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Multiple-Model Model Predictive Control for Nonlinear Systems

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Abstract – **In this paper we present a multi-parametric algorithm for control of nonlinear systems. Recently proposed algorithms for hybrid control of nonlinear systems introduce big computational burden. In order to reduce the optimization problem we use the multiple-model approach in. The algorithm is based on linearization of the nonlinear plant in multiple operational points. These operational points are chosen after detailed analysis of the plant's behaviour and the set of referent inputs. Then we construct model based predictive controller for each defined linearized model of the system, and we connect these states into one switched MPC. This controller automatically switches the prediction model according to the user instructions.**

Keywords – **Model predictive control, nonlinear control, switching control.**

I. Introduction

Process industries need an easy to setup predictive controller that costs low, and maintains an adaptive behavior which accounts for time-varying dynamics as well as potential plant miss-modeling. MPC has the ability to fulfill the expectation of the engineers and to successfully control complex processes.

As presented in [1] the essence of model-based predictive control (MBPC) or model predictive control (MPC) lies in optimization of the future process behavior with respect to the future values of the executive (or manipulated) process variables. Throughout this report the abbreviation MPC shall be used. The use of linear, non-linear, hybrid and time-delay models in model-based predictive control is motivated by the drive to improve the quality of the prediction of inputs and outputs, as well as to reduce the computer burden during the optimization [1-5].

This paper is organized as follows: we briefly present the latest developments in MPC theory, and especially nonlinear model predictive control in section 2. Then in section 3 we introduce the techniques for multiple-model MPC. In section 4 we present the simulation result performed on a model of a tank reservoir. At the end in section 5 we point out the conclusions from the paper and give some further research headings in the area of nonlinear and multiple-model model predictive control.

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II. The Basics of Model Predictive Control

A. The General Idea of MPC

The general idea behind MPC is simple indeed. If we have a reliable model of the system, represented as in (1) or similar, we can use it for predicting the future system behavior. At each consecutive time of sampling k the controls inputs (2) are calculated,

$$\begin{aligned} &x(k+1) = Ax(k) + Bu(k);\ x(0) = x_0 \\ &y(k+1) = Cx(k) + Du(k); \end{aligned} \tag{1}$$

$$\begin{aligned} u(k) = [&u(k\,|\,k),\ u(k+1\,|\,k),... \\ &u(k+N_u-2\,|\,k),\ u(k+N_u-1\,|\,k)] \end{aligned} \tag{2}$$

where A,B,C,D are the system matrices; N_u represents the length of the control horizon and the notation $u(k+p\,|\,k)$ means the prediction of the control input value for the future time $k+p$ calculated at time k. These control inputs are calculated in such a way as to minimize the difference between the predicted controlled outputs $y(k+p\,|\,k)$ and foreseen set points $r(k+p\,|\,k)$ for these outputs, over the prediction horizon $N_y, (p=1,2,...N_y)$. Then only the first element from the calculated control inputs is applied to the process, i.e. $u(k)=u(k\,|\,k)$. At the next sample time $(k+1)$, we have a new measurement of the process outputs and the whole procedure is repeated.

The most common used cost function is the Quadratic, and it can be formulated as:

$$\begin{aligned} J(k) = &\sum_{p=N_1}^{N_y} \left\| w(k+p\,|\,k) - y(k+p\,|\,k) \right\|^2 + \\ &\lambda \sum_{p=0}^{N_u-1} \left\| \Delta u(k+p\,|\,k) \right\|^2 \end{aligned} \tag{3}$$

B. Nonlinear MPC

When we control nonlinear systems, usually we consider the following definition

$$\dot{x}(t) = f(x(t),u(t)), \quad x(0) = x_0 \in \mathrm{X}_0 \tag{4}$$

subject to the input and state constraints

$$u(t) \in U, \quad x(t) \in X, \quad \forall t \geq 0, \tag{5}$$

where $x(t) \in X \subseteq \Re^n$ is the system state and $u(t) \in U \subset \Re^m$ is the input applied to the system. In this case we need to use more complex nonlinear programming in order to compute the optimal solution of the nonlinear control problem.

