinct subclasses. This reflects the richness of WDNs, and the most appropriate one have to be chosen for structure at hand. Till now, stepped-impedance filters, nonuniform structures with linearly tapered lines, and stub-line structures are analyzed by use of suggested one-dimensional (1D) wave digital approach [5-12].

The basic idea of the 1D wave digital approach is to treat the complex structure as a typical connection of several uniform segments. The delays of uniform segments vary from one another, and because of this each segment has to be represented as cascade connection of a certain number of UEs. A way of determining a minimal number of sections in WDN is described in paper [7].

The wave digital model of two cascade-connected transmission lines, which is presented in Section II, represents the background to the modeling strategies that follow.

The WDN is formed directly in the Simulink toolbox of MATLAB environment. Signal flow diagrams are the basis for block-oriented simulation programs such as Simulink. In Section III, a block-based wave digital model of the structure is described.Response in WDN can be found by use of formed block-diagram network and some basic MATLAB functions.

Finally, a simulation validation of the proposed modeling and analysis approaches provided by means of an Elliptic filter is presented and discussed in Section IV.

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II. CASCADED TRANSMISSION LINES AND THEIR WAVE DIGITAL MODEL

In this section, the equivalence between cascade connection of a transmission line and an open stub and its wave digital model is described.

Consider now cascade connection of two transmission lines where the second one is an open stub, Fig. 1.

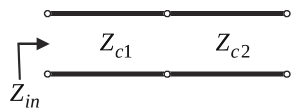


Fig. 1. Cascaded transmission lines

An input impedance of a lossles transmission line terminated by impedance Z_l has complex value

$$Z_{in} = Z_c \cdot \frac{Z_l + S \cdot Z_c}{Z_c + S \cdot Z_l}. \tag{1}$$

According to equation (1) and the fact that $Z_l \to \infty$ for an open line, an input impedance of the casacaded lines shown in Fig. 1 is

$$Z_{in} = \frac{Z_{c1}}{Z_{c1} + Z_{c2}} \cdot \left(S \cdot Z_{c1} + \frac{Z_{c2}}{S} \right). \tag{2}$$

Billinear frequency transformation is

$$S = \left(1 - z^{-1}\right) / \left(1 + z^{-1}\right), \tag{3}$$

where $z = e^{ST}$ is a complex variable and $S = j\Omega$ is a normalized complex frequency.

By replacing the normalized complex frequency in the relation (2) and assigning a new coefficient as

$$\alpha = \left(Z_{c1} - Z_{c2}\right) / \left(Z_{c1} + Z_{c2}\right), \tag{4}$$

the relation for input impedance can be written in form

$$Z_{in} = Z_{c1} \cdot \frac{1 - 2\alpha z^{-1} + z^{-2}}{1 - z^{-2}}. \tag{5}$$

Finally, the reflection coefficient is

$$\frac{B}{A} = \frac{Z_{in} - Z_{c1}}{Z_{in} + Z_{c1}} = \frac{(z^{-1} - \alpha) \cdot z^{-1}}{1 - \alpha \cdot z^{-1}}. \tag{6}$$

Further, a wave digital network depicted in Fig. 2 is observed.

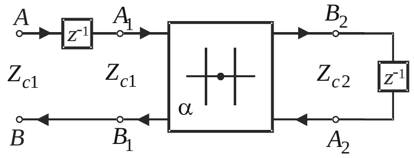


Fig. 2. Wave digital network

An equation system for two-port series adaptor is

$$B_1 = -A_2 - \alpha \cdot (A_1 + A_2),$$
$$B_2 = -A_1 + \alpha \cdot (A_1 + A_2), \tag{7}$$

where the adaptor's coefficient is

$$\alpha = (Z_{c1} - Z_{c2})/(Z_{c1} + Z_{c2}). \tag{8}$$

For the network, the additional relations can be written

$$A_2 = z^{-1} \cdot B_2, \quad A_1 = z^{-1} \cdot A \quad \text{and} \quad B = B_1. \tag{9}$$

The reflection coefficient is found by solving the new equation system obtained by substitution of the additional relations into equation system given by (7)

$$\frac{B}{A} = \frac{(z^{-1} - \alpha) \cdot z^{-1}}{1 - \alpha \cdot z^{-1}}. \tag{10}$$

Comparing the reflection coefficients given by the relations (6) and (10), it can be concluded that cascade connection of two transmission lines (Fig. 1) can be transform into wave digital network (Fig. 2) by using billinear transformation.

