Effects of Carrier Phase Error on QPSK Receiver over Nakagami-*m* Fading/Gamma Shadowing

Goran T. Djordjevic[1], Dejan N. Milic[2], Bojana Z. Nikolic[3] and Aleksandra M. Cvetkovic[4]

***Abstract* – In this paper, taking the imperfect reference signal extraction into account we analyze the reception of quaternary phase-shift keying (QPSK) signals over the composite fading channel. The composite fading channel is described by Nakagami-*m* probability density function (PDF) whose average power is also a random process with gamma distribution. The effects of multipath fading severity, shadowing and imperfect reference signal recovery on bit error rate (BER) performance are studied.**

***Keywords* – Bit error rate, Fading, Shadowing.**

I. Introduction

In wireless communication channels, short-term (multipath) fading caused by combination of delayed, reflected scattered and diffracted signal components, and long-term fading (shadowing) due to local topography in receiver surrounding occur simultaneously leading to the composite fading [1]-[3]. Composite fading is often modeled by Nakagami-*m* probability density function (PDF) whose mean power is random process with lognormal distribution. However, a composite PDF, obtained in this way, is in integral form and it is not convenient for further analysis and mobile radio system design. For this reason, equivalent gamma distribution is more often used for describing slow fading. In [2], [3], the relations between lognormal and equivalent gamma distribution was determined. Consequently, random fast fluctuations of the signal envelope in composite fading channel can be described by Nakagami-*m* PDF while the average power is also a random process with gamma PDF.

In previous papers, concerning coherent signal detection in composite fading channel [1], [2], it was strictly assumed that reference signal carrier is perfect i.e. the phase of the reference signal carrier is perfectly extracted. It is, however, clear that results, obtained in this way, are idealized and they should be taken as the most optimistic in given circumstances. In practical realizations of the receivers, due to the presence of a noise and fading in a channel, the extraction of the reference signal carrier is imperfect i.e. there is a certain difference between the phase of the incoming signal and the phase of the extracted carrier.

The influence of the imperfect reference signal carrier extraction has been considered in previous papers only in multipath fading channels, without taking into account a shadowing effect [4], [5]. In [4] the error probability has been determined in the case of digital binary and quaternary phase-shift keying (BPSK and QPSK) signal detection in the presence of Gaussian noise, Nakagami-*m* fading and imperfect reference signal carrier extraction. It was assumed that phase error has uniform distribution. In [5] the derivation procedure of the general explicit analytical expression for average symbol error probability for MPSK signal detection in the presence of Nakagami-*m* fading, Gaussian noise and imperfect phase reference has been presented. It was assumed that reference signal recovery is performed by phase-locked loop (PLL) from unmodulated pilot signal.

The contribution of this paper is in the estimation of the bit error rate (BER) in detection of QPSK signal, propagating over the composite fading channel. A reference signal carrier extraction is performed by PLL from unmodulated signal and is assumed imperfect. A certain phase error exists and represents the difference between the phase of the incoming signal and the phase of the recovered reference signal carrier. This phase error is a random process whose instantaneous values have Tikhonov PDF [5]. The BER performance of QPSK receiver are obtained and discussed.

II. Channel and Receiver Model

A. Channel model

The PDF of the envelope *r* in the Nakagami-*m* channel is

$$p_r(r)=\frac{2}{\Gamma(m_m)}\left(\frac{m_m}{\Omega_m}\right)^{m_m} r^{2m_m-1}\exp\left(-\frac{m_m}{\Omega_m}r^2\right) \qquad (1)$$

where $\Gamma(.)$ is the Euler Gamma function [6], m_m is the severity parameter of multipath fading and Ω_m is the mean envelope power $\Omega = E\left\{r^2\right\}$, where $E\{.\}$ denotes the expectation operator. The more the value of m_m, the less the fading severity is. When there is no shadowing, the average power

[1]Goran T. Djordjevic is with Faculty of Electronic Engineering, Department of Telecommunications, Aleksandra Medvedava 14, 18000 Nis, Serbia, E-mail: goran@elfak.ni.ac.rs

[2]Dejan N. Milic is with Faculty of Electronic Engineering, Department of Telecommunications, Aleksandra Medvedava 14, 18000 Nis, Serbia, E-mail: dejan.milic@elfak.ni.ac.rs

[3]Bojana Z. Nikolic is with Faculty of Electronic Engineering, Department of Telecommunications, Aleksandra Medvedava 14, 18000 Nis, Serbia, E-mail: bojana.nikolic@elfak.ni.ac.rs

