Full Perturbation Analysis of the Discrete-time LMI Based H_∞ Quadratic Stability Problem

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***Abstract* – The quadratic H_∞ control ensuring closed-loop performance γ can be implicitly realized by the solutions *Q, Y* of a system of linear matrix inequalities (LMIs). The paper is concerned with performing linear full perturbation analysis for the discrete-time LMI based H_∞ quadratic stability problem. The sensitivity analysis of the perturbed matrix inequalities is considered in a similar manner as for perturbed matrix equations, after introducing a suitable right hand part, which is slightly perturbed. The proposed approach leads to tight linear perturbation bounds for the LMIs' solutions to the H_∞ quadratic stability problem. Numerical example is also presented**

***Keywords* – Full perturbation analysis, H_∞ quadratic stability, LMI based synthesis, Linear systems.**

I. Introduction

In many control problems, the design constraints have a simple reformulation in terms of linear matrix inequalities (LMIs). This is hardly surprising, given that LMIs are direct byproducts of Lyapunov based criteria, and that Lyapunov techniques play a central role in the analysis and control of linear systems, see [1,2] and the literature therein.

The H_∞ control problem is a good illustration of this point. Indeed, the H_∞ constraints can be expressed as a single matrix inequality via the bounded real lemma [3]. Even though the H_∞ control problem has a solution in terms of Riccati equations [4], the LMI approach remains valuable for several reasons. First it is applicable to all plants without restrictions on infinite or pure imaginary invariant zeros. Secondly, it offers a simple and insightful derivation of the Riccati based solvability conditions [5]. In addition, the LMI based H_∞ control is practical thanks to the availability of efficient convex optimization algorithms, based on the interior point method [6], and software [7].

In this paper we propose an approach to perform full linear perturbation analysis of the LMI based H_∞ quadratic stability problem via introducing a suitable right hand part in the considered matrix inequalities.

We use the following notations: $R^{m\times n}$ - the space of real $m\times n$ matrices; $R^n = R^{n\times 1}$; I_n - the identity $n\times n$ matrix; e_n - the unit $n\times 1$ vector; M^T - the transpose of M ; $M^\dagger$ - the pseudo inverse of M ; $\|M\|_2 = \sigma_{\max}(M)$ - the spectral norm of M , where $\sigma_{\max}(M)$ is the maximum singular value of M ; $vec(M)\in R^{mn}$ - the column-wise vector representation of $M\in R^{m\times n}$; $\Pi_{m,n}\in R^{mn\times mn}$ - the vec-permutation matrix, such that $vec(M^T)=\Pi_{m,n}vec(M)$; $M\otimes P$ - the Kroneker product of the matrices M and P . The notation “:=” stands for “equal by definition”.

The remainder of the paper is organized as follows. In Section 2 we shortly present the problem set up and objective. Section 3 describes the performed linear full perturbation analysis of the LMI based H_∞ control problem. Section 4 presents a numerical example before we conclude in Section 5 with some final remarks.

II. Problem Set up and Objective

Consider the linear discrete-time system

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k),\\ y(k) &= Cx(k)+Du(k) \end{aligned} \qquad (1)$$

where $x(k)\in R^n$, $u(k)\in R^m$,and $y(k)\in R^r$ are the system state, input and output vectors respectively, and A, B, C, D are constant matrices of compatible size.

We consider an LMI approach to solve the H_∞ quadratic stability problem, as stated in [8]

$$\begin{bmatrix} -P^{-1} & A & B & 0\\ A^T & -P & 0 & C^T\\ B^T & 0 & -\gamma I & D^T\\ 0 & C & D & -\gamma I \end{bmatrix} < 0, P > 0, \qquad (2)$$

which is actually an Eigenvalue Problem (EVP) with respect to the variables P and γ. Here we assume that the optimal closed-loop performance γ_{opt} of the system (1) is already obtained.

In order to obtain quadratic H_∞ stability and to ensure closed-loop performance γ it is necessary to design a state-feedback control $u=Kx$. To transform LMI (2) we apply Schur complement argument [9] to obtain the following inequality:

$$\begin{bmatrix} -P^{-1} & (A+BK) & 0 & 0\\ (A+BK)^T & -P & 0 & (C+DK)^T\\ 0 & 0 & -\gamma I & 0\\ 0 & C+DK & 0 & -\gamma I \end{bmatrix} < 0, P > 0, \qquad (3)$$

with respect to the variables K, P and γ. We pre-and post-multiply inequality (3) by $diag\{I, P^{-1}, I, I\}$ and introduce change of variables such that $Q=P^{-1}$ and $Y=KP^{-1}$ to obtain the following system of LMIs:

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$$\begin{bmatrix} -Q & (AQ+BK) & 0 & 0 \\ (AQ+BY)^T & -Q & 0 & (CQ+DY)^T \\ 0 & 0 & -\gamma I & 0 \\ 0 & CQ+DY & 0 & -\gamma I \end{bmatrix} < 0, Q > 0, \quad (4)$$

The main objective of the paper is to perform a linear sensitivity analysis of the LMI system (4) near the optimal value of γ, needed to solve the H_∞ quadratic stability problem.

