Structure Related FD BPM Design of Photonic Devices

Dušan Ž. Djurdjević[1]

Abstract – **The structure related finite difference beam propagation method (SR-FD-BPM) is recently developed simulation technique for photonic and optoelectronic design. This paper reviews some key issues related to this conceptually straightforward and flexible numerical simulation technique. SR-FD-BPM is based on the numerical solution of the 3D vector Helmholtz's equation subject to open boundary conditions. Structure related co-ordinate transformation approach allows the comfortable analysis of wide variety of geometrically complex light-wired photonic structures. Some illustrative design examples are presented in the paper.**

Keywords – **Beam propagation method, Finite difference, Photonics, Optoelectronics, CAD design, Numerical simulations.**

I. Introduction

Advances in photonics and optoelectronics in the 21st century have been mostly driven by the demands of the telecom and datacom boom. Stringent demands in ever-changing photonics industry are to enable production of high-quality components, to increase efficiencies and reduce costs. The research in modelling techniques and the developments of computer-aided design (CAD) software for modelling photonic and optoelectronic components and systems play the crucial and extremely significant role behind the industrial and commercial scene.

The most of integrated photonics and optoelectronic devices and components are built in dielectric waveguide technology. The numerical simulation has become a mandatory approach providing excellent results in photonic design. Amongst several developed numerical techniques, the beam propagation method (BPM) is the most widely used numerical simulation technique for modelling photonic devices. BPM is a particular approach for numerical solving of appropriate approximation of an exact vector Helmholtz's equation (known as the vector paraxial or Fresnel's equation).

FE-BPM, MoL-BPM, FDTD-BPM algorithm, etc., have been developed in recent two decades [1]. One of the most commonly used BPM simulation algorithm is the frequency-domain based finite difference beam propagation method (FD-BPM), [1-4]. FD-BPM gives a numerical solution of vector (or scalar) Fresnel's equation for monochromatic waves (fundamental or higher order mode fields) propagating in the waveguide based photonic structure. Most commercial BPM software is based on FD-BPM.

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FD-BPM is usually implemented in a rectangular co-ordinate system (standard FD-BPM), [2]. If the structure under analysis contains oblique or curved interfaces or when the structure is changing in the direction of the propagation, the inevitable staircase approximation of the boundaries causes serious problems and certain restrictions of the method. To avoid using the fine meshes and small propagation steps, FD-BPM has been reformulated in non-orthogonal structure related (SR) co-ordinate systems, [5]. In SR co-ordinate schemes for FD-BPM the discretisation procedure exactly matches the local geometry of the structure, thus eliminating non-physical scattering due to the staircasing effect.

SR approach is used nowadays to design tapered, oblique and bi-oblique shaped waveguide-based devices, such as z-variant directional waveguide couplers, y-branches, optical interconnects, waveguide polarizers, optical modulators and similar components that include waveguide bends. Some illustrative examples of SR design, including BPM analysis of a sloped walled rib waveguide and 3D SR-BPM analysis of an "S"-curved directional waveguide coupler, are given.

II. Structure related FD-BPM formulation

Attempts to overcome the presence of the staircasing in the FD related numerical methods have been resulted, during the last two decades, in the developing of the so-called improved FD-BPM schemes. The improved FD approach takes into account the boundary conditions for the field and its derivatives near the dielectric interfaces. Contrary to the improved FD approach, SR co-ordinate systems naturally follow sloped or curved geometry of dielectric interfaces. In SR FD-BPM algorithm coarser mesh sizes can be used for the same accuracy, in comparison to the rectangular FD schemes.

A. SR-FD-BPM in the transverse plane

For z-invariant structures (constant cross-section in the transverse plane), the transverse plane (x, y) is transformed in appropriate non-orthogonal SR co-ordinates. As an example of the SR-FD-BPM approach in the transverse plane, some results of the sloped walled rib waveguide analysis are given. The staircasing approximation of the sloped waveguide wall, obtained with the standard FD-meshing procedure in the rectangular co-ordinate system, is shown in Fig. 1.

