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Discrete Model of Isotropic Excitable Media for Image Processing

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***Abstract*-In this paper is proposed one model of discrete excitable media for wave propagation. Model provides a constant wave velocity along all directions, which gives possibility for effective parallel image processing. The state of element of discrete media is described by the vector S=(X,Y,O). During the n-tm step of wave propagation every element receives state vectors of it's neighbour elements and calculates the minimal Euclidean distance to the source of excitation. During the process of changing the state of elements, every element will switch to active state after a time, that depends of it's Euclidean distance to the source of excitation. Thus, having only local relations, discrete media provides a spherical front of wave propagation.**

***Keywords*: texture generation, shapes modulation, covering of surface**

I. Introduction

Wave processes in excitable media (WPEM) are non-linear wave processes. For continuum they are described by partial differential equations of type:

$$\frac{du}{dt} = D.\frac{d^2u}{dx^2} + f(u) \qquad (1)$$

Equation (1) describes change of state of element u as a function of partial co-ordinates x,y and time t. It is derivative of equation, which describes process of diffusion – spatial and time changing of concentration, known as Fick's second laws of diffusion [2]. Discrete model of excitable media is cellular automata, in wich the state of a particular cell in the next time step depends on the state of the neighbours cells at the current time. Discrete models of WPEM gives possibilty for effective parallel image processing: recovering of contours, smoothing of contours, obtaining sceleton, etc.

II. Binary cellular model of WPEM

In the simplest binary model of excitable discrete media every element has only two possible states (we can denote them with 0 and 1). Algorithm of wave propagation can be described as follows:

If at the given time step one element has state 0 and at least one neighbour element has step 1, in the next time step this element switch to state 1. in opposite case element keeps it's state.

This model, despite it's simplicity, gives possibility to examinate wavefront of WPEM. However, models of excitable medium need more then two states of elements – at least one additional state – refractory state -- the element has recently been excited and has not yet been through the refractory period. Refractory period can continue one or more steps of wave propagation.

III. Type of wavefront in discrete periodical set of elements

Algorithm of binary WPEM, described above, defines the changing element's state rule as a simple logical function on neighbour elements state. To define precisely this function, it's need to define wich are the neighbour elements of given element. Most often descrete medium is considered obtained from continuum using rectangular mesh for discretization.

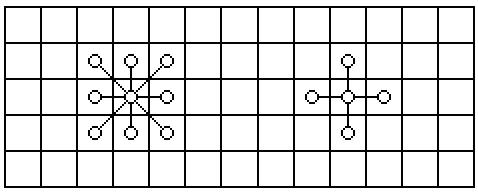

Fig. 1. 8-Connectivity and 4- Connectivity between elements of rectangular mesh

This definition of connectivity in a digital pattern need clarification: which pixels are neighbor pixels? Answer of this question (the need of which is a specific problem of digital images) is related to definition of neighborhood of elements in rectangular mesh. The figure below shows two types of connection;

- 4- Connectivity – neighbors are the elements, which share an edge
- 8- Connectivity – neighbors are the elements, which share an vertex

Difference between two types of connectivity deterninates difference between wavefronts of WPEM from point source. Wavefront from point source for WPEM with 4-connectivity for k steps of wave propagation is presented as follows:

$$Y4(n1,n2,k)= \delta[|n1+|n2|-k] , \qquad (2)$$

Wavefront from point source for WPEM with 8-connectivity for k steps of wave propagation is presented as follows:

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$$Y8(n1,n2,k)= \delta[\max(|n1|,|n2|)-k] \qquad (3)$$

where:
n1 and n2 are co-ordinates of element in discrete media.
к – discrete time step.
δ{…} – discrete impuls function,
max(|n1|,|n2|) dnotes the biggist of two co-ordinates, given as absolute values.

