Design of Gyrator Active Filters with Equalized Maxima of the Amplifier Output Voltages

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***Abstract* – New simpler method for equalizing the output voltage maxima of the amplifiers in gyrator active filter is proposed. The method is based on a proper variation of the gyrator transconductances. First is outlined the general approach and then it is applied consistently for deriving a set of equivalent circuits for replacing of each impedance of the LC ladder prototype by its gyrator counterparts.**

***Keywords* – active filters, gyrators, dynamic range, OTA**

I. INTRODUCTION

The dynamic range optimization of the active filters is a long-time studied problem [1], [2], maintained by the continuous demands for reducing the supply voltages in the integrated circuit (IC) and by the increased frequency range of operation. The dynamic range is two-sided problem: equalizing the maxima of the amplifier output voltages and minimizing the filter output noise. The first task is treated for 40 years (see e.g. [3], [4]) and there are satisfactory solutions for the most classes of the active filters. Exclusions are the filters, derived from passive ladder LC filters by substituting of the inductors with gyrator active circuits. The main difficulty is the preserving the structure of the passive filter, which keeps the nodal AC voltages in the gyrator filters the same as they are in the passive one. The problem was long time mentioned as unsolved [1] and a solution was proposed recently [4] as a consequence of more general method for optimizing the dynamic range of OTA-C filters.

This paper proposes an alternative approach, which is simpler and requires fewer calculations. It is based on a model of a floating inductor replaced by two gyrators and a capacitor. The necessary data for the equalization are received by single analysis of the initial LC ladder and then the parts of the LC ladder are directly replaced by their active counterparts, which elements can be calculated very easy.

II. THE PROPOSED METHOD

An admittance Y connected across between two gyrators in cascade with arbitrary transconductances is equivalent to longitudinal impedance Z in cascade with an artificial element called scalor (Fig. 1) [5], [6], which two port equations are $V_1 = K_V V_2$; $I_1 = K_I(-I_2)$. The relationships between the elements in Fig. 1 are

$$Z = Y/(g_{f1}g_{b1}); K_V = g_{b2}/g_{f1}; K_I = g_{b1}/g_{f2}. \qquad (1)$$

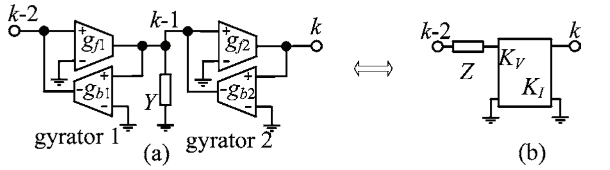

Fig. 1. An admittance between two gyrators (a) and its model (b).

The scalor can be moved freely when it is in cascade with other elements. When it is moved from output to the input of the circuit, it multiplies all "over-jumped" impedances by K_I/K_V; all over-jumped voltages by $1/K_V$ and all over-jumped currents by $1/K_I$. When the scalor motion is in opposite direction, the multiplying coefficients are reciprocal. This property keeps unchanged the relationship between input and output voltages of the whole circuit.

The voltages and currents in the LC ladder can be determined by AC analysis. First step in the design of the gyrator filter is to insert two scalors with reciprocal parameters before the last node (Fig. 2(b)). The scalors do not change the circuit due to the reciprocity of their parameters. Then the first scalor is moved leftward and placed before node 2 (Fig. 2(c)).

