Comparison of the Structures of the Inverse Difference and Laplacian Pyramids for Image Decomposition

R. Kountchev[1], R. Kountcheva[2]

Abstract – **In this work are compared and evaluated the computational structures of the Inverse Difference Pyramid, IDP (already presented in earlier publications of the authors) and the famous Laplacian Pyramid, LP for discrete images decomposition. On the basis of the comparison of the substitution graphs which represent the recursive calculation of the 3-layer decompositions and of the evaluation of their structures complexity, are outlined the basic IDP advantages for pipeline image processing. The results obtained could be used for design of IDP coders for image compression, aimed at real-time applications.**

Keywords – **Pyramidal decompositions, Laplacian Pyramid, Inverse Difference Pyramid**

I. Introduction

The contemporary computer world involves the management and processing of huge amounts of visual information: still images, video, and multimedia. The efficient storage and compression of this information requires the use of various techniques for image representation. The primary form for digital image presentation, which is not compressed, is the *matrix* [1]. This approach ensures unchanged image quality, but, its storage requires significant resources, and the processing – high computational power. The secondary forms for image representation are obtained from the primary one and could be pyramids, multi-dimensional vectors, orthogonal transforms, tree structures, algebraic models, models for visual information perception, etc. The main attention in this paper will be given to the pyramidal representations and their efficiency.

In general, the *pyramidal* representation [1-4] describes the image with progressively increased resolution, which corresponds to the layers of the Gaussian-Laplacian Pyramid. The derivatives of this representation are the Reduced Sum/Difference pyramid; the S-transform pyramid, the Hierarchy-Embedded Differential Pyramid; the Least Square Pyramid, the Morphological Pyramid, etc. This group of pyramids is called over complete [6] because the needed memory is larger than that for the non-compressed image.

[1]Roumen K. Kountchev is with the Faculty of Telecommunications, Technical University, Kl. Ohridsky 8, 1000 Sofia, Bulgaria, E-mail: rkountch@tu-sofia.bg

[2]Roumiana At. Kountcheva is with T&K Engineering, 1712 Sofia, Pob 12, Bulgaria, E-mail: kountcheva_r@yahoo.com

The Orthogonal pyramids are non-over complete. They are usually based on Wavelets [1] or Contourlets [7] functions and have higher efficiency and computational complexity than pyramids from the first group. The *spectral* image representation [1-4] is based on orthogonal transforms of different kind: statistical (Karhunen-Loeve Transform, Principle Component Analysis, Independent Component Analysis, Singular Value Decomposition) and determined (Discrete Fourier Transform, Discrete Cosine Transform, Walsh-Hadamard Transform-WHT, Hartley Transform, Lapped Orthogonal Transform, etc.). In this group could be included the new algebraic image transform [8] based on 2D angular windowing functions, which is suitable for the synthetic shape local phase and orientation evaluation. The transforms from the first group have higher computational complexity than these from the second one. Another approach for image representation is the *perceptual* one [9], based on anisotropic filtration controlled by the Human Visual System (HVS) visual attention model. The *knowledge-based* models for image representation are used mostly in the systems for Visual Information Retrieval. The main approach for image representation used is a pyramid model of 4 layers, which contain correspondingly: the primary matrix, the features vectors, the description of the relations between the features and the semantic image structure.

The general class of linear transform decomposes the image into various components by multiplication with a set of transform functions. Some examples are the Discrete Fourier and Discrete Cosine Transforms, the Singular Value Decomposition, and finally, the Wavelet Transform, of which the Laplacian Pyramid and other subband transforms are simple ancestors.

In this paper the IDP decomposition [5] is compared with the Laplacian pyramid (LP), because LP is the fundamental technique in this area.

The paper is arranged as follows: Section 2 is devoted to the analysis of the quantization noise on the IDP and LP decompositions; in Section 3 are presented the graphs, representing the calculation rules of IDP and LP and Section 4 is the Conclusion.