III. Multiple-Model Model Predictive Control for Nonlinear Systems

For control the process is approximated with p linear affine models that built a hybrid PWA state space model as presented in [6]

$$\begin{aligned} & x(k+1) = A^i x(k) + B^i u(k) + f^i \\ & y(k) = C^i x(k) + D^i u(k) + g^i \\ & \text{if } \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in P^i, i \in \{1, ..., p\} \end{aligned} \tag{6}$$

where k is the discrete time index, A^i, B^i, C^i, D^i state space matrices, f^i, g^i the affine vectors, $u \in \Re^m$ input, $x \in \Re^n$ state, and P^i valid region of the state+input space in $\Re^{m+n}$. The system is subject to input and state constraints. For each region P^i a model exists and for it the corresponding mp-MPC controller is designed. The currently active model is determined by Model selection algorithm from estimated state values. Each time step the active controller computes the control signal. The control scheme is presented in Fig. 1.

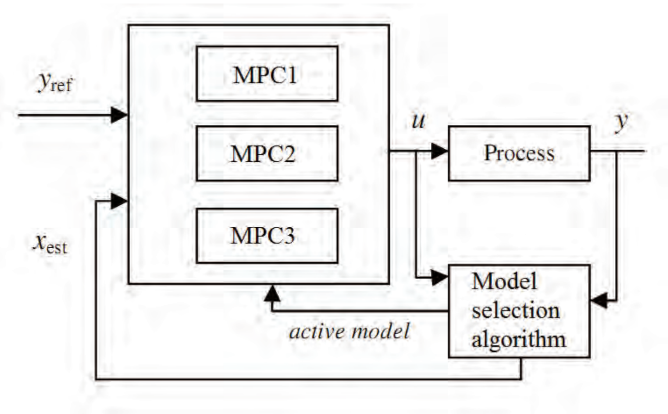

Fig. 1. Multiple-model predictive control scheme

This model predictive controllers use linear models of the nonlinear system to predict the future behavior. The models are linearized around different working points of the plant.

Model selection algorithm is the most important part of the multiple-model MPC. Usually it is a function depending on the inputs and outputs of the system which results with appropriate model of the system. In more complicated systems Kalman filter is used to estimate the system states, and afterwards the algorithm selects the appropriate model.

IV. Simulation of MMMPC on a Water Tank Model

In this paper we will use multiple-model MPC algorithm control a nonlinear reservoir tank model. This model is originally proposed in MathWorks Inc. MATLAB software.

Water enters a tank from the top and leaves through an orifice in its base. The rate that water enters is proportional to the voltage, V, applied to the pump. The rate that water leaves is proportional to the square root of the height of water in the tank. The water tank is illustrated on Fig. 2.

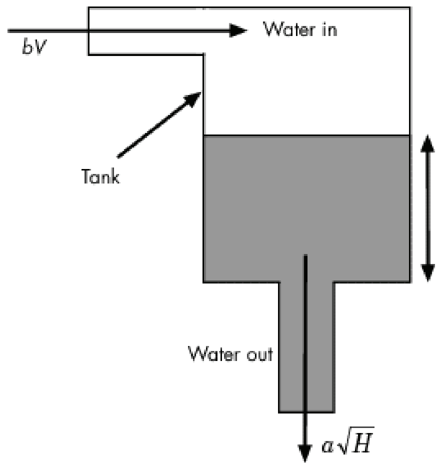

Fig. 2. Schematic Diagram for the Water-Tank System

The dynamics of the process can be described with the following equation

$$\frac{d}{dt} Vol = A \frac{dH}{dt} = bV - \sqrt{H} \tag{7}$$

where Vol is the volume of water in the tank, A is the cross-sectional area of the tank, b is a constant related to the flow rate into the tank, and a is a constant related to the flow rate out of the tank. The equation describes the height of water, H, as a function of time, due to the difference between flow rates into and out of the tank.

The equation contains one state, H, one input, V, and one output, H. It is nonlinear due to its dependence on the square-root of H.

The tank does have physical limitations. Maximum height ca ne 10 meter (that is the height of the tank), and the input voltage (which is proportional to the fluid flow in the tank) can vary between 0 and 24 VDC.

The linearization of the water tank model is realized using Matlab functions from the Control Systems Toolbox in five separate points depending of the only state of the system – height of the water tank H. Since the height is limited with both minimal and maximal value, we can easily decide which linearization points we should use. In this example we use linearization point at H=2m; H=4m; H=6m; H=8m; H=10m.

