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Construction of a Hybrid Quantizer with Huffman Coding for the Laplacian Source

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Abstract – **This paper presents a hybrid model of a speech signal quantizer consisted of the uniform scalar quantizer and the optimal Lloyd-Max's quantizer. The proposed hybrid quantizer is developed for a small number of quantization levels *N*. The paper also gives an overview of the numerical results that indicate the advantages of the developed hybrid quantizer over the conventional uniform and nonuniform quantizer, where in all three cases was used Huffman coding method.**

Keywords – **Hybrid quantizer, Huffman coding, Uniform and nonuniform quantizer**

I. INTRODUCTION

Speech signal compression involves quantization process, i.e. rounding off the previously sampled values of the signal from an infinite set to a values from the finite set, and the coding process, i.e. representing quantized signal values with bit sequences (code words).

There are a number of signal compression techniques, i.e. preparation of signals for transmission and storage. Compression techniques can be harshest divided into lossy (with losses) and lossless (with no losses) [5], [7]. The usage of lossy compression techniques leads to a loss of information about the signal which is compressed through a process of quantization, i.e. rounding off on the previously determined values (levels). Error added by the quantization process is measured by the distortion, while the quality of the compressed signal is measured by the value of signal-to-quantization noise ratio.

In order to raise the quality of voice signal it is important to choose an adequate quantizer, and a specific coding technique with it, in order to realize the lossless type of compression. For the realization of lossless types of compression the entropic codes are often used, and they are used for coding of a finite set of quantized signal values. The implementation of these codes brings additional complexity in the process of compression.

Although there are many realizations of quantizers, there is still a plenty of room for development of new models, especially when we need to raise the quality of the signal. In addition, with appropriate coding technique it should be easy to produce a digital signal which has a smaller impact on a channel range over which the it will be transferred (i.e. to reduce the average bit rate).

By following these requirements, in this paper we suggested a model of hybrid quantizer which output is coded with entropic Huffman code. Hybrid quantizer is consisted of the uniform and the optimal Lloyd-Max's scalar quantizer. It is known that the uniform quantizer with Huffman coding achieves lower average bit rate, while the nonuniform quantizer with Huffman coding achieves better signal-to-quantization noise ratio [4], [5], [7]. The proposed model of the hybrid quantizer incorporate both of the advantages mentioned above, which makes the hybrid quantizer better than the uniform and nonuniform quantizer. The speech signal at the entrance of the hybrid quantizer is modeled by the Laplacain probability density function. With this model of the hybrid quantizer and with Huffman coding we wanted to raise the quality of the signal, i.e. increase the value of signal-to-quantization noise ratio for the specific bitrate and simultaneously tended to reduce the average bit rate for a given value of signal-to-quantization noise ratio. On the other hand, the introduction of the coding techniques with variable length code words resulted in increased complexity of the proposed process of compression [4], [5], [7].

In this paper, beside the basic characteristics of the hybrid quantizer, will be presented the advantages achieved by applying Huffman coding algorithm. The advantages of the proposed hybrid quantizer over the standard uniform and nonuniformni scalar quantizer will be pointed out with a serie of numerical results. In order to compare the quantizers properly, the Huffman coding algorithm was applied in all three cases.

II. MODEL OF THE HYBRID QUANTIZER

In this paper is proposed a model of the hybrid scalar quantizer which is consisted of the simple uniform scalar quantizer and the optimal Lloyd-Max's scalar quantizer [1], [2], [3]. The model was developed for a small number of quantization levels N = 8, 16 and 32. Hybrid quantizer with N segments, was carried out in a way which means that the uniform quantizer part occupies the central part of the range of hybrid quantizer and include N-2L segments of equal width, which boundaries are defined by the decision levels x_i