II. Analysis of the Quantization Noise on the IDP and LP Decompositions

The still image is initially represented as a matrix [B] of size N×N ($N=2^n$) and elements B(i,j). In order to simplify the IDP decomposition analysis [5] is assumed that in the transform of each sub-image is retained one spectrum coefficient only. In case that this coefficient is with spatial frequency (0,0) the IDP is represented by the relation:

$$B(i,j)=\overline{B}(i,j)+\sum_{p=1}^{n-1}\overline{E}_{p-1}^{k_p}(i,j)+E_{n-1}(i,j),\qquad k_p=1,2,..,4^p,\quad i,j=1,2,..,N. \tag{1}$$

where

- for level p = 0

$$\overline{B}(i,j)=I_0(\overline{B})=\overline{B}$$

$$\overline{B}=M_0[B(i,j)]=(2^n\times 2^n)^{-1}\sum_{i=1}^{2^n}\sum_{j=1}^{2^n} B(i,j)$$

$$E_0(i,j)=B(i,j)-\overline{B}(i,j),$$

- for level p = 1,2,.., n -1

$$\overline{E}_{p-1}^{k_p}(i,j)=I_p(\overline{E}_{p-1}^{k_p}),$$

$$\overline{E}_{p-1}^{k_p}=M_p[E_{p-1}^{k_p}(i,j)]=(2^{n-p}\times 2^{n-p})^{-1}\sum_{(i,j)\in W_{k_p}}\sum E_{p-1}(i,j),$$

$$E_{p-1}(i,j)=E_{p-2}(i,j)-\overline{E}_{p-2}^{k_{p-1}}(i,j),$$

- for level p = n

$$E_{n-1}(i,j)=E_{n-2}(i,j)-\overline{E}_{n-2}^{k_{n-1}}(i,j).$$

W_{k_p} is a window of size $2^{n-p}\times 2^{n-p}$ and k_p is the number of the mean difference $\overline{E}_{p-1}^{k_p}$ or interpolated $\overline{E}_{p-1}^{k_p}(i,j)$ image in the IDP layer p.

The IDP components are quantizated starting from the layer p = 0 up to layer p = n. It is assumed that the influence of the quantization noises could be represented by linear additive model. Then Eq. (1) is transformed into:

$$B'(i,j)=\overline{B}'(i,j)+\sum_{p=1}^{n-1}\overline{E}'^{k_p}_{p-1}(i,j)+E'_{n-1}(i,j), \qquad (2)$$

where:

$$\overline{B}'(i,j)=I_0\{Q_0^{-1}[Q_0(\overline{B})]\}; \qquad \overline{B}=M_0[B(i,j)],$$

$$\overline{E}'^{k_p}_{p-1}(i,j)=I_p\{Q_p^{-1}[Q_p(\overline{E}_{p-1}^{k_p})]\},\ \overline{E}_{p-1}^{k_p}=M_p[E_{p-1}^{k_p}(i,j)],$$

$$E'_{n-1}(i,j)=Q_n^{-1}\{Q_n[E_{n-1}(i,j)]\}.$$

Here $Q_p(.)$ and $Q_p^{-1}(.)$ are the corresponding operators for quantization/dequantization of the components in the decomposition layer p. With the mark $(.)'$ are indicated the terms, restored after dequantization, which contain additive noise components obtained in result of the quantization.

The dequantizated IDP component for the decomposition layer p=0 is represented by the sum:

$$\overline{B}'(i,j)=\overline{B}(i,j)+\varepsilon_0(i,j), \qquad (3)$$

where $\varepsilon_0(i,j)$ is the noise ingradient.

For the next decomposition layers p = 1,2,...,n is obtained correspondingly:

$$\overline{E}'^{k_p}_{p-1}(i,j)=\overline{E}_{p-1}^{k_p}(i,j)+M_p\{\varepsilon_{p-1}(i,j)+M_{p-1}[\varepsilon_{p-2}(i,j)+\ldots +M_1[\varepsilon_0(i,j)]]\}+\varepsilon_p(i,j). \qquad (4)$$

Тук $e_p(i,j)$ is the quantization noise for the IDP component p.

The restored image (Eq. 2) could be expressed using the original one and Eqs. 3 and 4):

$$B'(i,j)=B(i,j)+\varepsilon_\Sigma(i,j) \qquad (5)$$

where $\varepsilon_\Sigma(i,j)$ is the total quantization error for the layer p=n:

$$\varepsilon_\Sigma(i,j)=\sum_{p=0}^{n}\varepsilon_p(i,j)+M_1[\varepsilon_0(i,j)]+M_2\{\varepsilon_1(i,j)+ +M_1[\varepsilon_0(i,j)]\}+\ldots+2M_{n-1}\{\varepsilon_{n-2}(i,j)+ +M_{n-2}[\varepsilon_{n-3}(i,j)+\ldots+M_1[\varepsilon_0(i,j)]\}+\varepsilon_{n-1}(i,j) \qquad (6)$$

For the LP decomposition [2] the total quantization error for the layer p=0 (which corresponds to IDP layer p=n), is defined as follows:

$$\varepsilon_\Sigma^{PL}(i,j)=\sum_{p=0}^{n}\varepsilon_p(i,j)+F[\varepsilon_n(i,j)]+F\{\varepsilon_{n-1}(i,j)+ +F[\varepsilon_n(i,j)]\}+\ldots+F\{\varepsilon_1(i,j)+F[\varepsilon_2(i,j)+\ldots+F[\varepsilon_n(i,j)]\} \qquad (7)$$

where F(.) is an operator, representing the filtration of the corresponding LP component.