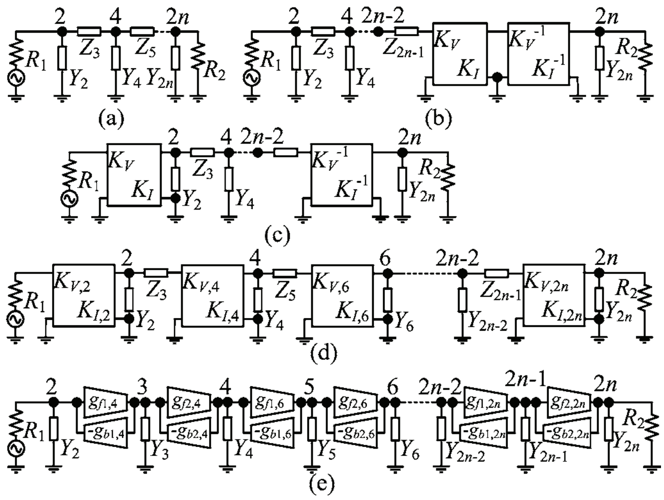

Fig. 2. (a) The LC ladder prototype. (b) Placing two scalors before the last node. (c) Moving the first scalor before the node 2. (d) The circuit after adding all scalors. (e) Replacement the series impedances and associated scalors by gyrator pairs and cross admittances.

The next steps are similar: New scalor pairs with mutually reciprocal parameters are placed before the second scalor in Fig 2(c) and the first scalors from the pairs are moved leftward and placed before nodes 4, 6, etc. At the end there will be a scalor before every node of the ladder (Fig 2(d)). All

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second scalors from the pairs, connected in cascade before node $2n$, are equivalent to a scalor with parameters $K_{V,2n} = 1/(K_{V,2}K_{V,4}...K_{V,2n-2})$, $K_{I,2n} = 1/(K_{I,2}K_{I,4}...K_{I,2n-2})$.

The scalor moving multiplies all over-jumped voltages by $1/K_V$, which allows to use the K_V parameters for equalizing the maxima of the voltages in the nodes 2, 4, 6, etc. to the maximum of the output voltage. The node 2 is over-jumped only by the scalor $[K_{V,2}\ K_{I,2}]$ and the maxima of V_2 and V_{2n} will be equal if $K_{V,2} = \max|V_2|/\max|V_{2n}|$. The other ladder nodes are over-jumped by more scalors, intended for equalizing the voltage maxima of the previous nodes. The last scalor, passed through node k before adding the scalor associated with this node, is the scalor, making $\max|V_{k-2}| = \max|V_{2n}|$. Thus, the equalizing of the maxima at the nodes before k changes V_k to $V_k\max|V_{2n}|/\max|V_{k-2}|$. Then K_V of the scalor associated with node k must be:

$$K_{V,k} = \max|V_k| / \max|V_{k-2}| \qquad (2)$$

After equalizing the voltage maxima in all ladder nodes, every longitudinal impedance has an associated scalor (Fig. 2(d)). They can be replaced by gyrator pairs and cross impedances according Fig. 1 as it is shown in Fig. 2(e). This step creates new nodes (with odd numbers in Fig. 2(e)), which voltage maxima must be equalized too.

The voltage at node k-1 in Fig. 1(a) is equal to $(g_{f1}V_{k-2} - g_{b2}V_k)/Y$ and it can be transformed in the following way, if take into account formulas (2):

$$V_{k-1} = \left(g_{f1}V_{k-2} - g_{b2}V_k\right)/\left(g_{f1}g_{b1}Z\right) = \left(V_{k-2} - K_V V_k\right)/\left(g_{b1}Z\right). \qquad (3)$$

The voltage $K_V V_k$ is the voltage V_k' at node k before inserting the scalor associated with the impedance Z, when the nodes k-2 and k are connected by the impedance Z only. Thus the voltage V_{k-1} is equal to I_Z'/g_{b1}, where I_Z' is the current through Z. This current is affected by all scalors passed through it for equalizing the voltage maxima at the nodes before k-2 and it is equal to the current I_Z through the impedance Z in the LC prototype, divided by the K_I's of all scalors, passed through node k-2. The requirement for equal maxima of the voltages V_{k-1} and V_{2n} gives the following expression for $g_{b1,k}$

$$g_{b1,k} = \max|I_Z| / \left(K_{I,2}K_{I,4}...K_{I,k-2}\max|V_{2n}|\right). \qquad (4)$$

The first scalor $[K_{V,2}\ K_{I,2}]$ can be processed in a simpler way. It can be moved leftward again. Then it jumps over the input resistance R_1, multiplying its value by $K_{I,2}/K_{V,2}$, and over the signal source and disappears. The multiplication of the source voltage by $1/K_V$ can be compensated by additional amplifier if necessary.