III. MATLAB/SIMULINK MODEL

To understand the modeling principle, consider for example the microstrip structure which comprises one parallel branch with cascaded transmission lines. Uniform segments have to be connected as depicted in Fig. 3.

A wave digital network which corresponds to the observed segment connection is depicted in Fig. 4. The uniform segments have different characteristic impedances Z_{ck}, $k = 1,2,3,4$. All uniform segments are modeled by several cascaded UE ($n_k \times T$ blocks), i.e. several sections.

Models of uniform segments UTL1, UTL2 and UTL4 are connected by use of one three-port parallel adaptor with port 2 being dependent. Models of the cascaded transmission line UTL2 and the open stub UTL3 are connected by use of one two-port series adaptor. The cascade-connected segments are connected to adaptor's dependent port 2.

Adaptors are memoryless devices whose task is to perform transformations between pairs of wave variables that are referred to different levels of port resistance.

A network of three-port parallel adaptor with port 2 being dependent is depicted in Fig. 5, [1-4]. The adaptor coefficients α_1 and α_2 are shown explicitly next to the ports 1 and 3, respectively. The coefficients are

$$\alpha_j = 2 \cdot G_j / (G_1 + G_2 + G_3), \qquad j = 1,2, \tag{11}$$

where the port conductances are $G_k = 1/Z_{ck}$, $k = 1,2,3$.

In the symbolic representation of a two-port series adaptor [1-4] given in Fig. 6, it is shown explicitly the parameter α next to the port 1.

Block diagram is a representation of physical structure using blocks. Individual blocks can be put together to

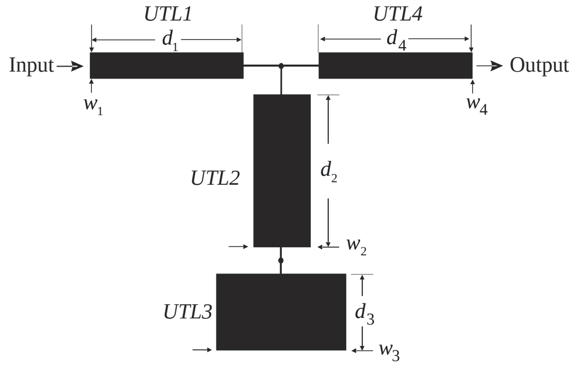


Fig. 3. Segment connection in microstrip structure

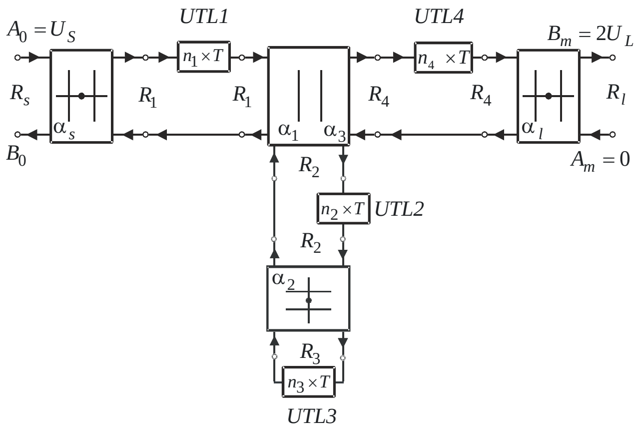
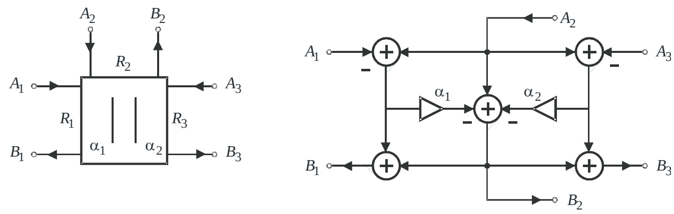
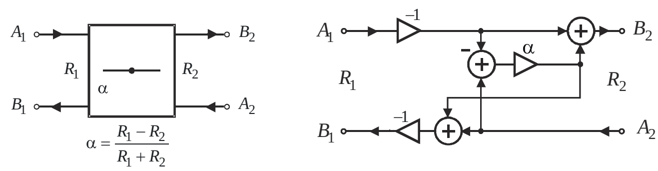