Suppose that the matrices *A, B, C, D* are subject to perturbations $\Delta A, \Delta B, \Delta C, \Delta D$ and assume that they do not change the sign of the LMI system (4). The full perturbation analysis of the discrete-time LMI based H_∞ quadratic stability problem is aimed at determining perturbation bounds of the LMIs (4) near the optimal value of γ, as functions of the perturbations in the data *A, B, C, D* and in γ_{opt}.

III. Linear Full Perturbation Analysis

We perform perturbation analysis of the LMI (4) for the discrete-time system (1)

$$\begin{bmatrix} -(Q+\Delta Q) & ABQY^T & 0 & 0 \\ ABQY & -(Q+\Delta Q) & 0 & CQDY^T \\ 0 & 0 & -(\gamma+\Delta\gamma) & 0 \\ 0 & CQDY & 0 & -(\gamma+\Delta\gamma) \end{bmatrix} < 0, \quad (5)$$

where $ABQY^T = (Q+\Delta Q)(A+\Delta A)^T + (Y+\Delta Y)^T(B+\Delta B)^T$

$ABQY = (A+\Delta A)(Q+\Delta Q) + (B+\Delta B)(Y+\Delta Y)$,

$CQDY^T = (Q+\Delta Q)(C+\Delta C)^T + (Y+\Delta Y)^T(D+\Delta D)^T$,

$CQDY = (C+\Delta C)(Q+\Delta Q) + (D+\Delta D)(Y+\Delta Y)$. We have to study the effect of the perturbations $\Delta A, \Delta B, \Delta C, \Delta D$ and $\Delta\lambda$ on the perturbed LMI solutions $Q^* + \Delta Q$ and $Y^* + \Delta Y$, where Q^*, Y^* and $\Delta Q, \Delta Y$ are the nominal solution of the inequality (4) and the perturbations, respectively. The essence of our approach is to perform perturbation analysis of the inequality (4) in a similar manner as for a proper matrix equation after introducing a suitable right hand part, which is slightly perturbed. Thus for LMI (5) we have:

$$\begin{bmatrix} -(Q^*+\Delta Q) & ABQY^{*T} & 0 & 0 \\ ABQY^* & -(Q^*+\Delta Q) & 0 & CQDY^{*T} \\ 0 & 0 & -(\gamma_{opt}+\Delta\gamma) & 0 \\ 0 & CQDY^* & 0 & -(\gamma_{opt}+\Delta\gamma) \end{bmatrix} = L^* + \Delta L_1 < 0, \quad (6)$$

where $ABQY^{*T} = (Q^*+\Delta Q)(A+\Delta A)^T + (Y^*+\Delta Y)^T(B+\Delta B)^T$,

$ABQY^* = (A+\Delta A)(Q^*+\Delta Q) + (B+\Delta B)(Y^*+\Delta Y)$

$CQDY^{*T} = (Q^*+\Delta Q)(C+\Delta C)^T + (Y^*+\Delta Y)^T(D+\Delta D)^T$,

$CQDY = (C+\Delta C)(Q^*+\Delta Q) + (D+\Delta D)(Y^*+\Delta Y)$ and L^* is obtained using the nominal LMI

$$\begin{bmatrix} -Q^* & Q^*A^T + Y^{*T}B^T & 0 & 0 \\ AQ^*+BY^* & -Q^* & 0 & Q^*C^T + Y^T{}^*D^T \\ 0 & 0 & -\gamma_{opt} & 0 \\ 0 & CQ^*+DY^* & 0 & -\gamma_{opt} \end{bmatrix} = L^* < 0, \quad (7)$$

The matrix ΔL_1 is due to the data and closed-loop performance perturbations, the rounding errors and the sensitivity of the interior point method that is used to solve the LMIs.