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(i=L,...,N-L), and Lloyd- Max's quantizer part which contains 2L segments and occupies the outer part of the complete range. We only discussed the values of 1, 2 and 4 for the number of segments L. In order to achieve the lossless effect of the compression, we applied the Huffman coding algorithm with variable length code words. The advantage of the hybrid quantizer, achieved by applying the Huffman coding algorithm, is reflected on the level of quality of the compressed signal, with reduction of the average bit rate. The complexity of the proposed compression method mainly comes from the variable length coding algorithm. The proposed hybrid quantizer is designed for the small number of quantization levels, and for that reason with the uniform quantizer is used optimal Lloyd-Max's scalar quantizer. It is known that for the realization of the hybrid quantizer for a grater number of quantization levels, the compander is preferred instead of the Lloyd-Max's quantizer, as nonuniform quantizer [1], [2], [3].

How much the process of quantization affects the quality of compressed voice signal with unit variance modeled by the Laplacian probability density function,

$$p(x) = \frac{1}{\sqrt{2}} e^{-|x|\sqrt{2}} \tag{1}$$

can be estimated on the basis of the value of distortion D, i.e. the error introduced by the quantization procedure

$$D = \frac{2}{3} \frac{x_{N-L}^2}{(N-2L)^2} \int_0^{x_{N-L}} p(x)dx + $$

$$+ 2 \sum_{i=N-L}^{N-1} \int_{x_i}^{x_{i+1}} (x - y_{i+1})^2 p(x)dx , \tag{2}$$

for N=1,...,32 and L=1,2,4. The x_{N-L} value, from the previous expression, is the decision threshold level that separates the range of the uniform quantizer from the range of the Lloyd-Max's quantizer within a range of the hybrid quantizer. Decision threshold level is determined in accordance with minimal distortion condition, i.e. equating the first derivate of distortion by the requested value with zero. When the value of the decision threshold level is known, the width of the segments in the uniform part of the range of the hybrid quantizer can easily be determined by dividing the double value of x_{N-L} with N-2L. The parameters, y_{i+1}, are known as the representation levels in the part of the range with Lloyd-Max's quantization [4]. The quality of the signal after compression can be estimated on the basis of the value of signal-to-quantization noise ratio

$$SQNR[dB] = 10 \log_{10} \frac{1}{D} \tag{3}$$

Analysing the dependence of the signal-to-quantization noise ratio $SQNR$ on the average bit rate $\overline{R}$ (calculated in accordance with the Huffman coding algorithm), easily can be noticed the advantages of the hybrid quantizer over the uniform and nonuniform quantizer. All about this, as well as all about the other features of the proposed hybrid quantizer can be found in more words in the section with numerical results.

III. HUFFMAN CODING PROCEDURE

In this paper is analyzed the Huffman coding algorithm, which is implemented using already worked out MATLAB program code. Huffman coding algorithm is used for coding of the representation levels from the segments of the hybrid quantizer, or simply, the segments (because each segment contains a single representation level).

The Huffman algorithm will be described in the following. Let's analyse a source with the symbols s_i (i=0,1,...,q), which probabilities of appearance are P_i (i=0,1,...q) [6]. In the particular case of hybrid quantizer as symbols s_i (i=0,1,...q, q=N-1) are taken the segments from the range of the hybrid quantizer. Probabilities P_i (i=0,1,...q, q=N-1) are the probabilities of appearance of segments within a range of the hybrid quantizer and they are defined as follows:

$$P_i = \int_{x_i}^{x_{i+1}} p(x)dx , \quad i=0,...,N\text{-}1 \tag{4}$$

where parameters x_i are the decision levels ($x_0 \to -\infty$, $x_n \to \infty$) [4], [5], [7]. The number of symbols is equal with number of segments (representation levels) N. The symbols should be regulated by the non-growing probabilities so that $P_0 \geq P_1 \geq ... \geq P_q$. If the symbols have the same probability, its order in the editing does not matter. Now, it is needed to replace two the least probable symbols by a single, equivalent symbol, i.e. to form their union. Probability of appearance of this symbol is equal to the sum of the probabilities of the symbols that are replaced. In this way, the reduction of the source S with q symbols on the source S_1 which has q-1 symbols, is performed. Now, it is needed to re-edit the symbols of the reduced source by non-growing probabilities (because the new symbol does not need to be the least probable) and make a new (second) reduction to obtain a source S_2 with q-2 symbols. This process continues until q-1 reductions are made and the S_{q-1} source with only two symbols is got. At each process of editing symbols by non-growing probabilities the order within the group of symbols with equal probabilities is not important. Now we proceed from the reduced source with two symbols and begin to write code words in such a manner so the one symbol gets 0 as the code word, and the other gets bit 1 (arbitrarily is taken to place 0 in the code word of the symbol written above and to place bit 1 in the code word of the symbol written below). Since these symbols are the unions of the symbols from the previous reduction, the next process is to go step by step backward and break up the unions. At each breaking, bit 0 or bit 1 are added to the appropriate places in the code words, until the original source is reached [6]. For example, in the case of the hybrid quantizer with N=8 levels (the source with eight symbols) and L=1, the calculated probabilities of the segments in the entire range are given, from the first to the eighth: P_0=0.027, P_1=0.045, P_2=0.118, P_3=0.31, P_4=0.31, P_5=0.118, P_6=0.045

and P_7=0.027, respectively. Before these probabilities are put in the MATLAB program code, it is required to set them in non-growing order. After the algorithm is over the length of the code words, which are used to represent the previosly mentioned segments, are calculated and given here: l_0=4, l_1=4, l_2=3, l_3=2, l_4=2, l_5=3, l_6=4 and l_7=4, respectively. The average length of the various length code words, was calculated as follows:

$$\overline{R} = \sum_{i=0}^{N-1} P_i l_i \tag{5}$$

In this particular example, the average length of various length code words, i.e. average bit rate is 2.524 bits/sample.

IV. The numerical results

In this section will be given all the numerical results of the analysis of the proposed hybrid quantizer with Huffman coding. In Table I are given the maximal values of signal-to-quantization noise ratio which are achieved by the proposed hybrid quantizer, while are in Table II exhibited average bit rate values at which the maximal values of signal-to-quantization noise ratio from Table I are achieved. It can be seen, from the displayed tables, that the better signal quality is achieved by the quantizer with a grater number of levels, but on the other hand this results in increased average bit rate. On the basis of the offered numerical results the optimum solution of the hybrid quantizer can be chosen in accordance with the requirements for better signal quality and/or lower average bit rate.

TABLE I
THE REWIEV OF MAXIMAL VALUES OF SIGNAL-TO-QUANTIZATION NOISE RATIO

SQNR (dB)	L=1	L=2	L=4
N=8	11.918	12.464	/
N=16	16.517	17.328	17.959
N=32	21.192	22.076	23.010

TABLE II
THE REWIEV OF AVERAGE BIT RATES AT WHICH THE MAXIMAL VALUES OF SIGNAL-TO-QUANTIZATION NOISE RATIO ARE ACHIEVED

$\overline{R}$ (bits/sample)	L=1	L=2	L=4
N=8	2.524	2.588	/
N=16	3.168	3.368	3.496
N=32	3.918	4.082	4.269

On Figs. 1, 2 and 3 are graphically illustrated functionalities of signal-to-quantization noise ratio from average bit rate for hybrid quantizer, when the Huffman coding algorithm is in use. By observing these graphics it can be seen that the higher quality of the signal can be achieved by using the quantizer with grater number of levels, which results in average bit rate increase (the same thing could be concluded on the basis of the numerical results from Table I and Table II). The aim of the analysis of these graphics is to see in which ranges of average bit rate the specific quantizer (referring to the values of N and L) gives the best quality of the signal at the output.