[4]Aleksandra M. Cvetkovic is with Faculty of Electronic Engineering, Department of Telecommunications, Aleksandra Medvedava 14, 18000 Nis, Serbia, E-mail: caleksandra@gmail.com

Ω_m is constant, but when shadowing is present, Ω_m is the stochastic variable, and consequently eq. (1) can be rewritten in the form

$$p_{r/\Omega_m}(r/\Omega_m)=\frac{2}{\Gamma(m_m)}\left(\frac{m_m}{\Omega_m}\right)^{m_m} r^{2m_m-1}\exp\left(-\frac{m_m}{\Omega_m}r^2\right), \quad (2)$$

where Ω_m has lognormal distribution

$$p_{\Omega_m}(\Omega_m)=\frac{1}{\sqrt{2\pi\sigma_{sh}^2}\,\Omega_m}\exp\left(-\frac{(\ln\Omega_m-\mu)^2}{2\sigma_{sh}^2}\right), \quad (3)$$

where $\mu = \ln P_r$ and σ are mean value and standard deviation of $\ln\Omega_m$. In this formula, μ and σ are expressed in nepers. The average received signal power is denoted by P_r. By σ_{sh} we denote the shadow standard deviation. Standard deviation expressed in decibels (shadowing spread) is given by $\sigma_{shdB} = 8.686\sigma_{sh}$.

The PDF of envelope when multipath fading and shadowing are simultaneously present is given by

$$p_r(r)=\int_0^\infty p_{r/\Omega}(r/\Omega)p_\Omega(\Omega)d\Omega \quad (4)$$

that is not a closed form solution and is inconvenient for further analysis. Therefore, the lognormal distribution is approximated by equivalent gamma PDF

$$p_{\Omega_m}(\Omega_m)=\frac{1}{\Gamma(m_s)}\left(\frac{m_s}{\Omega_s}\right)^{m_s}\Omega_m^{m_s-1}\exp\left(-\frac{m_s}{\Omega_s}\Omega_m\right). \quad (5)$$

By replacing (2) and (5) in (4), the PDF of the composite envelope of the Nakagami-*m*/gamma shadowing signal is derived as

$$p_r(r)=\frac{4}{\Gamma(m_m)\Gamma(m_s)}\left(\frac{m_m m_s}{\Omega_s}\right)^{(m_m+m_s)/2}$$

$$\times r^{m_m+m_s-1}K_{m_s-m_m}\left(2r\sqrt{\frac{m_m m_s}{\Omega_s}}\right), \quad (6)$$

where $K_\nu(.)$ is the modified Bessel function of the second kind and order ν [6].

The relations among parameters of original lognormal distribution (3) and equivalent gamma distribution (5) can be established by using two different approaches. In [3] by using moment matching method the relations among parameters of lognormal and gamma PDF are

$$m_s=\frac{1}{\exp(\sigma_{sh}^2)-1}, \quad \Omega_s=P_r\sqrt{\frac{m_s+1}{m_s}}. \quad (7)$$

i.e.

$$\sigma_{sh}=\sqrt{\ln\frac{m_s+1}{m_s}}, \quad \mu=\ln\left(\Omega_s\sqrt{\frac{m_s}{m_s+1}}\right). \quad (8)$$

In [2], the parameters of lognormal and gamma PDF are related through

$$\sigma_{sh}=\frac{10}{\ln 10}\sqrt{\psi'(m_s)}\ \text{(dB)}, \quad \mu=\frac{10}{\ln 10}\left(\ln\frac{\Omega_s}{m_s}+\psi(m_s)\right)\ \text{(dB)}, \quad (9)$$

where $\psi(.)$ is the digamma function and $\psi'(.)$ is the trigamma function defined as [6]

$$\psi(z)=\frac{d}{dz}\ln\Gamma(z)=\frac{\Gamma'(z)}{\Gamma(z)}, \quad \psi'(z)=\frac{d^2}{dz^2}\ln\Gamma(z). \quad (10)$$

The illustration of signal propagation between transmitter and receiver in the composite fading environment is given in Fig. 1. Three statistically phenomena should be noted: deterministic path loss, slow shadowing and fast multipath fading.