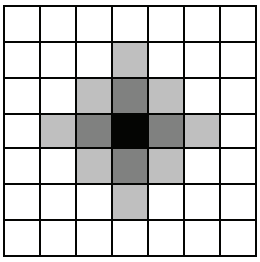


Fig. 2a - Wavefront from point source for 4 connectivity

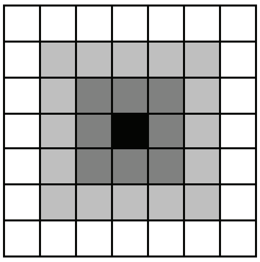


Fig. 2b - Wavefront from point source for 8 connectivity

Fig. 2 shows propagation of WPEM from source point in rectangular mesh of elements, having 4 and 8 connectivity. Image on Fig. 2a presents the state of discrete media after two steps of WPEM for 4 connectivity between elements, while image on Fig. 2b presents the state of discrete media after two steps of WPEM for 8 connectivity between elements. Comparing of wavefronts shows, that on the direction of axes n1 and n2 of coordinate system velocity of wavefront of WPEM for four-connectivity and eight-connectivity is the same – wavefront moves to the next discrete element on every step of wave propagation. However, along the rest directions, velosities are different: high velocity has propagation of wavefront на of WPEM for eight-connectivit.

The common link between the two wavefronts is their zero curvature, which shows anisotropic nature of examinated discrete excitable media. This vastly restrict its possibilities for image processing. In example using of WPEM for scalling of contour images gives as a result at the same time change of dimensions and change of form of the contour, because the contour line moves with the different velocity along different directions.

Fig. 3a shows one digital image, which defines initial state of elements in rectangular mesh. White color present elements, which are in initial (non-excited) state. Black color present elements, which are in active (excited) state. Gray color present refracter state (state, after which element switch to inactive state regardless the state of neighbour elements

As can be seen, contour, obtained after 10 steps of WPEM with 4 connectivity beteen elements not only scale the given contour, but change the form of contour. The cause of this result, as was told above, is thedifferent velocity of wave along different directions. Exactly this type of connectivity gives hier velosity along direction of axes X and Y of coordinate system, that’s why contour curve is more moved along these directions

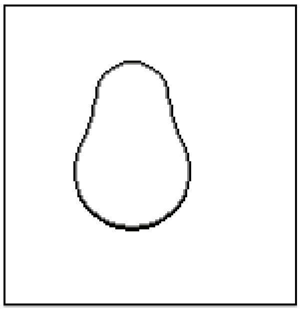


Fig. 3a – Given contour image

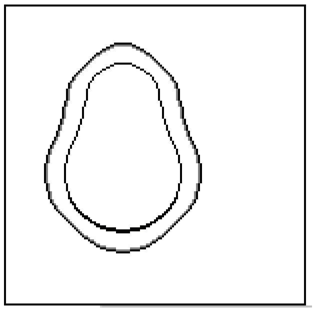


Fig. 3b - Given contour image and contour image, obtained after 10 steps wave process of with 4 connectivity

The simular result we will get using WPEM with 8 connectivity beteen elements. Fig. 4b shows the given contour and the contour, obtained after 10 steps of wave process in mesh with 8 connectivity. Result again is change of firm of contour, the cause of thedifferent velocity of wave along different directions. Exactly this type of connectivity gives hier velosity along direction of axes X and Y of coordinate system and along their’s angular bisectors, that’s why contour curve is more moved along these directions

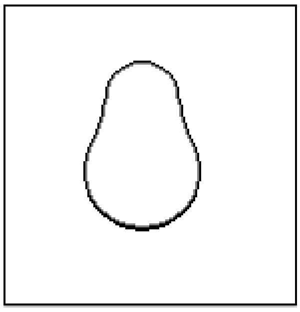


Fig. 4a – Given contour image

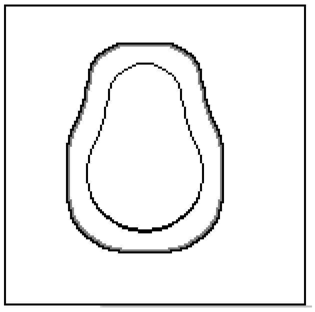


Fig. 4b - Given contour image and contour image, obtained after 10 steps wave process of with 8 connectivity

This report offers isotropic excitable discrete media, which can be used for some tasks of image processing.

IV. Model of isotropic excitable discrete media

The cause for the anisotropic behavior of the regular discrete mesh is the regular and local type of connectivity between elements. To have the same velocity in all directions, it is needed to have wavefront from point source, which looks as follows:

$$Y(n1,n2,m1,m2,k) =$$

$$\delta\left(k-\sqrt{(n1-m1)^2+(n2-m2)^2}\right) \quad (4)$$

where:
n1 and n2 are co-ordinates of element in discrete media.
к – discrete time step.
δ{…} – discrete impuls function,
m1 and m2 are co-ordinates of point source.