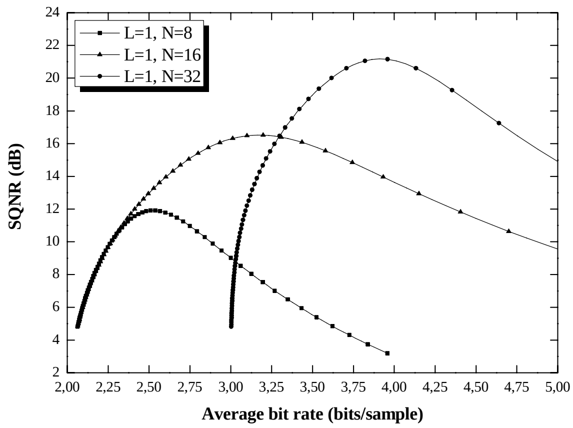

Fig. 1. The graphical interpretation of functionality of signal-to-quantization noise ratio from average bit rate for the hybrid quantizer, L=1 segment and N=8, 16 and 32 levels

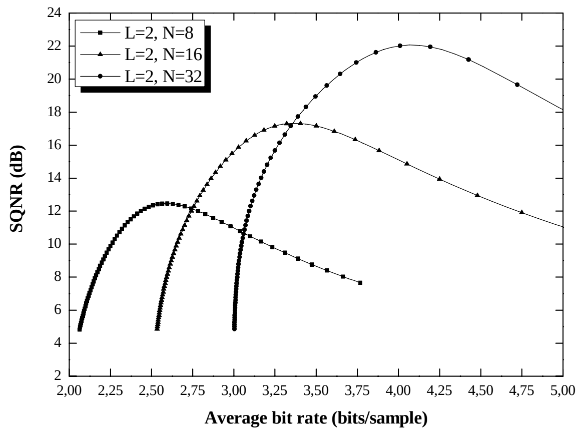

Fig. 2. The graphical interpretation of functionality of signal-to-quantization noise ratio from average bit rate for the hybrid quantizer, L=2 segments and N=8, 16 and 32 levels

Thus, from the Fig. 1 it can be determined that it is better to use the quantizer with N=8 levels for bit rates up to 2.315 bits/sample (for the mentioned range of the bit rate the quantizer with 8 levels gives the same quality as the quantizer with N=16 levels, but with less complexity), for the bit rates ranging from 2.315 to 3.298 bits/sample better quality is achieved with quantizer which has N=16 levels, while for the bit rates higher than 3.298 bits/sample it is better to use the quantizer with N=32 levels. By analyzing the graphics on Fig. 2 it can be determined that it is better to use the quantizer with

N=8 levels for bit rates up to 2.747 bits/sample, for the bit rates ranging from 2.747 to 3.36 bits/sample it is better to use the quantizer with N=16 levels, while for the bit rates higher than 3.36 bits/sample it is the best to use the quantizer with N=32 levels. In the same manner from the Fig. 3 can be concluded that it is better to use the quantizer with 16 levels for the bit rates up to 3.518 bits/sample, while for the higher bit rates better quality is achieved by using the quantizer with N=32 levels.

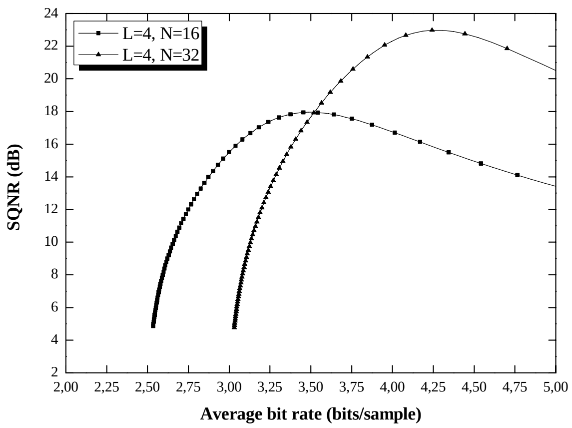

Fig. 3. The graphical interpretation of functionality of signal-to-quantization noise ratio from average bit rate for the hybrid quantizer, L=4 segments and N=8, 16 and 32 levels

It is known that the signal-to-quantization noise ratio of the standard uniform quantizer increases 6.02 dB with each increase of the bit rate for one bit, when the coding technique with constant length of the code words is used [4]. The thing which interested us, is the value of the average bit rate, of the hybrid quantizer, to which it can be increased by one bit, so that the gain in quality still be higher than that achieved with the standard uniform quantizer, or just to be greater than 6.02 dB. Specific values are shown in Table III.