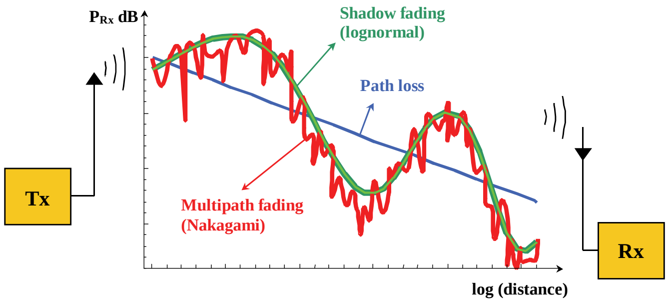

Fig. 1. Illustration of composite fading channel

B. Receiver model

The input signal of the receiver can be written in the form

$$s(t)=r(t)\cos(\omega_0 t+\gamma(t)+\phi_n)+n(t), \quad (14)$$

where $r(t)$ is a signal envelope, $\gamma(t)$ is a random phase of the received signal, which appeared due to multipath propagation and ϕ_n is a information bearing signal that can take one of the following values: {π/4, 3π/4, -3π/4, -π/4}. Additive white Gaussian noise (AWGN) with zero mean value and variance σ^2 is denoted by $n(t)$. Since the signal propagates over the composite fading channel, the envelope $r(t)$ of that signal is a random process and its instantaneous values have PDF given by (6).

The multipath Nakagami-*m* fading parameter is denoted by m_m. A greater parameter m_m indicates a lower fading severity. The range of this parameter's values is: $0.5 \le m_m < \infty$. The shadowing parameter is denoted by m_s. The greater the parameter m_s the smaller the shadowing is. The average signal power is defined as $\overline{r^2} = \int_0^\infty r^2 p_r(r) dr = \Omega_s$. It can be shown that the PDF of the instantaneous signal-to-noise ratio (SNR) is given by

$$p_\gamma(\gamma) = \frac{2}{\Gamma(m_m)\Gamma(m_s)}\left(\frac{m_m m_s}{\gamma_0}\right)^{(m_m+m_s)/2} \times \gamma^{\frac{m_m+m_s-2}{2}} K_{m_s-m_m}\left(2\sqrt{\frac{m_m m_s}{\gamma_0}\gamma}\right), \quad (15)$$

where γ_0 is an average symbol SNR. The relation between the average symbol and bit SNR is: $\gamma_0 = \gamma_{0b} \log_2 M$, where M denotes the number of phase levels and it is worth $M = 4$ for QPSK. The average bit SNR is represented by γ_{0b}.

The signal $s(t)$ is first filtered by a band pass filter and then led into the upper and lower branch of the receiver. In upper branch the signal is multiplied by the recovered signal from the PLL, $2\cos(\omega_0 t + \hat{\gamma}(t))$, and the resulting signal is then filtered by a low pass filter. The signal in the lower branch is multiplied by the recovered signal from the PLL, phase shifted for the value of π/2, $-2\sin(\omega_0 t + \hat{\gamma}(t))$, and then filtered by the low pass filter. It should be noticed that the estimated phase is denoted by $\hat{\gamma}(t)$. A difference between the phase of the incoming signal and the estimated phase is $\varphi(t) = \gamma(t) - \hat{\gamma}(t)$. This phase difference is a random process. Based on the signal values in the upper and lower branch of the receiver, a decision which symbol is sent by the transmitter has been made. The signals in the upper and lower branch of the receiver are

$$z_u(t) = r(t)\cos(\phi_n + \varphi(t)) + x(t), \quad (16)$$

$$z_l(t) = r(t)\sin(\phi_n + \varphi(t)) + y(t). \quad (17)$$

where $x(t)$ and $y(t)$ are Gaussian noise components with zero mean value and variance σ^2.

After the analysis of the receiver operating, an expression for the conditional BER can be derived

$$P_{e/\gamma,\varphi}(\gamma,\varphi) = 0.25\left\{erfc\left(\sqrt{\gamma/2}(\cos\varphi - \sin\varphi)\right),\right. \left. + erfc\left(\sqrt{\gamma/2}(\cos\varphi + \sin\varphi)\right)\right\}, \quad (18)$$

where erfc(.) is the complementary error function [6]. This is the BER conditioned on the phase error, φ, and the instantaneous SNR, γ.

When the reference signal carrier phase is extracted from unmodulated pilot signal, the phase error has Tikhinov PDF [5]

$$p_\varphi(\varphi) = \frac{\exp(\gamma_{PLL}\cos\varphi)}{2\pi I_0(\gamma_{PLL})}, \quad -\pi \le \varphi < \pi, \quad (19)$$

where γ_{PLL} is the SNR in PLL circuit and it is related to standard deviation of phase error by

$$\gamma_{PLL} = \frac{1}{\sigma_\varphi^{\,2}}. \quad (20)$$

The average BER can be obtained by averaging the conditional BER over the instantaneous SNR, γ, and the phase error, φ,

$$P_e = \int_0^\infty \int_{-\pi}^{\pi} P_{e/\gamma,\varphi}(\gamma,\varphi) p_\gamma(\gamma) p_\varphi(\varphi) d\varphi d\gamma. \quad (21)$$

III. Numerical results

In this Section, we present the numerical results obtained by applying numerical integration in (21) and simulation results obtained by Monte Carlo simulations.