Fig. 4. WDN of the structure given in Fig. 1



(*a*) (*b*)

Fig. 5. Three-port parallel adaptor with port 2 being dependent: (a) symbol, and (b) WDE



(*a*) (*b*)

Fig. 6. Two-port series adaptor: (a) symbol, and (b) WDE

represent the structure in block diagram form. Individual blocks can be the basic blocks or they can be subsystems. They are considered in the text given below.

The formed Simulink model of WDN is depicted in Fig. 7. The blocks **TLine_1**, **TLine_2**, **Stub_3** and **TLine_4** represent uniform segments. The blocks **ADP-S**, **ADP-S23** and **ADP-L** represent two-port series adaptors, and block **ADP_T1S23T4** three-port adaptor. The two-port adaptors at the ends are used for matching source and load resistances to the rest of the WDN. The adaptor coefficients for the blocks **ADP-S** and **ADP-L** in the WDN are

$$\alpha_s = (R_s - Z_{c1})/(R_s + Z_{c1}), \tag{12}$$
$$\alpha_l = (Z_{c2} - R_l)/(Z_{c2} + R_l).$$

For the blocks **TLine_1**, **TLine_2**, **Stub_3**, and **TLine_4**, the Integer Delay blocks from Simulink/Discrete Library are used. The Integer Delay block delays its input by N sample periods. The block accepts one input and generates one output. Its mask is given in Fig. 8.

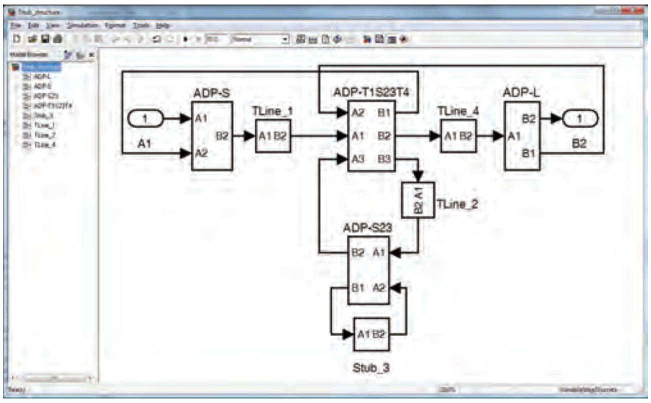


Fig. 7. Simulink model of WDN given in Fig. 4

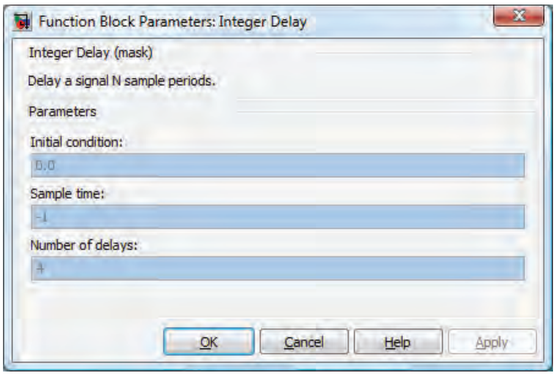


Fig. 8. A mask of the Integer Delay block

The study of discontinuities associated with the circuit (step, T-junction and open stub) is a very important. The simplest modeling approaches of the equivalent discontinuity networks involve changing line lengths. Simulink model given in Fig. 7 can be used in both cases where discontinuity effects aren't callculated and where they are modeled as described in the papers [7], [12], and [14].