TABLE III
VALUES UP TO WITCH IT MAKES SENSE TO INCREASE THE AVERAGE BIT RATE

$\overline{R}_{\lim.}$ (bits/sample)	L=1	L=2	L=4
N=8	2.492	2.466	/
N=16	2.821	3.126	3.201
N=32	3.676	3.744	3.953

The analysis was performed for all combinations of the number of levels N and the number of segments L. From the mathematical point of view, it means that the slope of the curve $SQNR(\overline{R})$, at any point up to the value of the limit bit rate $\overline{R}_{\lim.}$, is greater than 6.02.

To indicate the advantages of the hybrid model, we compared its performances regarding to the signal quality and average bit rate, with the standard uniform and nonuniform quantizer.

In order to make the comparation properly, we applied the Huffman coding algorithm in all three cases. The advantages of the hybrid quantizer versus the uniform and nonuniform quantizer can be reflected in the fact of how much is lower the average bit rate achieved by the hybrid quantizer when the value of signal-to-quantization noise ratio is fixed, or what is the gain in quality achieved by the hybrid quantizer, when the value of the average bit rate is fixed. The values of the signal-to-quantization noise ratio and average bit rate, which are fixed, refer to the uniform and nonuniform quantizer, and these fixed values are maximal signal quality and average bit rate at which it is achieved. Compared to the uniform quantizer, the hybrid quantizer achieves the quality of 16 dB at a bit rate lower for 0.15 bits/sample and achieves the rise in quality of 0.82 dB at a bit rate of 3.82 bits/sample. Compared to the nonuniform quantizer, the hybrid quantizer achieves the quality of 17.55 dB at a bit rate lower for 0.152 bits/sample and achieves the rise in quality of 1.36 dB at a bit rate of 2.606 bits/sample. The above results demonstrate that the adequate selection of the number of quantization levels N, the proper distribution of the segments in the entire range of the hybrid quantizer, and proper coding algorithm can result in a better signal quality and better compression, at the same time.

V. CONCLUSION

In this paper is proposed the hybrid quantizer which output is coded using Huffman coding algorithm with variable length code words. Based on the theoretical results obtained by the proper analysis of the proposed hybrid quantizer we showed that its advantages clearly can be seen in terms of signal quality as well as in the value of the average bit rate. Therefore, we believe that the proposed hybrid quantizer with Huffman coding technique, represent an efficient solution when the desired signal quality is needed to be achieved with a lower average bit rate.

REFERENCES

[1] Z. Perić, M. Dinčić, M. Petković, “Design of a Hybrid Quantizer with Variable Length Code”, IOS Press, Fundamenta Informaticae, vol. 98, no. 2-3, pp. 233-256, 2010.

[2] Z. Perić, J. Nikolić, D. Pokrajac, “Hybrid scalar quantizer for the Laplacian source”, WSEAS Transactions on Communications, vol. 6, no. 1, pp. 60-65, 2007.

[3] Z. Perić, L. Velimirović, J. Lukić, A. Jocić and D. Denić, "Konstrukcija kvantizera promenljive dužine i male bitske brzine za Laplasov izvor", TELFOR'09, Conference Paper, pp.576-579, Belgrade, Serbia, 2009.

[4] N. Jayant, *Digital Coding of Waveforms, Principles and Applications to Speech and Video*, New Jersey, Prentice Hall, 1984.

[5] D. Salomon, *A Concise Introduction to Data Compression*, London, Springer, 2008.

[6] D. Drajić, *Uvod u teoriju informacija i kodovanje*, Beograd, Akademska misao, 2004.

[7] K. Sayood, *Introduction to Data Compression*, Amsterdam, Elsevier, 2006.