Using the relation (7) the perturbed equation (6) may be written as

$$\Delta_Q + \Omega_Q = \Delta L_1, \quad (8)$$

where

$$\Delta_Q = \begin{bmatrix} -\Delta Q & \Delta Q A^T & 0 & 0 \\ A\Delta Q & -\Delta Q & 0 & \Delta Q C^T \\ 0 & 0 & 0 & 0 \\ 0 & C\Delta Q & 0 & 0 \end{bmatrix},$$

$$\Omega_Q = \begin{bmatrix} 0 & Q^*\Delta A^T + \Delta Y^T B^T + Y^{*T}\Delta B^T & 0 & 0 \\ \Delta A Q^* + B\Delta Y + \Delta B Y^* & 0 & 0 & Q^*\Delta C^T + \Delta Y^T D^T + Y^{*T}\Delta D^T \\ 0 & 0 & -\Delta\gamma I & 0 \\ 0 & \Delta C Q^* + D\Delta Y + \Delta D Y^* & 0 & -\Delta\gamma I \end{bmatrix},$$

Here the terms of second and higher order are neglected. The relation (8) may be written in a vector form as

$$vec(\Delta_Q) + vec(\Omega_Q) = vec(\Delta L_1), \quad (9)$$

where

$$vec(\Delta_Q) = [-I, A\otimes I, 0, 0, I\otimes A, -I, 0, C\otimes I, 0,0,0,0,0, I\otimes C, 0, 0]^T$$
$$* vec(\Delta Q) = T\Delta q,$$

$vec(\Omega_Q) =$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ (I\otimes Q^*)\Pi_{n^2} & (B\otimes I)\Pi_{n\times m} & (I\otimes Y^{*T})\Pi_{m^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (Q^*\otimes I) & (I\otimes B) & (Y^*\otimes I) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (D\otimes I)\Pi_{n\times m} & 0 & (I\otimes Q^*)\Pi_{n^2} & 0 & (I\otimes Y^*)\Pi_{m^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (I\otimes D) & 0 & (Q^*\otimes I) & 0 & (Y^*\otimes I) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e & 0 \end{bmatrix} \times$$

$$\times\begin{bmatrix} vec(\Delta A) \\ vec(\Delta Y) \\ vec(\Delta B) \\ vec(\Delta C) \\ \Delta\gamma \\ vec(\Delta D) \end{bmatrix} = \left[T_{t1}, T_{t2}, T_{t3}, T_{t4}, T_{t5}, T_{t6}\right]\Delta_{aybcd\gamma} = T_t\Delta_{aybcd\gamma}.$$

Further we obtain the expression

$$\begin{aligned} &T\Delta q + T_{t1}vec(\Delta A) + T_{t2}vec(\Delta Y) + T_{t3}vec(\Delta B) + T_{t4}vec(\Delta C) + T_{t5}\Delta\gamma + \\ &\quad + T_{t6}vec(\Delta D) = vec(\Delta L_1). \end{aligned} \tag{10}$$

Finally the relative perturbation bound for the solution Q^* of the LMI (4) has the form

$$\begin{aligned} \frac{\|\Delta q\|_2}{\|vec(Q^*)\|_2} \le &\frac{1}{\|vec(Q^*)\|_2}\left(T_1\frac{\|vec(\Delta A)\|_2}{\|vec(A)\|_2} + T_2\frac{\|vec(\Delta Y)\|_2}{\|vec(Y^*)\|_2} + T_3\frac{\|vec(\Delta B)\|_2}{\|vec(B)\|_2}\right) \\ &+ \frac{1}{\|vec(Q^*)\|_2}\left(T_4\frac{\|vec(\Delta C)\|_2}{\|vec(C)\|_2} + T_5\frac{|\Delta\gamma|_2}{|\gamma_{opt}|_2} + T_6\frac{\|vec(\Delta D)\|_2}{\|vec(D)\|_2}\right) \\ &+ \frac{1}{\|vec(Q^*)\|_2}\left(L_1\frac{\|vec(\Delta L_1)\|_2}{\|vec(L^*)\|_2}\right) \end{aligned} \tag{11}$$

where

$$\frac{T_1}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|T_{t1}\|_2\|vec(A)\|_2}{\|vec(Q^*)\|_2}, \quad \frac{T_2}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|T_{t2}\|_2\|vec(Y^*)\|_2}{\|vec(Q^*)\|_2},$$

$$\frac{T_3}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|T_{t3}\|_2\|vec(B)\|_2}{\|vec(Q^*)\|_2}, \quad \frac{T_4}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|T_{t4}\|_2\|vec(C)\|_2}{\|vec(Q^*)\|_2},$$

$$\frac{T_5}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|T_{t5}\|_2|\Delta\gamma|_2}{\|vec(Q^*)\|_2}, \quad \frac{T_6}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|T_{t6}\|_2\|vec(D)\|_2}{\|vec(Q^*)\|_2},$$

$$\frac{L_1}{\|vec(Q^*)\|_2} = \frac{\|T^\dagger\|_2\|vec(L^*)\|_2}{\|vec(Q^*)\|_2}.$$

may be considered as individual relative condition numbers of the LMI (4) with respect to the perturbations $\Delta A, \Delta B, \Delta C, \Delta D, \Delta Y$ and $\Delta\gamma$.