Not all inductors in the circuit can be replaced by gyrators and capacitors during the above steps since: 1) The LC filter could has inductors in the cross branches. 2) Fig. 3 shows two cases of longitudinal LC branches in the prototype, in which the replacement the inductors only is not desirable. In fact, if a longitudinal branch in the ladder is of degree 2 or higher, it must be replaced at once by gyrators according Fig. 1, which creates new inductors. Even single longitudinal capacitors (in high-pass filters) create cross inductors when the voltage maxima in the ladder nodes are equalized.

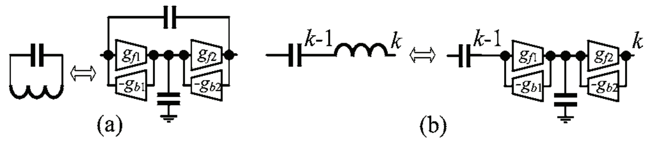

Fig. 3. Two transformations, which can not be applied: (a) The gyrator circuit is not equivalent to the tank if g_m's differ [7], [8]; (b) The voltage maximum in node k-1 can not be equalized.

A single grounded inductor is transformed by the classical way (Fig.4(a)) and the condition for equal voltage maximum in the new node m to the other voltage maxima is $g_b = \max|I_L|/\max|V_{2n}|$. A new scalor is inserted in a grounded series LC tank (Fig. 4(b)) for equalizing the voltage maxima in the intermediate nodes. Then the inductor and the scalor are replaced by gyrator pair and a capacitor C_L and the following conditions hold for having the voltage maxima in the nodes m_1 and m_2 equal to the others:

$$K_V = \max|V_m| / \max|V_{2n}|; \quad g_{b1} = \max|I_{LC}| / \max|V_{2n}| \qquad (5)$$

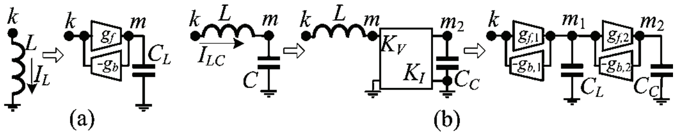

Fig. 4. Replacement of (a) single grounded inductor; (b) grounded series LC tank.

Grounded inductors and series LC tanks can appear in the structure in Fig. 2(e) in two ways: they could exist in the cross branches in the LC prototype (then k is even); or they can be a result from transformation of floating capacitors or parallel LC tanks according Fig. 1. The transformation in the second case converts impedances in admittances and vice verse, voltages into currents and vice verse as well. This is illustrated in Fig. 5. The relationships between element, voltage and current values in these equivalent circuits can be derived easily from the basic gyrator properties

$$\begin{aligned} &I_L = g_{f1}V_C;\ I_{LC} = g_{f1}V_{LC};\ V_m = I_L/g_{b1}; \\ &L_C = C/\left(g_{f1}g_{b1}\right);\ C_L = LC/L_C \end{aligned} \qquad (6)$$

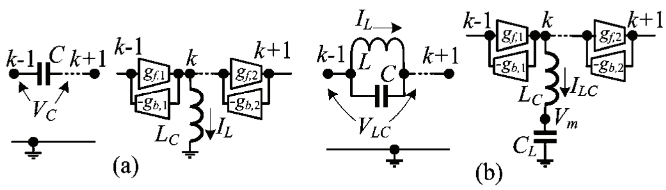

Fig. 5. Transforming of longitudinal elements: (a) capacitor; (b) parallel LC tank.

Of course, the element values and the voltages and currents in the right-hand sides of equations (5) and (6) are affected also by the scalors passed through these elements.

III. Catalogue for Replacement of the Basic Branches in LC Ladder Circuits

The method outlined above allows to create a catalogue containing the basic branches in an LC ladder circuit, their gyrator counterparts and the corresponding design equations ensuring equal voltage maxima at all amplifier outputs. Fig. 6 gives the basic ladder blocks and their gyrator equivalents.