Fig. 3 presents the influence of standard deviation of phase error on BER performance. It should be noticed that stochastic phase error causes appearance of irreducible error floor. When SNR tends to infinity this BER floor remains constant and depends only on phase noise standard deviation.

Fig. 4. presents the influence of multipath fading severity on BER performance of partially coherent QPSK receiver. The BER values decrease with m_m increasing (severity decreasing).

Fig. 5. illustrates the influence of shadowing on BER performance. The shadowing has considerable influence on the BER. In the case of $\sigma_\varphi = 10^o$, for $\gamma_{0b} = 22$ dB, BER is 8.6×10^{-5} when $m_s = 10$ (light shadowing) and 3.6×10^{-3} when $m_s = 1$ (heavy shadowing). It should be also noticed that as m_s increases, the BER performance becomes less sensitive to m_s.

Fig. 6 shows the average SNR values in the channel that are required for achieving BER of 10^{-4}, for different severities of multipath fading and shadowing intensities. For example, if the $m_m = 3$ and $m_s = 5.25$, the minimum required value of SNR in the channel is 19 dB.

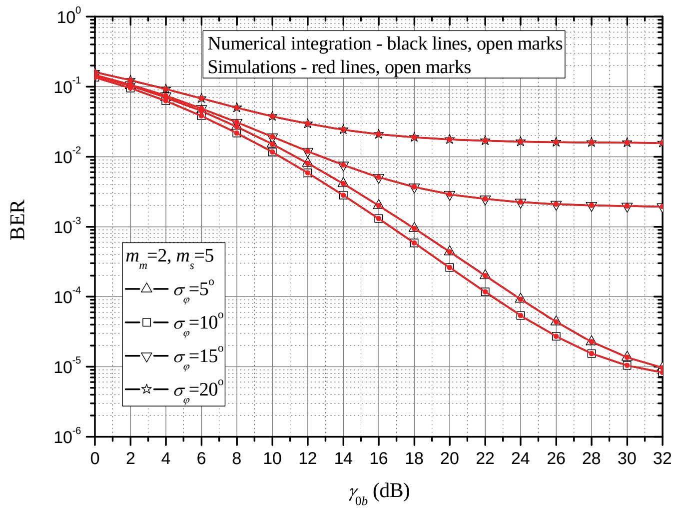

Fig. 1. BER versus SNR for different values of standard deviation of phase noise

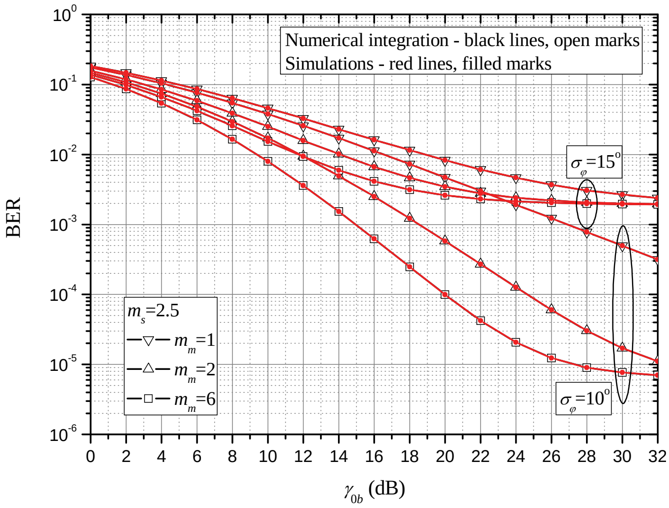

Fig. 4. BER versus SNR for different values of multipath fading severity

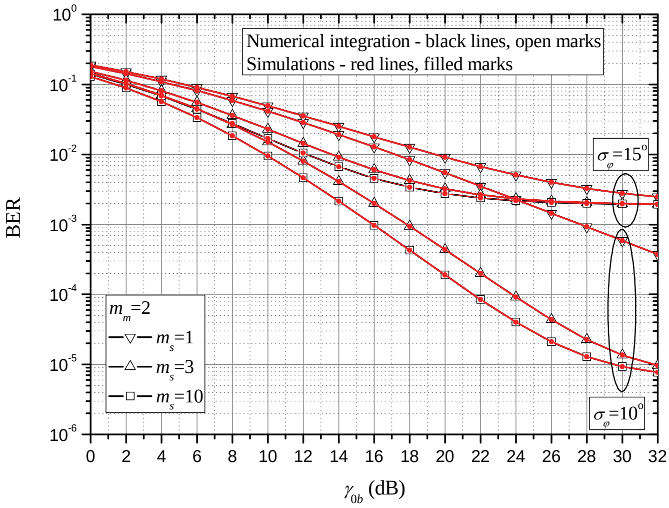

Fig. 5. BER versus SNR for different values of shadowing

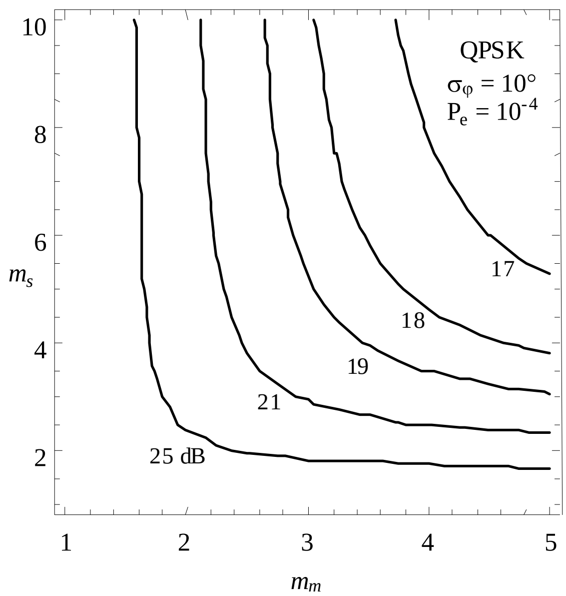

Fig.6. Required values of average SNR in order to achieve BER of 10^{-4}

IV. CONCLUSION

In this paper, we have presented theoretical basics and simulation model for composite fading channel. We have observed Nakagami-*m*/gamma shadowing channels. Furthermore, the error performance of QPSK receiver operating over composite fading channel has been determined. The BER degradations caused by simultaneous influences of imperfect reference signal extraction, multipath fading severity and shadowing have been determined. There are good match between results obtained by numerical integration and results obtained by Monte Carlo simulations.