The simplest case of the scalar Helmholtz's equation in the rectangular co-ordinate system (x, y, z), reads

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2n^2\right]\mathbf{\Phi_t}=0, \qquad (1)$$

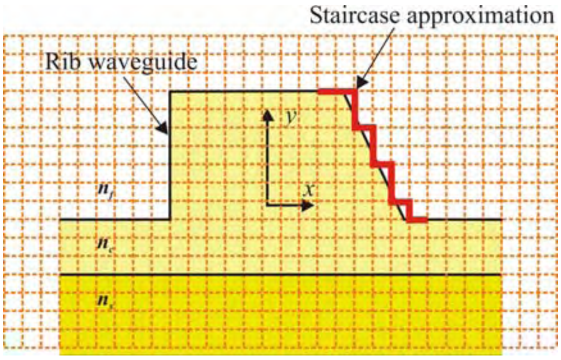

Fig. 1. Staircase approximation forced onto a rectangular grid in the transverse plane, causes non-physical numerical noise in simulation.

where $k = 2\pi/\lambda$ is the free space wave number, λ is the operating wavelength, $n = n(x, y)$ is the z invariant refractive index and $\mathbf{\Phi_t}(x, y, z)$ represents the scalar transverse electric, $\mathbf{E_t}$, or magnetic field, $\mathbf{H_t}$. Eq. (1) is not suitable for obtaining stable numerical algorithms and it is usually replaced with its first order paraxial approximation by assuming a slowly-varying envelope approximation of the transverse field,

$$\mathbf{\Phi_t}(x, y, z) = \mathbf{F_t} e^{-j\beta z}, \tag{2}$$

where $\beta = kn_0$ is an imposed propagation constant for the scalar field envelope $\mathbf{F_t}$. By neglecting the $\partial^2/\partial z^2$ term, the one-way scalar paraxial wave equation, or Fresnel's equation, in the Cartesian orthogonal co-ordinate system is obtained [2],

$$j2kn_0 \frac{\partial \mathbf{F_t}}{\partial z} = \frac{\partial^2 \mathbf{F_t}}{\partial x^2} + \frac{\partial^2 \mathbf{F_t}}{\partial y^2} + k^2(n^2 - n_0^2)\mathbf{F_t} = 0, \tag{3}$$

where n_0 denotes a reference refractive index. Eq. (3) can be rewritten in any non-orthogonal transverse co-ordinate system. If we choose the non-orthogonal oblique co-ordinate system (u, v) in the transverse plane, in which $u = x \mp y \tan\theta$, $v = y$, one can derive the scalar paraxial wave equation in the oblique co-ordinate system as, [5-7],

$$\sigma \frac{\partial \mathbf{F_t}}{\partial z} = \sec^2\theta \frac{\partial^2 \mathbf{F_t}}{\partial u^2} \pm 2\tan\theta \frac{\partial^2 \mathbf{F_t}}{\partial u \partial v} + \frac{\partial^2 \mathbf{F_t}}{\partial v^2} + \kappa^2 \mathbf{F_t}, \tag{4}$$

where $\sigma = j2kn_0$, $\kappa^2 = k^2(n^2 - n_0^2)$, the envelope of the field $\mathbf{F_t} = \mathbf{F_t}(u, v, z)$ and the refractive index $n = n(u, v)$ are functions of the oblique co-ordinates u and v. A sloped angle θ is measured with respect to the negative direction of the y axis. In Eq. (4) the sign "+" stands for the right-hand side type of the oblique co-ordinate system.

In the case of a tapered co-ordinate system (t, v) in the transverse plane, in which $t = \tan\theta$, $x = t(y - y_0)$, $v = y$, the scalar paraxial wave equation can be expressed as, [5-7],

$$\sigma \frac{\partial \mathbf{F_t}}{\partial z} = \frac{\partial^2 \mathbf{F_t}}{\partial v^2} - \frac{2t}{v - v_0} \frac{\partial^2 \mathbf{F_t}}{\partial v \partial t} + \frac{1}{(v - v_0)^2} \frac{\partial}{\partial t}\left[(1 + t^2)\frac{\partial \mathbf{F_t}}{\partial t}\right] + \kappa^2 \mathbf{F_t}, \tag{5}$$

where the envelope of the scalar field $\mathbf{F_t} = \mathbf{F_t}(t, v, z)$ and the refractive index $n = n(t, v)$ are functions of the tapered co-ordinates t and v. The origin of tapered co-ordinate system (x_0, y_0) is given parametrically as $v_0 = y_0 = x_0 \cot\theta$.