In a similar way the relative perturbation bounds for the solution Y^* of the LMI (4) may be obtained using the following expression

$$\Delta_Y + \Omega_Y = \Delta L_2, \tag{12}$$

where

$$\Delta_Y = \begin{bmatrix} 0 & \Delta Y^T B^T & 0 & 0 \\ B\Delta Y & 0 & 0 & \Delta Y^T D^T \\ 0 & 0 & 0 & 0 \\ 0 & D\Delta Y & 0 & 0 \end{bmatrix},$$

$$\Omega_Y = \begin{bmatrix} -\Delta Q & \Delta Q A^T + Q^*\Delta A^T + Y^{*T}\Delta B^T & 0 & 0 \\ A\Delta Q + \Delta A Q^* + \Delta B Y^* & 0 & 0 & \Delta Q C^T + Q^*\Delta C^T + Y^{*T}\Delta D^T \\ 0 & 0 & -\Delta\gamma I & 0 \\ 0 & C\Delta Q + \Delta C Q^* + \Delta D Y^* & 0 & -\Delta\gamma I \end{bmatrix},$$

Here the terms of second and higher order are neglected. The relation (12) may be written in a vector form as

$$vec(\Delta_Y) + vec(\Omega_Y) = vec(\Delta L_2), \tag{13}$$

where

$$\begin{aligned} vec(\Delta_Y) = &[0, (B\otimes I)\Pi_{n\times m}, 0, 0, (I\otimes B), 0, 0, (D\otimes I)\Pi_{n\times m}, 0, 0, 0, 0, 0, (I\otimes D), 0, 0]^T \\ &* vec(\Delta Y) = W\Delta y, \end{aligned}$$

$$vec(\Omega_Y) =$$

$$= \begin{bmatrix} 0 & -I & 0 & 0 & 0 & 0 \\ (I\otimes Q^*)\Pi_{n^2} & (A\otimes I) & (I\otimes Y^{*T})\Pi_{m^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (Q^*\otimes I) & (I\otimes A) & (Y^*\otimes I) & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (C\otimes I) & 0 & (I\otimes Q^*)\Pi_{n^2} & 0 & (I\otimes Y^*)\Pi_{m^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (I\otimes C) & 0 & (Q^*\otimes I) & 0 & (Y^*\otimes I) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e & 0 \end{bmatrix} \times$$

$$\times\begin{bmatrix} vec(\Delta A) \\ vec(\Delta Q) \\ vec(\Delta B) \\ vec(\Delta C) \\ \Delta\gamma \\ vec(\Delta D) \end{bmatrix} = \left[W_{t1}, W_{t2}, W_{t3}, W_{t4}, W_{t5}, W_{t6}\right]\Delta_{aqbcd\gamma} = W_t\Delta_{aqbcd\gamma}.$$

Further we obtain the expression

$$\begin{aligned} &W\Delta y + W_{t1}vec(\Delta A) + W_{t2}vec(\Delta Q) + W_{t3}vec(\Delta B) + W_{t4}vec(\Delta C) + W_{t5}\Delta\gamma + \\ &\quad + W_{t6}vec(\Delta D) = vec(\Delta L_2). \end{aligned} \tag{14}$$

Finally the relative perturbation bound for the solution Y^* of the LMI (4) has the form

$$\frac{\|\Delta y\|_2}{\|vec(Y^*)\|_2} \le \frac{1}{\|vec(Y^*)\|_2}\left(W_1\frac{\|vec(\Delta A)\|_2}{\|vec(A)\|_2} + W_2\frac{\|vec(\Delta Q)\|_2}{\|vec(Q^*)\|_2} + T_3\frac{\|vec(\Delta B)\|_2}{\|vec(B)\|_2}\right)$$
$$+\frac{1}{\|vec(Y^*)\|_2}\left(W_4\frac{\|vec(\Delta C)\|_2}{\|vec(C)\|_2} + W_5\frac{|\Delta\gamma|_2}{|\gamma_{opt}|_2} + W_6\frac{\|vec(\Delta D)\|_2}{\|vec(D)\|_2}\right)$$
$$+\frac{1}{\|vec(Y^*)\|_2}\left(L_2\frac{\|vec(\Delta L_2)\|_2}{\|vec(L^*)\|_2}\right) \quad (15)$$