The comparison of the quantzation noise distribution on IDP and LP decomposition layers shows that for the IDP the noises for the layers with icreasing numbers p=1,2, ... are much lower than these for the LP layers with decreasing numbers p = n-1, n-2,...

The noise relation for layers p=1 for IDP and p=n-1 for LP could be represented as follows:

$$\varepsilon_{1(i,j)}+M_1[\varepsilon_0(i,j)] \ll \varepsilon_{n-1}(i,j)+F[\varepsilon_n(i,j)], \qquad (8)$$

because

$$\frac{F[\varepsilon_n(i,j)]}{M_1[\varepsilon_0(i,j)]}=(2^{n-2}\times 2^{n-2})\frac{\sum\sum_{(i,j)\in W(2\times 2)}\varepsilon_n(i,j)}{\sum\sum_{(i,j)\in W_{k_1}(2^{n-1}\times 2^{n-1})}\varepsilon_0(i,j)} \gg 1 \qquad (9)$$

Similarly, for IDP and LP layers p = 2 and p = n-2 correspondingly this relation is:

$$\varepsilon_2(i,j)+M_2\{\varepsilon_1(i,j)+M_1[\varepsilon_0(i,j)]\} \ll \ll \varepsilon_{n-2}(i,j)+F\{\varepsilon_{n-1}(i,j)+F[\varepsilon_n(i,j)]\}, \qquad (10)$$

because

$F\{\varepsilon_{n-1}(i,j)+F[\varepsilon_n(i,j)]\} \gg M_2\{\varepsilon_1(i,j)+M_1[\varepsilon_0(i,j)]\}$, etc.

Then in result of the comparison of Eqs. (6) and (7) follows, that

$$\varepsilon_\Sigma(i,j) \ll \varepsilon_\Sigma^{PL}(i,j), \qquad (11)$$

i.e. the influence of the quantization noise on the restored image is much lower in IDL than in LP.

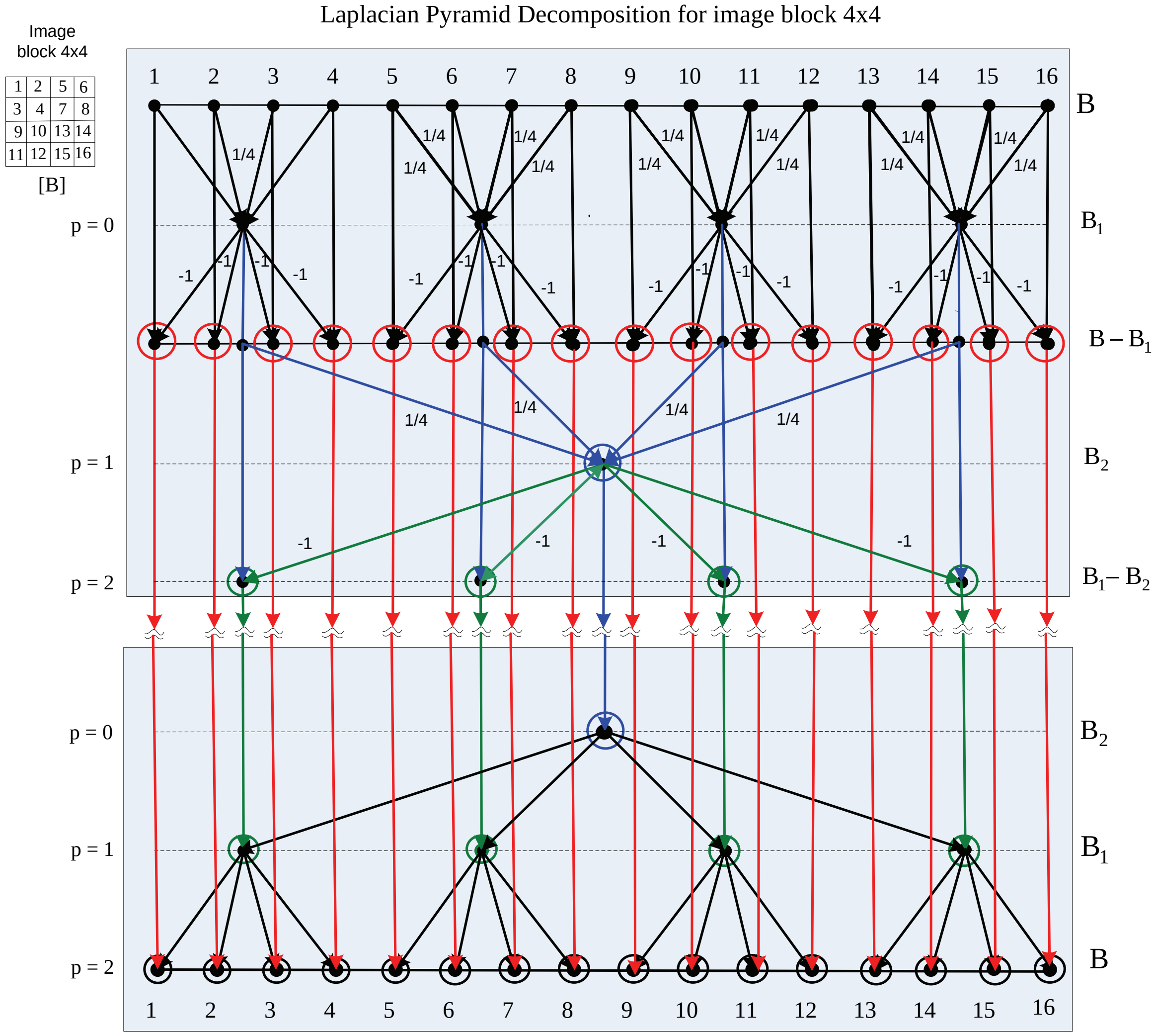
III. Graphs, Representing the IDP and LP Decompositions