A very simple method of analysis of the WDN is a block-diagram method. Each Simulink model represents a series of MATLAB and Simulink commands and functions which are used for its creation. Response is obtained directly in the time domain, and Fourier transformation is used for frequency response calculation.

A major part here is formed Simulink model (mdl-file), whereby the mdl-file is run by m-file that is provided for initialization, response calculation and plotting.

MATLAB built-in functions *dlinmod.m*, *dimpulse.m* and *fft.m* are employed to find a response of observed WDN. *Dlinmod.m* function obtains linear model from discrete system described by block-diagram. *Dimpulse.m* function gives impulse response of discrete-time linear systems. Function *fft.m* is the discrete Fourier transform, i.e. it gives response in the frequency domain.

IV. Approach Verification - An Elliptic Filter

The objective of this section is to prove the accuracy of the proposed modeling and analyzing approaches.

To demonstrate the main idea and approach, a microstrip Elliptic lowpass filter with a bandpass frequency of 1100 *MHz* [15] is depicted. The layout is shown in Fig. 9. The substrate dielectric constant is $\varepsilon_r = 2.55$, and the board thickness is $h = 558.80\,\mu m$. Metalisation is cooper and the metal thickness is $t = 61.4680\,\mu m$. Simulink model of WDN for Elliptic filter is depicted in Fig. 10.

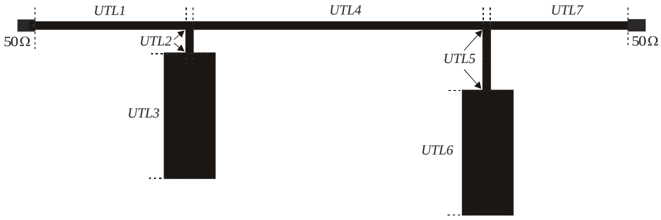


Fig. 9. Layout of lowpass Elliptic filter

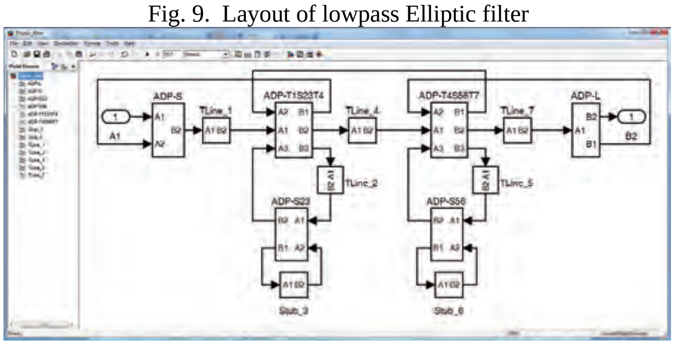


Fig. 10. Simulink model of Elliptic filter

The Elliptic filter is approximated by connection of seven uniform segments with parameters given in the Tables I and II. In order to absorb discontinuity effects new line lengths are counted. T-junction discontinuities are modeled by decreasing lengths of the lines in junctions (UTL1, UTL2, UTL4 in the first junction, and UTL4, UTL5, UTL7 in the other one) [12]. The effects of the open stubs are compensated by increasing lengths of the segments UTL3 and UTL6, [14]. According to the parameters given in the Tables, it can be concluded that the number of transmission line is the same in both cases, but their physical lengths differ one another and beacuse of that their delays differ either. The step discontinuities are modeled by increasing lengths of the lines in junctions (UTL2, UTL3 and UTL5, UTL6 segments are modeled). The characteristic impedances are chosen to be: for the transmission line used for approximation of the series inductance $150\,\Omega$, and for the line used for approximation of the parallel capacitance $5\,\Omega$ [7].

