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Switched Fuzzy Control Systems: Concepts for Simulation

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Abstract **–In this paper we propose a concept for building simulation algorithms for switched fuzzy systems. Using these concepts we have shown the whole process of modeling, stability analysis and design of stabilizing controllers for the hovercraft-vehicle, as a typical nonholonomic system. MATLAB Simulator (Simulink model) and special MATLAB functions for the switched fuzzy controller of the hovercraft vehicle, along with the numerous simulation results, verify the correctness of the proposed concepts.**

Keywords **– Control, fuzzy systems, hybrid systems, switched systems, switching, hovercraft.**

I. INTRODUCTION

Switched system is special class of hybrid dynamical systems, which consist of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching among them. The last couple of decades have witnessed an enormous growth of interest of the class of switched systems in combination with the even larger class of hybrid systems [4][5][6][7], as these systems have a wide range of potential applications. From the middle of the 1980's, there have appeared a number of analysis/synthesis problems for Takagi-Sugeno (T-S) fuzzy systems [1][2][3].

Following the remarkable developments in theory, applications, and the industrial implementations of fuzzy control systems, recently [8] switched systems have been extended further to encompass *switched fuzzy systems*. In general, a switched fuzzy system involves fuzzy systems among its sub-systems or an alternative fuzzy-switching law, or the both (least explored case). Resent developments in this area promote a new direction in the control of dynamic systems [9] [10] [11] [12] [13] [14] [15], and it is clear that the field of switched fuzzy systems is becoming very popular.

To our best knowledge it may well be found that, up to know the most of the research in this area is focused on representation modeling, stability analysis and controller design, that guarantees stabilization and certain system performance. We are expecting that these results can be used as a good platform for solving real world control problems. Thus, in this paper we are exploring the performance of switched fuzzy control systems, and also we are proposing the simulation scheme algorithms, that can be used in modelling and design of switched fuzzy controllers for real world systems. Hovercraft vehicle, as a typical nonholonomic system, is used for verifying the simulations.

To begin with, in Section 2 we present the basic concepts of switched fuzzy systems with levels of structure, their representation modelling and stability analysis, taken from [9][10][11][12]. In Section 3 we are proposing the simulation scheme algorithms. In Section 4 we present the whole process of modelling and design of the switched fuzzy controllers, using the previously suggested simulation schemes, for the hovercraft vehicle. In Section 5 we give the essential conclusions of this study.

II. CONCEPTS OF SWITCHED FUZZY SYSTEMS, REPRESENTATION MODELLING AND STABILITY ANALYSIS

Since in [15] we have given a detailed overview of the achievements in the field of switched fuzzy systems, followed by the comparative study for this kind of systems, here, we will only provide a discussion of some of the key principles of the joint concept, using the representation modelling given in [9],[10],[11],[12], the so-called "switched fuzzy system with levels of structure".

The switching fuzzy model from [9][10][11][12] is given with:

Region Rule i :

IF $z_1(t)$ **is** N_{i1} **and** $\ldots$ **and** $z_p(t)$ **is** N_{ip},

THEN (1)

Local Plant Rule l :

IF $z_1(t)$ **is** M_{il1} **and** $\ldots$ **and** $z_p(t)$ **is** M_{ilp},

$$\textbf{THEN}\begin{cases}\dot{x}(t)=A_{il}x(t)+B_{il}u(t),\\ y(t)=C_{il}x(t),\end{cases}\quad l=1,2,\ldots,r\quad i=1,2,\ldots,m$$

Here, m is the number of regions partitioned on the premise parts space. $N_{ij}(z(t))$ is a crisp set, where $N_{ij}(z(t))=\begin{cases}1 & ,z(t)\in N_{ij}\\ 0 & ,o.w\end{cases}$; r is the number of rules of the local models; M_{ilj} is fuzzy set; $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input vector, $y(t)\in R^q$ is the output vector, $A_{il}\in R^{n\times n}$, $B_{il}\in R^{n\times m}$, and $C_{il}\in R^{q\times n}$; $z(t)=[z_1(t),\ldots,z_p(t)]$ are known premise variables that can be functions of the state variables, external disturbances, and/or time.

Region Rule i :

IF $z_1(t)$ **is** N_{i1} **and** $\ldots$ **and** $z_p(t)$ **is** N_{ip},

THEN (2)

Local Control Rule l :

IF $z_1(t)$ **is** M_{il1} **and** $\ldots$ **and** $z_p(t)$ **is** M_{ilp},

$$\textbf{THEN}\quad u(t)=-F_{il}x(t),\qquad l=1,2,\ldots,r\quad i=1,2,\ldots,m.$$

From (1), it is clear that the switching fuzzy model has two levels of structure: region rule level and local fuzzy rule level. The switching fuzzy model (1) is inferred by fuzzily blending the linear system models $\dot{x}(t)=A_{il}x(t)+B_{il}u(t)$ and switching the global T-S fuzzy models, defined on every region.

In [9], authors propose a new PDC to design a stable switching fuzzy controller for the switching fuzzy system (1). The structure of

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the PDC fuzzy controller is given with (2), where the design purpose is to determine the local feedback gains F_{il} in the consequent parts.

Along with the representation modelling of switched fuzzy systems, authors in [9][10] present LMI stability (relaxed stability) conditions for the system that consists of the switching fuzzy model (1) and the appropriate PDC controller, given with (2). Except the stability conditions, authors in [9]-[12] present the conditions for the constraints on control inputs.

III. Proposed Simulation Algorithms

The whole process for design of switched fuzzy controller using switched fuzzy model with levels of structure, for the certain nonlinear system is shown on Fig. 1.