ACKNOWLEDGEMENT

This work was supported in part by the Ministry of Science of Serbia within the Project "Development and realization of new generation software, hardware and services based on software radio for specific purpose applications" (TR-11030).

REFERENCES

[1] P. S. Bithas, N. C. Sagias, P. K. Mathiopoulos, G. K. Karagiannidis, A. A. Rontogiannis, "On the performance analysis of digital communications over generalized-K fading channels", IEEE Communications Letters, vol. 10, no. 5, pp. 353-355, May 2006.

[2] P. M. Shankar, "Error rates in generalized shadowed fading channels", Wireless Personal Communications, vol. 28, no. 4, pp. 223-238, Feb. 2004.

[3] I. M. Kostic, "Analytical approach to performance analysis for channel subject to shadowing and fading", IEE Proceedings, vol. 152, no. 6, pp. 821-827, Dec. 2005.

[4] C. M. Lo, W. H. Lam, "Average BER of BPSK and QPSK systems with noisy phase reference over Nakagami-m fading channels", IEICE Transactions on Communications, vol. E84-B, no. 6, pp. 1687-1689, 2001.

[5] I. M. Kostic, "Average SEP for M-ary CPSK with noisy phase reference in Nakagami fading and Gaussian noise", European Transactions on Telecommunications, vol. 18, no. 2, pp. 109-113, 2007.

[6] I. S. Gradsteyn, I. M. Ryzhik, *Table of integrals, series and products*, 7th ed. San Diego, CA: Academic, 1994.

[7] I. M. Kostic, "A method for error rate analysis in the presence of rapid and slow fading", Proc. XLVI ETRAN Conference, vol. II, pp. 62-64, Banja Vrucica – Teslic, June 4-7, 2002.

[8] G. Marsaglia, W. W. Tsang, "A simple method for generating gamma variables", ACM Transactions on Mathematical Software, vol. 26, no. 3, pp. 363-372, September 2000.

[9] D. Kundu, R. D. Gupta, "A convenient way of generating gamma random variables using generalized exponential distribution", Computational Statiscics & Data Analysis, vol. 51, no. 6, pp. 2796-2802, March 2007.

[10] M. C. Jeruchim, P. Balaban, K. Sam Shanmugan, Simulation of Communication Systems – Modeling, Methodology, and Techniques, New York: Kluwer, 2000.