Table I. Segment parameters without modeled discontinuities

nv	d [mm]	w [mm]	Zc [Ohm]	Tv [ps]
1	15.2554	0.4229	92.9455	71.6036
2	2.2298	0.4229	92.9455	10.4659
3	11.7203	3.9569	24.9115	58.6098
4	30.8920	0.4229	92.9455	144.9967
5	6.2308	0.4229	92.9455	29.2454
6	10.0432	3.9569	24.9115	50.2232
7	12.6924	0.4229	92.9455	59.5738

TABLE II. SEGMENT PARAMETERS WITH MODELED DISCONTINUITIES

nv	d [mm]	Zc [Ohm]	Tv [ps]
1	15.0439	92.9455	70.6112
2	2.2403	92.9455	10.5151
3	12.0681	24.9115	60.3492
4	30.4691	92.9455	143.0119
5	6.2413	92.9455	29.2946
6	10.3910	24.9115	51.9625
7	12.4809	92.9455	58.5814

For given error of $n\_er = 0.001\%$, a total minimal number of sections in *WDN* is $n_t = \sum_{k=1}^{7} n_k = 3309$. The numbers of sections in individual segments $n_k = round[q \cdot T_k / T_{\min}]$ are 551, 82, 471, 1115, 228, 405, and 457, respectively. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min} / q = 424.3260\ ps$ where $q = 82$ is a multiple factor and $T_{\min} = \min\{T_1, T_2, ..., T_7\} = 10.5151\ ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{7} T_k = 424.3235\ ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_S = n_t / T_t = 7798.2945\ GHz$. In this case, a relative error of delay is $er = \frac{T_\Sigma - T_t}{T_\Sigma} \cdot 100\ \% = 0.0005685\ \%$. According to the relation (11), the three-port adaptor coefficients are $\alpha_1 = 0.6667$ and $\alpha_2 = 0.5773$. The two-port adaptor coefficients are $\alpha = 0.6667$, and $\alpha_S = -\alpha_L = -0.3004$.

The results obtained by direct analysis of WDN, by linear simulations in the programs GENESYS and ADS (Advanced Design System), and by electromagnetic simulation done in ADS, are depicted in Fig. 11. It can be concluded that WDN and ADS curves are practically identical.

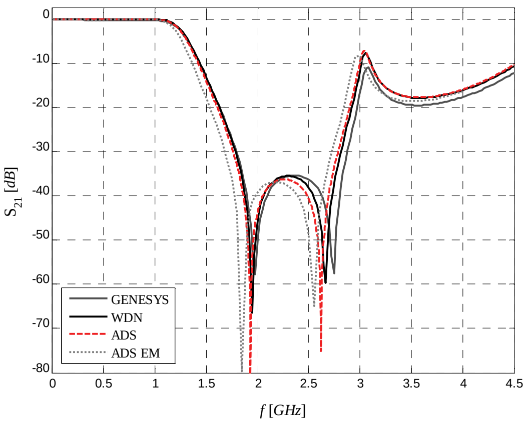


Fig. 11. Frequency response comparison

V. CONCLUSION

After an extensive search in the literature it was found an application of ADS to simulations of different microstrip structures based on their wave digital network representations [13]. The equivalent circuit model is solved through a circuit simulator. The adaptors with many multiply elements were used there.

The main objective of this paper is to give an original and general method to characterize the behaviour of microstrip structures in both the frequency and time domains by use of the wave digital networks in MATLAB/Simulink.

A wave digital approach into MATLAB provides easy simulation of microwave layouts such as microstrip structures of different geometries. Also, allows the user to get the fastest return out of 3D electromagnetic tool investments.

The broadband accuracy of the suggested procedure is validated by an Elliptic filter realized in the microstrip line technique. The analysis result obtained by WDN has shown a very good agreement with those obtained by other programs mentioned above.

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