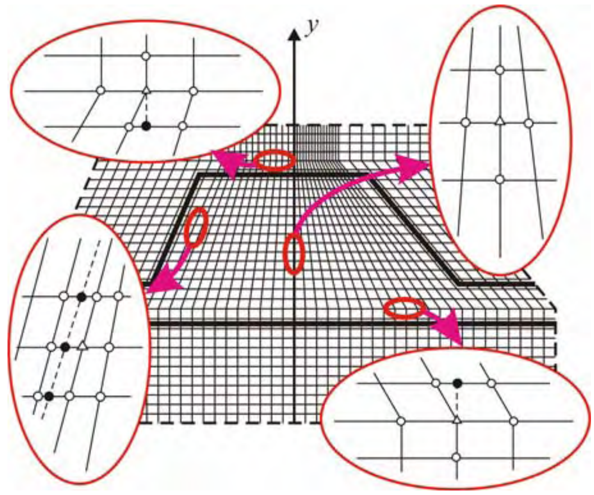

Fig. 2. SR "ROTOR" scheme of FD meshing, [6]. Interpolated field regions are shown in the small ellipses and interpolation procedures are shown in the zoomed ellipses (solid circles denote interpolated and empty circles denote directly sampled field values).

Each discretisation scheme offers potential benefits. By using hybrid SR "ROTOR" (**r**ectangular-**o**blique-**t**apered-**o**blique-**r**ectangular) FD-scheme, where non-orthogonal systems are coupled with rectangular one, FD-discretisation, where oblique interfaces are modelled exactly, can be obtained (co-ordinate SR lines are parallel with dielectric interfaces). Fig. 2 shows FD-discretisation with "ROTOR"-type scheme of an asymmetrical sloped walled rib waveguide. The deficiency of the hybrid FD-schemes is the necessity to join different co-ordinate systems. However, this can be done by using a simple linear approximation. Interpolation procedures are illustrated in Fig. 2 and explained in [6].

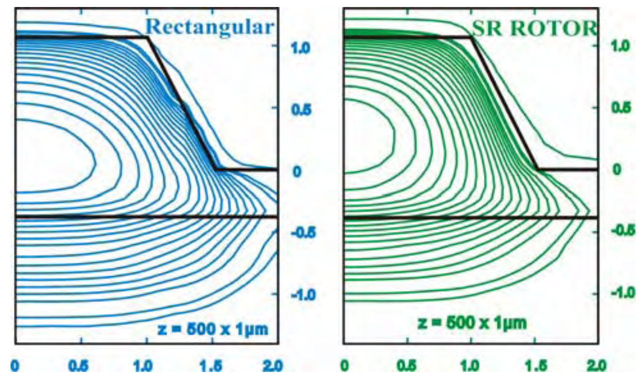

Fig. 3. The filed plots obtained with the standard rectangular and SR "ROTOR" FD-BPM numerical simulations, [6].

By FD-discretising transverse waveguide field $\mathbf{F_t}$ (Eqs. (3)) in the appropriate waveguide regions, an FD-BPM algorithm can be easily developed. BPM field plots obtained by standard rectangular and SR "ROTOR" numerical simulation algorithm (both emloying a standard Crank-Nicolson (CN) method, an imaginary distance (ID) mode solver and the TBC boundary conditions at the edges of the computational window, [6]), after 500 BPM steps ($\triangle z = 1\mu$m), with $\triangle x = \triangle y = 0.1\mu$m, are given in Fig. 3.

Field plots are calculated for a symmetrical sloped walled rib waveguide ($\theta = 25^0$, $n_f = 1$, $n_c = 3.44$, $n_s = 3.34$), with $n_{reff} = n_0 = 3.40483$ obtained by ID mode solver. The deterioration of the field obtained in the standard FD-BPM simulation near the oblique boundaries is clearly indicated. SR-FD-BPM allows simulations with coarser meshes to a prescribed accuracy in comparison to the standard rectangular BPM, offering savings in computational time and memory.

B. SR-FD-BPM in the longitudinal direction

SR-FD-BPM approach is successfully applied to the photonic waveguide structures employing curved waveguide sections in the propagation direction. The 3D plot of the "S"-curved directional waveguide coupler is given in Fig. 4. 3D geometry of the curved directional "Y"-branch is presented in Fig. 5. Staircase approximation forced onto a rectangular grid in the longitudinal direction is shown in Fig. 6.