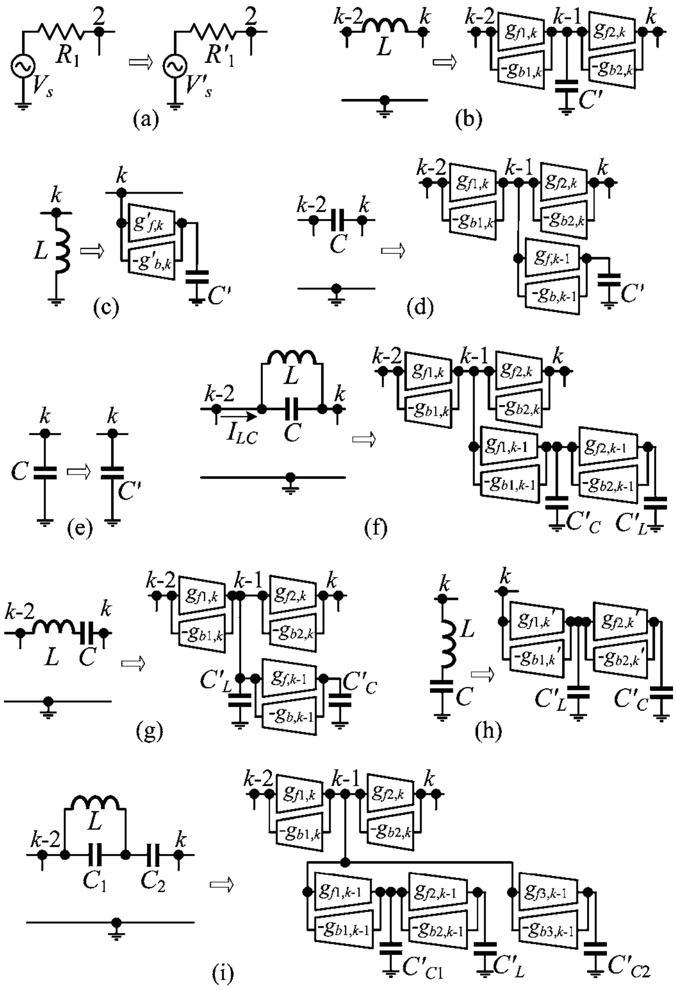

Fig. 6. Basic fragments of LC ladder filters and their gyrator counterparts: (a) signal source and its internal resistance; (b) longitudinal inductor; (c) cross inductor; (d) longitudinal capacitor; (e) cross capacitor; (f) longitudinal parallel LC tank; (g) longitudinal series LC tank; (h) cross series LC tank (i) third order longitudinal LC circuit.

The design equations for the circuits in Fig. 6 are:
Fig. 6(a):

$$R_1' = R_1 \frac{\max|V_o|}{\max|V_2|} K_{I,2};\ V_s' = V_s \frac{\max|V_o|}{\max|V_2|}$$

Fig. 6(b): C' must be chosen arbitrarily and

$$g_{b1,k} = \frac{\max|I_L|}{K_{I,2}K_{I,4}...K_{I,k-2}\max|V_o|};\ g_{f1,k} = \frac{C'}{L}\frac{\max|V_{k-2}|}{\max|I_L|};$$

$$g_{f2,k} = \frac{g_{b1,k}}{K_{I,k}};\ \ g_{b2,k} = g_{f1,k}\frac{\max|V_k|}{\max|V_{k-2}|}$$

Fig. 6(c): C' must be chosen arbitrarily and

$$g'_{b,k} = \frac{\max|I_L|}{K_{I,2}K_{I,4}...K_{I,k}\max|V_o|};\ g'_{f,k} = \frac{C'}{L}\frac{\max|V_k|}{\max|I_L|};$$