On Figs.1 and 2 are shown the graphs, representing the calculations of IDP and PL for image of size 4×4 pixels (N = 4). The comparison of these graphs shows that:

- The information in the IDP and LP components is identical, but arranged in inverse order;

- The volume of the accumulated quantization noises in the restored image is lower in IDP than in LP. This conclusion follows from the analysis of the noise distribution for both pyramids in accordance with Eqs. (2-10);
- IDP better corresponds to the requirement for progressive image transfer (PIT) [9] than LP, because the calculation of the IDP components starts from the pyramid top and continues to its base, while for the PL this sequence is in inverse order;
- The IDP graph (Fig.2) shows that after calculating the layer p = 0 the input data could be substituted with the so obtained, etc. – for the next layers (p = 1,2...). From this follows that the memory needed for the IDP pipline implementation is two times smaller than that for LP.

Laplacian Pyramid Decomposition for image block 4x4



Reconstruction of image block 4x4

Fig. 1. LP structure graph

$$[B(i, j)] = [\,\overline{B}(i, j)\,]+[\overline{E}_0^{k_1}(i, j)] + [\,E_1(i, j)]; \quad k_1 = 1,2,3,4.$$

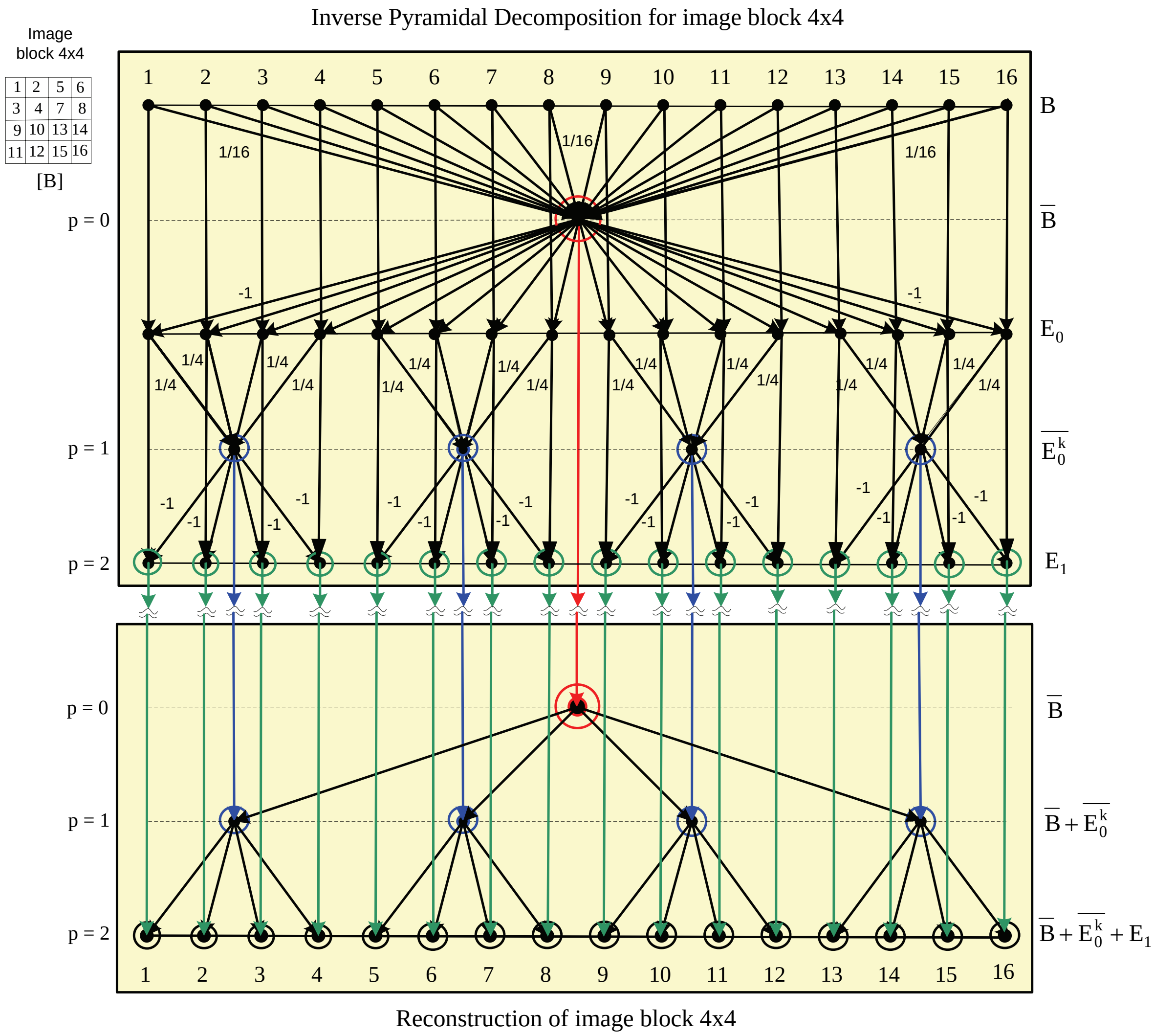


Fig.2. IDP structure graph

ACKNOWLEDGEMENT

This work was supported by the National Fund for Scientific Research of the Bulgarian Ministry of Education and Science, Contract VU-I 305.