The approach of the longitudinally dependent SR-FD-BPM is illustrated by formulating the one-way scalar paraxial SR wave equation in the general 3D non-orthogonal co-ordinate system (u, v, w). If we choose the co-ordinates (u, w) as $x = f(u, w)$, $z = w$ and $y = v$, under the one-way paraxial approximation, assuming that propagation occurs in the w direction, $+w$, (i.e. in the $+z$ with rectangular co-ordinates),

$$\mathbf{\Phi_t}(u, v, w) = \mathbf{F_t}(u, y, w)e^{-j\beta w}, \tag{6}$$

we obtain the scalar SR wave equation from Eq. (1), [5], [9],

$$L\frac{\partial}{\partial w}\mathbf{F_t} = M\mathbf{F_t}\,, \tag{7}$$

where the operators L and M are shown to be, [5],

$$L = 2j\beta C + B\frac{\partial}{\partial u}, \tag{8}$$

$$M = A\frac{\partial^2}{\partial u^2} + (Bj\beta - D)\frac{\partial}{\partial u} + C(k^2 - \beta^2) + \frac{\partial^2}{\partial y^2}. \tag{9}$$

In Eqs. (6) and (7) the field envelope $\mathbf{F_t} = \mathbf{F_t}(u, y, w)$ can be either $\mathbf{E_t} = \mathbf{E_t}(u, y, w)$ or $\mathbf{H_t} = \mathbf{H_t}(u, y, w)$. $A = A(u, w)$, $B = B(u, w)$, $C = C(u, w)$ and $D = D(u, w)$ are functions of the partial derivatives of $f(u, w)$, [5], [9],

$$A = 1 + \left(\frac{\partial f}{\partial w}\right)^2,\;\; B = 2\left(\frac{\partial f}{\partial u}\right)\left(\frac{\partial f}{\partial w}\right),\;\; C = \left(\frac{\partial f}{\partial u}\right)^2,$$

$$D = \frac{1}{\frac{\partial f}{\partial u}}\left[A\frac{\partial^2 f}{\partial u^2} - B\frac{\partial^2 f}{\partial w\,\partial u} + C\frac{\partial^2 f}{\partial w^2}\right]. \tag{10}$$

The SR approach allows to designer the flexibility in analysis to use any curvature function $x = f(u, w)$ providing optimal integrated optic requirements. A standard CN method can be easily introduced in the 3D SR-FD-BPM algorithm, and for the well-confined waveguide fields TBCs are typically applied in a standard way.

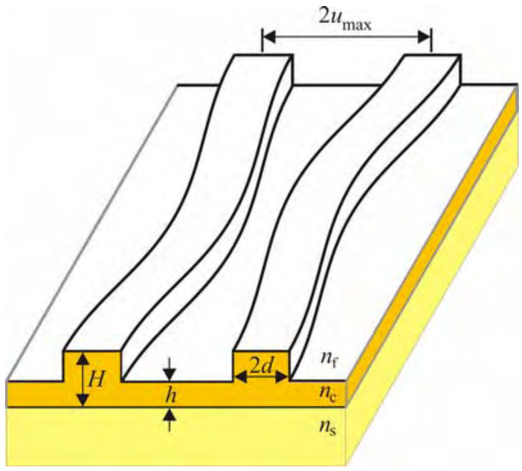

Fig. 4. Schematic diagram of the "S"-curved 3D coupler.

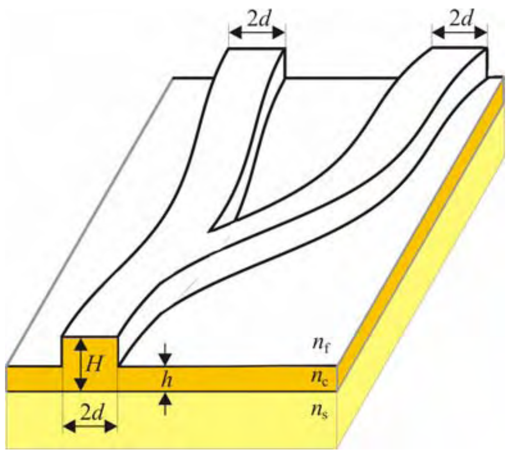

Fig. 5. Schematic diagram of the curved 3D "Y"-branch.

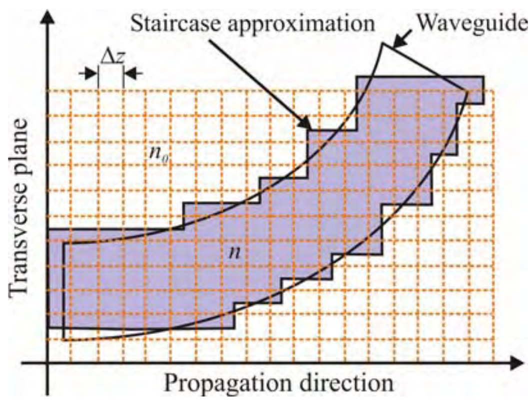

Fig. 6. Staircase approximation forced onto a rectangular grid in the longitudinal direction, causes non-physical scattering of the field.