To obtain this type of wavefront it is needed to have the time of change the state of elements proportional to Euclidean distance of that element to the source of excitation. It is easy to obtain this for discrete media, in which every element is connected to all others. If some element in moment 0 is in excitable state, every of all others elements can calculate it's own moment, in which have to change its state. Problem is , that this type of discrete mesh has too mush connections – it has quadratic relationships between the number of elements N and number or connections R

The state of element in discrete mesh can be presented by vector S=(X,Y,O) where X и Y are the distances to source point on direction of co-ordinate axes OX и OY , measured in number of elements, and O is the output signal of current element.

The steps of proposed algorithm are:

1) Wave process for changing the state of elements in mesh.
2) Change of output signals of elements as a function of its state and of its distance to the source.

Changing the state of elements is performed as follows: At the time of clock n of discrete wave process every one element receives from it's neighbour elements their state vectors and calculates it's minimal Euclidean distance to the source point of exitation. At the time of next step of algorithm – changing the state of elements, every element switch to active state after the time, proportional to Euclidean distance D of that element

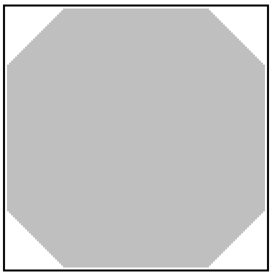
$$D = \sqrt{X^2 + Y^2} \quad (5)$$

to the source of excitation This way, having only connections between neighbour elements, proposed discrete media ce provide sзrerical form of wavefront.

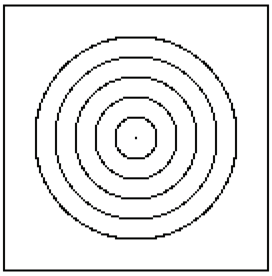
V. Practical results.

Has been created a program, which demonstrate using of algorithm, described above, for scaling of contour images.

Fig. 5 shows the results of wave process from source point in proposed discrete media. Fig. 5a shows area of elements, for which has been calculated the distance to the source point after 100 steps of wave process.



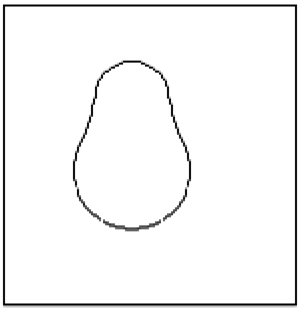
Фиг. 5а – Area of elements, whichdistance to source point has been calculated



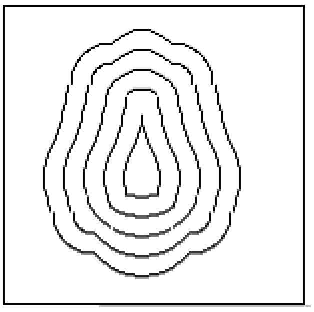
Фиг. 5b - Wavefronts from source point

Fig. 5b shows wavefront subsequently on 5-th, 10-th, 15-th, 20-th и 25-th step of process of change the output singnal of elements. As can be seen, wave fronts are spherical (circles) for all steps of wave propagation.

Fig. 6a shows given contour image, while Fig. 6b – given contour image and wavefronts on distances 10 и 20 pixels, obtained using proposed algorithm.



Фиг. 6а – Given contour image



Фиг. 6b – Given image and wavefronts on distance 10 and 20 pixels

This example demonstrates possibilities to obtain a contour curves on the given distance to the object in digital image.

VI. Conclusion

The proposed algorithm for wave propagation in discrete excitable media provide a spherical front of wave propagation from source point and identical velocity along all directions of wave propagation. However, it's importatn to take in consideration, that discrete media, wich works as it ti described above is not a linear system, which means, that it does not satisfied the superposition principle. That's why wave, obtained from two source points is not the sum of waves, wich can be obtained from this source points applied separately. This gives as a result specific distortion of wavefront, as can be seen in Fig. 6b. One possible decision of this problem is using of multilayer medium, in which at least neighbour source points "works' on different layers. In this model, one additional layer can make superposition of waves of waves of all layers, in which partial waves are propagated

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