REFERENCES

[1] R.Gonzalez, R. Woods, *Digital Image Processing*, Prentice Hall, 2001.

[2] P. Burt, E. Adelson. “The Laplacian Pyramid as a Compact Image Code”. IEEE Trans. on Communications, Vol. Com-3l, No. 4, April 1983, pp. 532-540.

[3] Al Bovik. *The Essential Guide to Image Processing*. Ch. 6, Multiscale Image Decomposition and Wavelets. Academic Press, NY, 2009, pp. 123-142.

[4] Z. Liu, Y. Ho, K. Tsukada, K. Hanasaki, Y. Dai, L. Li. “Using multiple orientational filters of steerable pyramid for image registration”. Information Fusion 3, 2002, Elsevier, pp. 203-214.

[5] R. Kountchev, V. Haese-Coat, J. Ronsin. “Inverse Pyramidal decomposition with multiple DCT”. Signal Processing: Image Communication, Elsevier, Vol. 17, Feb. 2002, pp. 201-218.

[6] V. Goyal, M. Vetterli, N. Thao, “Quantization of over complete expansions”, Proc. IEEE Data Compression Conf., Utah, USA, 1995, pp.13-22.

[7] M. Do, M. Vetterli, *Contourlets in Beyond Wavelets*, J. Stoeckler, G. Welland (eds.), Academic Press, New York, 2002.

[8] D. Zang, G. Sommer. “Algebraically Extended 2D Image Representation”. 17th Intern. Conf. on the Application of Computer Science and Mathematics in Architecture and Civil Engineering, Germany, July 2006, pp. 1-10.

[9] M. Mancas, B. Gosselin, B. Macq. “Perceptual Image Representation”, EURASIP Journal on Image and Video Processing, 2007, pp. 1-9.