Fig. 6(d): C' and $g_{f1,k}$ must be chosen arbitrarily and

$$g_{b1,k} = \frac{\max|I_C|}{K_{I,2}K_{I,4}...K_{I,k-2}\max|V_o|};$$

$$g_{f2,k} = \frac{g_{b1,k}}{K_{I,k}};\ \ g_{b2,k} = g_{f1,k}\frac{\max|V_k|}{\max|V_{k-2}|};$$

$$g_{f,k-1} = \frac{C'}{C}\frac{\max|I_C|}{\max|V_C|};\ g_{b,k-1} = g_{f1,k}\frac{\max|V_C|}{\max|V_{k-2}|}$$

Fig. 6(e):

$$C' = \frac{C}{K_{I,2}K_{I,4}...K_{I,k}}\frac{\max|V_k|}{\max|V_o|}$$

Fig. 6(f): C'_C and C'_L must be chosen arbitrarily and

$$g_{b1,k} = \frac{\max|I_{LC}|}{K_{I,2}K_{I,4}...K_{I,k-2}\max|V_o|};\ g_{f2,k} = \frac{g_{b1,k}}{K_{I,k}};$$

$$g_{f1,k} = \frac{C'_L K_{I,k-1}}{L}\frac{\max|V_{k-2}|}{\max|I_L|};\ g_{b2,k} = g_{f1,k}\frac{\max|V_k|}{\max|V_{k-2}|};$$

$$g_{f1,k-1} = \frac{C'_C}{C}\frac{\max|I_{LC}|}{\max|V_{LC}|};\ g_{b1,k-1} = \frac{C'_L K_{I,k-1}}{L}\frac{\max|V_{LC}|}{\max|I_L|};$$

$$g_{f2,k-1} = \frac{g_{b1,k-1}}{K_{I,k-1}};\ g_{b2,k-1} = \frac{C'_C}{C}\frac{\max|I_L|}{\max|V_{LC}|};$$

Fig. 6(g): C'_C and C'_L must be chosen arbitrarily and

$$g_{b1,k} = \frac{\max|I_L|}{K_{I,2}K_{I,4}...K_{I,k-2}\max|V_o|};\ g_{f1,k} = \frac{C'_L}{L}\frac{\max|V_{k-2}|}{\max|I_L|};$$

$$g_{f2,k} = \frac{g_{b1,k}}{K_{I,k}};\ \ g_{b2,k} = g_{f1,k}\frac{\max|V_k|}{\max|V_{k-2}|}$$

$$g_{b,k-1} = g_{f1,k}\frac{\max|V_C|}{\max|V_{k-2}|};\ g_{f,k-1} = \frac{C'_L C'_C}{g_{b,k-1}LC}$$

Fig. 6(h): C'_C and C'_L must be chosen arbitrarily and

$$g'_{b1,k} = \frac{\max|I_L|}{K_{I,2}K_{I,4}...K_{I,k}\max|V_o|};\ g'_{f1,k} = \frac{C'_L}{L}\frac{\max|V_k|}{\max|I_L|};$$

$$K'_{I,k} = \frac{1}{K_{I,2}K_{I,4}...K_{I,k}}\frac{C}{C'_C}\frac{\max|V_C|}{\max|V_o|};$$

$$g'_{f2,k} = \frac{g'_{b1,k}}{K'_{I,k}};\ \ g'_{b2,k} = g'_{f1,k}\frac{\max|V_C|}{\max|V_k|}$$

Fig. 6(i): C'_{C1}, C'_{C2}, $g_{f1,k}$ and $K_{I,k-1}$ must be chosen and

$$g_{b1,k} = \frac{\max|I_{C2}|}{K_{I,2}K_{I,4}...K_{I,k-2}\max|V_o|};\ g_{f2,k} = \frac{g_{b1,k}}{K_{I,k}};$$

$$g_{b2,k} = g_{f1,k}\frac{\max|V_k|}{\max|V_{k-2}|};\ C'_L = \frac{g_{f1,k}L}{K_{I,k-1}}\frac{\max|I_L|}{\max|V_{k-2}|};$$