Examples of 3D curved directional couplers design are presented to highlight the effectiveness of the SR FD-BPM. A symmetrical directional coupler made from two identical and adjacent but spatially separated curved input (I) and output (II) waveguides is shown in Fig. 4.

The curvature function $x = f(u, w)$ of an "S" curved coupler can be given parametrically or explicitly. A cosine type SR geometry was considered in [9], where functions A, B, C and D, Eq. (10), are easily obtained analytically, however, they can be even computed numerically. If the curvature function changes slowly with w, only the function C varies, thus we can introduce $A \simeq 1, B \simeq 0, D \simeq 0$, which is usually the case except for the sharp guide bends. The discretisation mesh in the (u, w) plane is shown in Fig. 7.

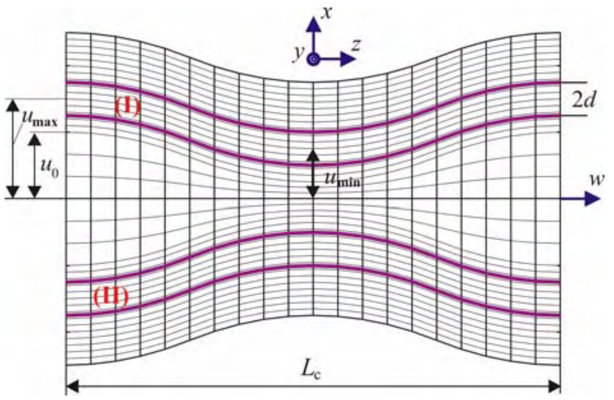

Fig. 7. Geometry of 3D coupler in SR co-ordinate system.

The total coupling length L_c is calculated, with both rectangular and SR based FD-BPM algorithms, for a 3D symmetrical rib waveguide coupler of the cosine type SR geometry and with (u, w) plane defined as in Fig. 7. Rib waveguides parameters are $n_f = 1$, $n_c = 3.44$, $n_s = 3.4$, $H = 1\mu$m, $h = 0.5\mu$m, $2d = 3\mu$m, $\lambda = 1.15\mu$m. The total coupling of the fundamental TE mode $(n_{ref_{TE}} = 3.41313)$ is obtained to be $L_{c_{TE}} = 4826\mu$m, with $u_{max} = 3.0\mu$m and $u_{min} = 1.8\mu$m. For TM-mode $(n_{ref_{TM}} = 3.41161)$ the total coupling length $L_{c_{TM}} = 4769\mu$m is obtained.

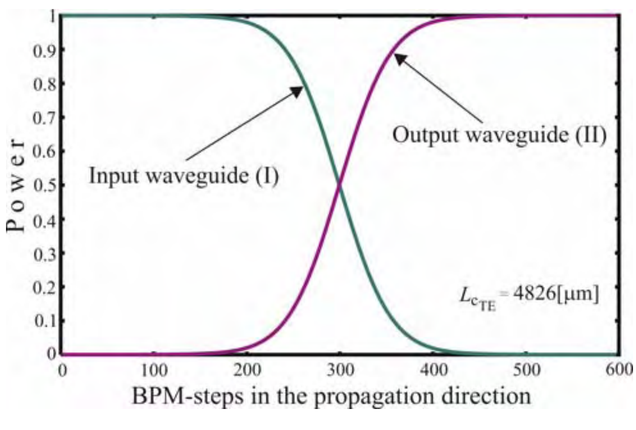

Fig. 8. Evolution of the total power transfer of the fundamental TE-mode field during the propagation in 3D directional coupler, [9].

The total power transfer evolution of the fundamental TE-field, using the *H*-field formulation of the algorithm, is shown in Fig. 8. The almost complete power transfer occurs in the coupler region where rib waveguides are closest to each other.

The SR scheme enables more accurate 3D simulations of the total coupling. In the SR scheme the simulation time is considerably shorter for the same order of accuracy in comparison to the rectangular FD-schemes. This leads to the conclusion that the accuracy of the BPM simulations depends strongly and mostly on the method used for the FD-discretisation of the dielectric waveguide boundaries.

III. Conclusion

Features of the co-ordinate transformation based structure related beam propagation method have been addressed and reviewed. The main advantage of the SR-based method is the exact modelling of the (transverse or/and longitudinal) geometry of the structure under analysis. Presented results obtained in SR-FD-BPM waveguide and directional coupler fundamental mode field simulations demonstrate the advantages and generality of SR over standard rectangular approach. The SR-FD-BPM method enable design flexibility, offering savings in both computational time and memory, thus it is an ideally suited method for optoelectronic CAD design.

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