The realization of the switched model predictive controller is done it Matlab Simulink platform. The Simulink file containing the Switched MPC and the controlled process if presented on

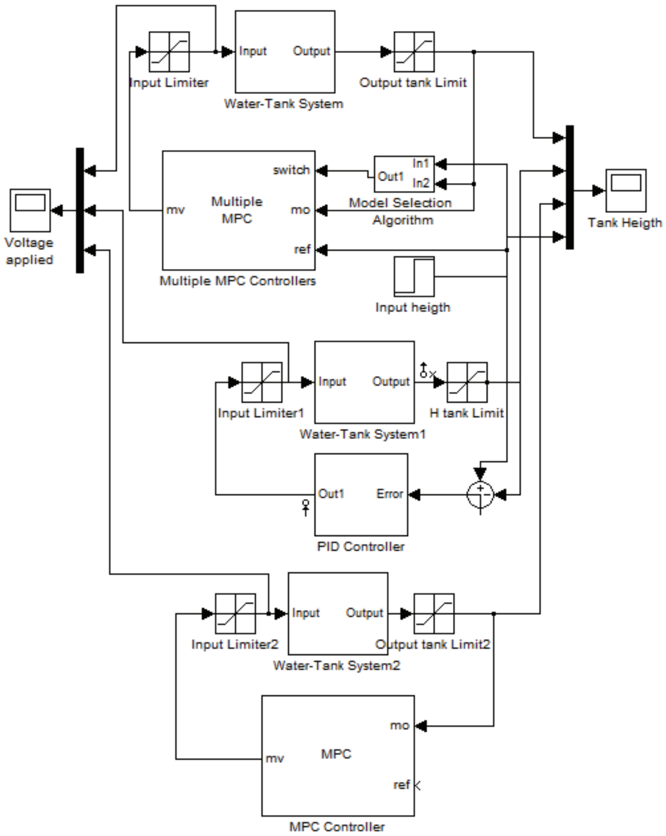

Fig. 3. Matlab Simulink model for simulation of the water tank system (M-M MPC, PID and MPC control systems, from top down)

We are presenting the results from an experiment for settling the water height at levels 2, 10 and 6 meters. The results are presents in figures 4, 5 and 6 retrospectively.

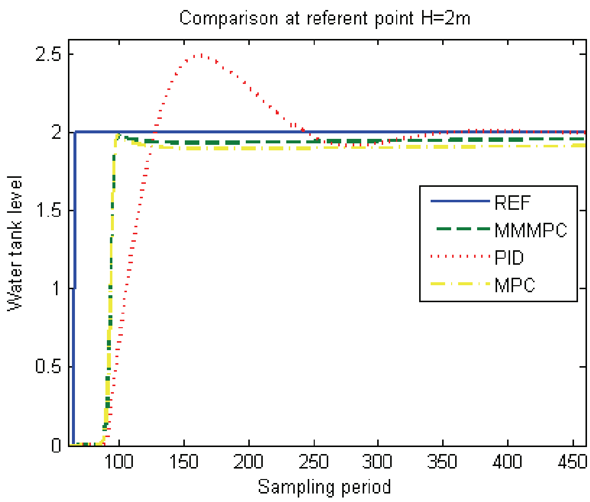

Fig. 4. Results from the simulation of the water tank system when we have referent height of 2 meters.

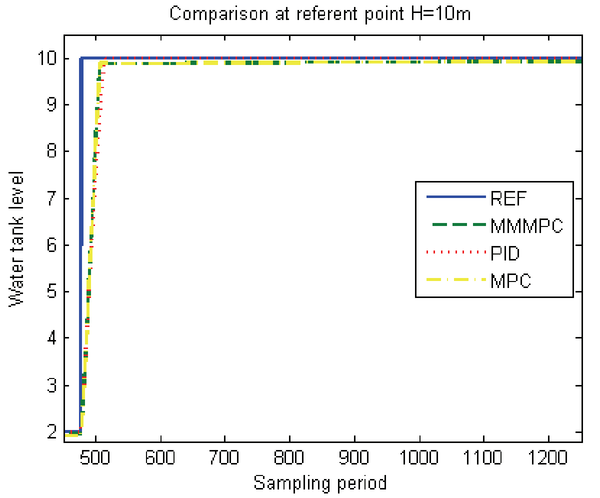

Fig. 5. Results from the simulation of the water tank system when we have referent height of 10 meters.