$$g_{f1,k-1} = g_{f1,k}\frac{\max|V_L|}{\max|V_{k-2}|};\ g_{f2,k-1} = \frac{g_{b1,k-1}}{K_{I,k-1}};$$

$$g_{f1,k-1} = \frac{C'_{C1}}{C_1}\frac{\max|I_{C2}|}{\max|V_L|};\ g_{b2,k-1} = g_{f1,k-1}\frac{\max|I_L|}{\max|I_{C2}|};$$

$$g_{f3,k-1} = \frac{C'_{C2}}{C_2}\frac{\max|I_{C2}|}{\max|V_{C2}|};\ g_{b3,k-1} = g_{f1,k}\frac{\max|V_{C2}|}{\max|V_{k-2}|}$$

In these equations V_o is the output voltage. The voltages and currents in the ladder elements are marked by V_Z and I_Z, where Z is L, C or LC depending on the case. V_k and V_{k-2} are the ladder nodal voltages. All voltage and current maxima in the formulas are from the LC prototype.

The catalogue and the corresponding formulas can be applied directly and the whole procedure with introducing and moving the scalors is hidden. From this point of view the K_I parameters, which are chosen arbitrarily, can be thought like some design constants.

In all cases is assumed that K_I parameters are freely chosen. This is not valid for the last longitudinal branch in the LC ladder. Its K_I (let say $K_{I,2n}$) is reciprocal to the product of all previous K_I's according the theory in section II:

$$K_{I,2n} = 1 \Big/ \prod_{k=1}^{n-1} K_{I,2k} \ . \tag{7}$$

IV. Example

The proposed method will be illustrated by a low-pass 5th order elliptic filter with normalized passband and stopband bounds of 1Hz and 1.2Hz respectively, passband ripples of 0.5dB and minimum stopband attenuation of 35.5dB. The circuit of the LC prototype is shown in Fig 7(a) and its normalized element values are: $R_1 = R_2 = 1\Omega$; $C_1 = 1.44$F; $L_2 = 0.975$H; $C_2 = 0.346$F; $C_3 = 1.77$F; $L_4 = 0.613$H; $C_4 = 1.07$F; $C_5 = 1.03$F. The AC analysis of the prototype is done by PSpice at 2V magnitude of the input source and it returns the following maxima of the voltages and currents of interest: $\max|V_6| = 1$V; $\max|V_2| = 1.724$V; $\max|V_4| = 1.866$V; $\max|V_{LC2}| = 3.342$V; $\max|V_{LC4}| = 2.419$V; $\max|I_{LC2}| = 2.175$A; $\max|I_{LC4}| = 1.418$A; $\max|I_{L2}| = 3.35$A and $\max|I_{L4}| = 3.949$A.

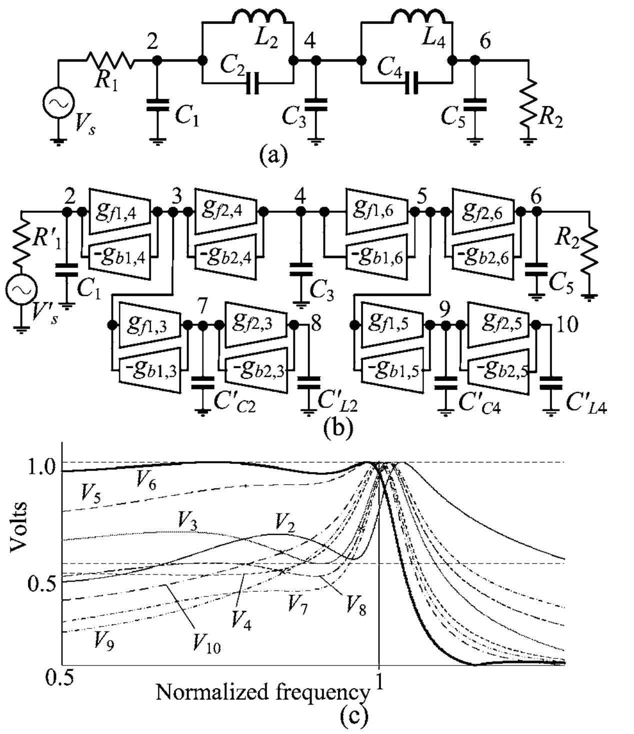

Fig. 7. (a) LC prototype for the example; (b) the gyrator filter; (c) amplifier output voltages in the gyrator circuit (PSpice simulation).