where

$$\frac{W_1}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|W_{t1}\|_2\|vec(A)\|_2}{\|vec(Y^*)\|_2}, \frac{W_2}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|W_{t2}\|_2\|vec(Q^*)\|_2}{\|vec(Y^*)\|_2},$$

$$\frac{W_3}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|W_{t3}\|_2\|vec(B)\|_2}{\|vec(Y^*)\|_2}, \frac{W_4}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|W_{t4}\|_2\|vec(C)\|_2}{\|vec(Y^*)\|_2},$$

$$\frac{W_5}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|W_{t5}\|_2|\Delta\gamma|_2}{\|vec(Y^*)\|_2}, \frac{W_6}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|W_{t6}\|_2\|vec(D)\|_2}{\|vec(Y^*)\|_2},$$

$$\frac{L_2}{\|vec(Y^*)\|_2} = \frac{\|W^\dagger\|_2\|vec(L^*)\|_2}{\|vec(Y^*)\|_2}.$$

may be considered as individual relative condition numbers of the LMI (4) with respect to the perturbations $\Delta A, \Delta B, \Delta C, \Delta D, \Delta Q$ and $\Delta\gamma$.

IV. Numerical Examples

Consider the discrete-time system (1), where

$$A=\begin{bmatrix} 1 & 0.01 \\ -0.0067 & 0.9966 \end{bmatrix}, B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.004 & -0.0007 & -0.001 & 0.0033 \end{bmatrix},$$
$$C=\begin{bmatrix} -0.6667 & -0.333 \\ 0 & 1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}, D=\begin{bmatrix} -0.4 & -0.0667 & -0.1 & 0.333 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The perturbations in the system matrices of the discrete-time system are chosen as

$$\Delta A = A\times 10^{-i}, \Delta B = B\times 10^{-i}, \Delta C = C\times 10^{-i},$$
$$\Delta D = D\times 10^{-i}, \Delta L_1 = L^*\times 10^{-i}, \Delta L_2 = L^*\times 10^{-i}, \Delta\gamma = \gamma_{opt}\times 10^{-i}$$
$$\Delta Q^* = Q^*\times 10^{-i}, \Delta Y = Y^*\times 10^{-i} \text{ for } i = 8, 7, \ldots, 4.$$

The perturbed solutions $Q^*+\Delta Q$ and $Y^*+\Delta Y$ are computed based on the method derived in [5] and using the software [7]. The relative perturbation bounds for the solutions Q^* and Y^* of the LMIs (4) are obtained by the linear bounds (11) and (15), respectively.

The results obtained for different values of i are shown in the following table

TABLE I

i	$\frac{\|\Delta q\|_2}{\|vec(Q^*)\|_2}$	Bound (11)	$\frac{\|\Delta y\|_2}{\|vec(Y^*)\|_2}$	Bound (15)
8	$3.59*10^{-8}$	$4.92*10^{-7}$	$1.54*10^{-8}$	$5.32*10^{-7}$
7	$3.59*10^{-7}$	$4.92*10^{-6}$	$1.54*10^{-7}$	$5.32*10^{-6}$
6	$3.59*10^{-6}$	$4.92*10^{-5}$	$1.54*10^{-6}$	$5.32*10^{-5}$
5	$3.59*10^{-5}$	$4.92*10^{-4}$	$1.54*10^{-5}$	$5.32*10^{-4}$
4	$3.59*10^{-4}$	$4.92*10^{-3}$	$1.54*10^{-4}$	$5.32*10^{-3}$

The obtained perturbation bounds (11) and (15), based on the presented solution approach, are close to the real relative perturbation bounds $\frac{\|\Delta q\|_2}{\|vec(Q^*)\|_2}$ and $\frac{\|\Delta y\|_2}{\|vec(Y^*)\|_2}$, thus they are good in sense that they are tight.

V. Conclusion

The linear full perturbation analysis of the discrete-time LMI based H_∞ control problem has been studied. Tight perturbation bounds, which are linear functions of the data perturbations, have been obtained for the matrix inequalities determining the problem solution. Based on these results we have presented numerical examples to explicitly reveal the performance and applicability of the proposed approach to analyze the sensitivity of the discrete-time LMI based H_∞ control problem.

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