Optimal Multi-Antenna Reception in Presence of Three-Dimensional Directional Multipath

Dimitar G. Valchev

Abstract – **In this paper the directional aspects of a spatial diversity reception in multipath wireless channels are analysed. The analysis implies that there is a three-dimensional orientation of the receiver multi-antenna array at which the highest possible signal-to-noise ratio (SNR) gain is achieved for a given number of antennas and a given distance between two adjacent antennas.**

Keywords – **Fading channels, Multipath channels.**

I. INTRODUCTION

A very important phenomenon in wireless communications is the small-scale (or multipath) fading channel formed by the interference of numerous waves incident to the receiver in a local volume. The fading causes a fluctuation in the received signal-to-noise ratio (SNR). This degradation is usually overcome by using a spatial diversity system at the receiver [1]. Studies show [2] that capacity increases linearly with the increase of the number of receive branches provided that the fading at the different branches is uncorrelated. However, in realistic channels there is a non-zero cross-correlation between two receiver branches placed at a limited distance. If the fading at two branches is highly correlated, a less diversity gain is achieved. Therefore it is important to study the impact on the resulting SNR using receiver branches with non-zero cross-correlation of the received faded signal envelope.

The multipath propagation channel has a generally non-uniform multipath angular power density (APD) function at the receiver due to the non-isotropic scattering within the wireless environment. In such channels the multipath is characterized by its *directivity*. The directional channel determines different width of the signal cross-correlation function between two branches in a multi-antenna system in different directions [3, 4]. Therefore, for achieving a high SNR gain by the multi-antenna system, while keeping a minimum distance between the antennas, the axis of the antenna array should be aligned with the direction with the most narrow cross-correlation function determined by the particular APD. Thus, two more degrees of freedom are introduced in designing spatial dicersity reception systems – the *azimuth* and *elevation* of the receiver array axis.

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II. APD AND RECEIVER ARRAY TOPOLOGY

The spatial diversity reception is achieved by using a multi-antenna array. For the present analysis a uniform linear array is assumed with uniform angular gain for each antenna with polarization mismatch being neglected. The distance between any two adjacent antennas in the array is d. The axis of the array has an azimuth θ and an elevation ϕ.

The APD function around the receiver may be estimated by using the same multi-antenna array, configured in a direction-of-arrival estimation mode. It may be rotated in the horizontal plane and in a vertical plane for each azimuth. Alternatively, the APD function may be estimated using other array topologies [5]. Once created, the APD function $p(\theta,\phi)$ shows the power that comes to the receiver at a particular azimuth-elevation pair. The geometrical configuration of the receiver multi-antenna array is shown in Fig. 1. The optimal direction of the array axis is the direction of the most narrow cross-correlation function of the received signal given the APD around the receiver as shown in the next section.

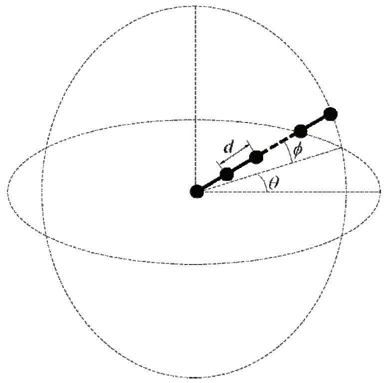

Fig. 1. Multiantenna receiver array geometry

III. SPATIAL CROSS-CORRELATION OF THE RECEIVED SIGNAL ENVELOPE

For a three-dimensional propagation the following approximate and mathematically tractable cross-correlation function has been derived in [4]:

$$\rho(d,\theta,\phi) \approx e^{-15.33\sigma(\theta,\phi)\left(\frac{d}{\lambda}\right)^2} \tag{1}$$

where the quantity $\sigma(\theta,\phi)$ is termed *fading rate variance* of the received signal. The increase of the fading rate variance as a measure of the fluctuation rate leads to a decreased width of the fading signal cross-correlation function. The fading rate variance can be expressed as [4]

$$\sigma(\theta,\phi)=\Upsilon^2\begin{bmatrix}\frac{2}{3}+\xi\left(2\sin^2\phi-\frac{2}{3}\right)\\+\chi\sin 2\phi\cos\left(\theta-\theta_{\phi 45^{\circ}}^{\max}\right)\\+\zeta\cos^2\cos\left(\theta-\theta_{\phi 0}^{\max}\right)\end{bmatrix} \quad (2)$$

where the parameters Υ, ξ, χ, ζ, $\theta_{\phi 45^{\circ}}^{\max}$ and $\theta_{\phi 0}^{\max}$ are termed *multipath shape factors* [4] defined in terms of the multipath APD around the mobile receiver $p(\theta,\phi)$ and its spherical harmonic coefficients S_l^m of *l*-th degree and *m*-th order [6]:

Angular Spread:

$$\Upsilon=\sqrt{1-\frac{S_1^{0\,2}+\left|S_1^1\right|^2}{S_0^{0\,2}}};\quad 0\le\Upsilon\le 1 \quad (3)$$

Elevational Constriction:

$$\xi=\frac{1.5S_2^0S_0^0-\left(S_1^{0\,2}-0.5\left|S_1^1\right|^2\right)}{S_0^{0\,2}-S_1^{0\,2}-\left|S_1^1\right|^2};-0.5\le\xi\le 1 \quad (4)$$

45˚-*Inclined Constriction*:

$$\xi=\frac{2\left|S_2^1S_0^0-S_1^0S_1^1\right|^2}{S_0^{0\,2}-S_1^{0\,2}-\left|S_1^1\right|^2};\quad 0\le\chi\le 1 \quad (5)$$

Azimuthal Constriction:

$$\zeta=\frac{2\left|S_2^2S_0^0-S_1^{1\,2}\right|}{S_0^{0\,2}-S_1^{0\,2}-\left|S_1^1\right|^2};\quad 0\le\zeta\le 1 \quad (6)$$

Azimuth of Maximum Fading at 45˚ *Elevation*:

$$\theta_{\phi 45^{\circ}}^{\max}=\arg\left\{S_2^1S_0^0-S_1^0S_1^1\right\};\quad 0\le\theta_{\phi 45^{\circ}}^{\max}\le 2\pi \quad (7)$$

Azimuth of Maximum Fading at Zero Elevation:

$$\theta_{\phi 0}^{\max}=\arg\left\{S_2^2S_0^0-S_1^{1\,2}\right\};\quad 0\le\theta_{\phi 0}^{\max}\le\pi \quad (8)$$

The expressions for the necessary particular spherical harmonic coefficients S_l^m are

$$S_0^0=\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}p(\theta,\phi)\cos\phi\,d\phi\,d\theta\,, \quad (9)$$

$$S_1^0=\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}p(\theta,\phi)\sin\phi\cos\phi\,d\phi\,d\theta\,, \quad (10)$$

$$S_1^1=\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}p(\theta,\phi)\cos\phi e^{j\theta}\cos\phi\,d\phi\,d\theta \quad (11)$$

$$S_2^0=\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}p(\theta,\phi)\left(\sin^2\phi-\frac{1}{3}\right)\cos\phi\,d\phi\,d\theta \quad (12)$$

$$S_2^1=\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}p(\theta,\phi)\cos\phi\sin\phi e^{j\theta}\cos\phi\,d\phi\,d\theta \quad (13)$$

$$S_2^2=\int_0^{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}p(\theta,\phi)\cos^2\phi e^{j2\theta}\cos\phi\,d\phi\,d\theta \quad (14)$$

It is seen that there is an explicit angular dependence in the expression for the autocorrelation of the received fading signal envelope in (1). This determines an explicit angular dependence for the SNR gain.

For the angular spread Υ=0 there is only one path incoming to the mobile receiver, the expression in (1) reduces to a constant one, implying no capacity gain when using a multi-antenna system compared to a single antenna. For a positive angular spread the correlation and hence the SNR gain depends on the angular orientation of the multi-element antenna. The combined action of the multipath shape factors determines an azimuth-elevation three-dimensional direction of maximum fading rate variance. This is the direction of the most narrow cross-correlation function given by (1). Therefore, the multi-antenna array should be aligned with this direction in order to achieve maximum SNR gain.

IV. Conclusion

This paper addresses the spatial dependence of the SNR gain achieved by a multi-antenna receiver in multipath fading channels. The results show that as the multipath power incoming to the receiver becomes more concentrated around a single direction, less SNR gain is obtained when using receive diversity techniques. For a single faded path the receiver spatial diversity is useless. For a non-zero angular spread of the received multipath power, and for a desired SNR gain, the receiver elements spacing is determined by the pair wise spatial cross-correlation functions of the received signal envelope which depend on the angular distribution of the multipath power, and which influence the resulting received SNR in the spatially diversified channel.

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