The LC prototype includes blocks, which gyrator counterparts are given in Fig. 6(a), 6(e) and 6(f). The gyrator circuit is given in Fig 7(b) and its elements are calculated by using the corresponding formulas in Section III. The signal source and the input terminating resistance are changed to $V'_s = 1.16$V, $R'_1 = 0.58\Omega$. The new values of the capacitors C_1 and C_3 are $C_1 = 2.483$F and $C_3 = 3.303$F. The capacitor C_5 and the output terminating resistance are unchanged. The elements of circuit, replacing the tank L_2C_2, have values: $g_{f1,4} = 2.174$S, $g_{b1,4} = 2.175$S, $g_{f2,4} = 2.175$S, $g_{b2,4} = 2.354$S, $g_{f1,3} = 2.739$S, $g_{b1,3} = 4.215$S, $g_{f2,3} = 2.737$S, $g_{f2,3} = 2.737$S. The following parameters are chosen for their calculation: $C'_{C2} = 1.456$F, $C'_{L2} = 2.675$F, $K_{I,2} = K_{I,4} = 1$, $K_{I,3} = 1.54$. The circuit, replacing the tank L_4C_4, is calculated in similar way: $K_{I,6} = 1$, $g_{f1,6} = 1.419$S, $g_{b1,6} = 1.418$S, $g_{f2,6} = 1.418$S, $g_{b2,6} = 0.76$S, $g_{f1,5} = 0.66$S, $g_{b1,5} = 1.839$S, $g_{f2,5} = 0.66$S, $g_{b2,5} = 1.837$S, with the following choice of the other values and parameters $K_{I,5} = 2.785$, $C'_{C4} = 1.204$F, $C'_{L4} = 0.661$F. A PSpice simulation confirms of the equality of the maxima of all amplifier output voltages (Fig 7(c)).

V. Conclusion

A modification of the method for design of active filters based on direct element substitution in LC prototype with gyrator circuits is proposed. The basic advantage of the new method is its simplicity. The active filter is received by block-by-block replacement in the LC prototype with corresponding active circuits and applying of simple formulas for calculation of the elements. The formulas are modified in such a way, which ensures equal maxima of all amplifier output voltages. This is the first step for complete optimization of the filter dynamic range. Some of the elements and the parameters are chosen freely in the design procedure and they can be used in the next minimization of the output noise.

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References

[1] T. Deliyannis, Y. Sun, J. K. Fidler, *Continuous-Time Active Filter Design*, Boca Raton, CRC-Press, 1999.

[2] R. Schaumann, M. E. Van Valkenburg, *Design of Analog Filters*, Oxford University Press, 2001.

[3] S. Halfin, An Optimizatin Method for Cascaded Filters, *The Bell System Techn. Journal*, 49, No. 2 (February 1970), pp. 185-190.

[4] Y. Palaskas, Y. Tsividis, Dynamic Range Optimization of Weakly Nonlinear, Fuly Balanced, Gm-C Filters with Power Dissipation Constraints, *IEEE Trans. on Circuits and Systems – II: Analog and Digital Signal Processing*, vol. 50, No 10, Oct. 2003, pp. 714-727.

[5] S. I. Koutzarov, I. S. Uzunov, Equivalent Circuit for an Ungrounded Gyrator if Realized by Two Real Gyrators, *AEÜ*, vol. 26, pp. 408-409, September 1972.

[6] I. S. Uzunov, Theoretical Model of Ungrounded Inductance Realized With Two Gyrators, *IEEE Trans. on Circuits and Systems – II: Express Briefs*, vol. 55, No. 10, Oct. 2008, pp. 981-985.