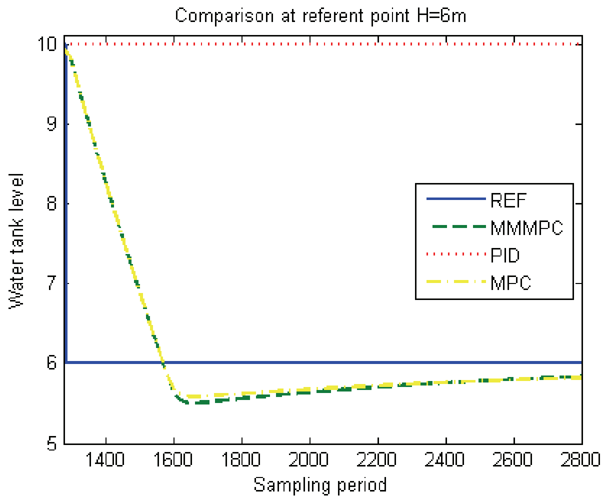

Fig. 6. Results from the simulation of the water tank system when we have referent height of 6 meters.

It is obvious that the multiple-models MPC has the best performance comparing with the PID and the MPC with only one point of linearization. On Fig. 4. we can notice that the PID controller (which is properly tuned) has significant overshot compared to the MPC controllers. Also the MMMPC has better performance compared with MPC linearized only in operating point with height 10 m.

On Fig. 5. we can notice that all tree controller have almost the same behavior. Nevertheless the PID controller is not “aware” of the physical limitation of the water tank, which can be seen from the control signal during this period. The input voltage is high compared to the control signals of the MMMPC and MPC controllers. During this period of time, the water tank is full, and yet the control signal is high, which results with overflowing of the water tank. Here we must point out that the model based predictive controllers have big advantage compared to the conventional PID, because of the possibility to enter the input and output constraints.

The results presented on Fig. 6. are directly affected by the previous state of the system. The MMMPC and the MPC, have similar behavior, but as time passes, the MMMPC leans towards smaller steady state error than the MPC. In this case the PID controller continuously has the value of 10 m, which is result on an integrated value of the controller during the foregoing period.

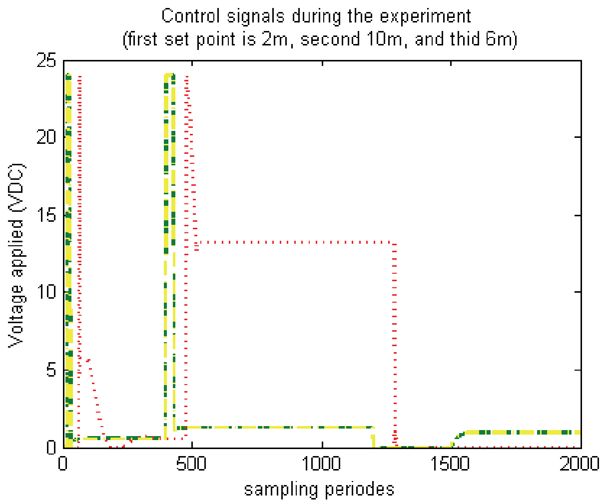

Fig. 7. Control signals during the whole experiment.

Fig 7. presents us the control signals during the experiment. On this picture we can notice that the control signals for the MPC and MMMPC are almost equivalent, but there is big difference compared to the PID control signal. The reasons why are explained before in the paper.

V. Conclusions

The model predictive control techniques have great advantage compared to the conventional control techniques, but still they are model-dependent. This means that the performance of the MPC is directly affected by the precision of the model. When we are controlling nonlinear processes and plants, the model mismatch will rather be bigger than smaller. In some particular cases we can easily partition the system and linearize in multiple points. We have shown that these linearized models can be used for constructing a multiple-model MPC, which has better characteristic compared to the MPC linearized around only one operating point.

There are possibilities for further research especially in defining proper model selection algorithm and determining the proper operation points of the system.

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