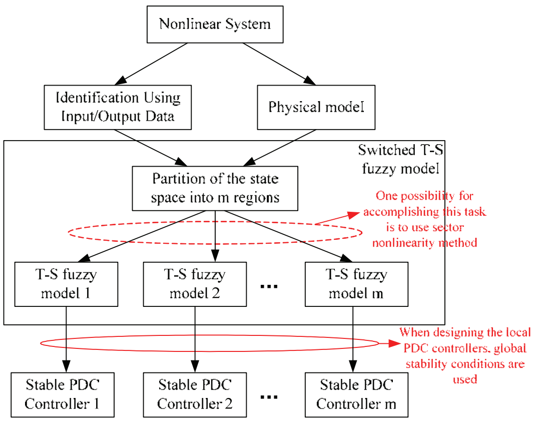

Fig. 1. Switched fuzzy controllers design.

In this paper, the simulations for performance analysis of the hovercraft-vehicle control will be made in few steps, which is shown on Fig. 2. This scheme for constructing the simulation environment can be also used for other nonlinear systems, if the parameters for the switched nonlinear model with levels of structure of a system are previously calculated. The final aim is to generate stable controllers which will satisfy certain conditions given in a form of LMIs.

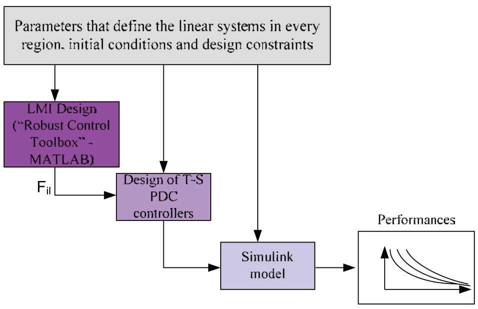

Fig. 2. Steps for controller design and performance evaluation for the given switched fuzzy model.

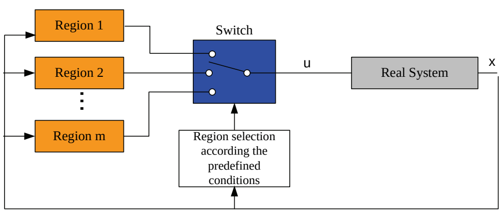

Fig. 3. Control of a real system with previously designed controllers, according the procedure given on Fig. 2.

After the local switched controllers are generated, according to the appropriate switched T-S fuzzy model with levels of structure, they can be used for controlling the real nonlinear system. Choosing the appropriate controller in certain moment (switching among these controllers) is made according the conditions which define the certain region selection. This is shown on Fig. 3. In every region there is an appropriate stable controller (in this case the controller is of PDC type).

IV. T-S Switched Fuzzy Model and Controller for the Hovercraft Vehicle

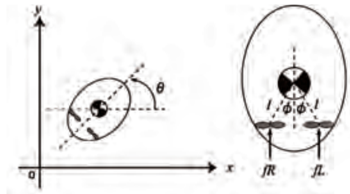

Fig. 4. Hovercraft model and its coordinate system [9].

Acording to hovercraft model of Fig. 4, the following state-space model of the hovercraft vehicle is given in [9]:

$$\dot{x}_1(t)=\ddot{y}(t)=\frac{1}{M}\sin\theta(t)f_1(t),\ \dot{x}_2(t)=\dot{y}(t)=x_1(t),$$

$$\dot{x}_3(t)=\ddot{\theta}(t)=\frac{l\sin\phi}{I}f_2(t),\ \dot{x}_4(t)=\dot{\theta}(t)=x_3(t) \qquad (3)$$

where $f_1(t)=f_R(t)+f_L(t)$, $f_2(t)=f_R(t)-f_L(t)$, θ is the angle of the vehicle; l is the distance between the gravity and fans; ϕ is the angle between the gravity and fans; f_R and f_L are the forces generated by the right and left side fans, respectively; M and I are the mass and the inertia, respectively. The control purpose is $\lim_{t\to\infty} y(t)=0$ and $\lim_{t\to\infty}\theta(t)=0$, by manipulating $f_R(t)$ and $f_L(t)$.

The hovercraft vehicle, is a typical nonholonomic system (the system is subject to constraints involving both, the position and velocity, whereupon it can not be stabilized by any continuous feedback law [4]). Further, we will show that this system can be stabilized with switching controller, based on the switched fuzzy-logic model.

A. Switched Fuzzy Model

Assuming that $\theta(t)\in[-179\quad 179]$, we divide the premise variable space into three regions with nonnegative constant d. We will use sector nonlinearity concept for determing the local linear models in every region [3].

Region 1: $\theta(t)\geq d$

$\sin\theta(t)=\sum_{i=1}^{2}h_{1i}(\theta(t))a_{1i}$, where $h_{11}(\theta(t))=\frac{\sin(\theta(t))-a_{12}}{a_{11}-a_{12}}$ and $h_{12}(\theta(t))=\frac{a_{11}-\sin(\theta(t))}{a_{11}-a_{12}}$, where $a_{11}=1$ and $a_{12}=\sin(179^0)\approx 0.01$.

Region 2: $-d<\theta(t)<d$

In this region, we fix the first input, i.e. $f_1(t)=C$, where C is a positive constant. This implies $\ddot{y}(t)=\frac{\sin\theta(t)}{M}C$.

$\sin\theta(t)=\left(\sum_{i=1}^{2}h_{2i}(\theta(t))a_{2i}\right)\theta(t)$, where $h_{21}(\theta(t))=\frac{\frac{\sin(\theta(t))}{\theta(t)}-a_{22}}{a_{21}-a_{22}}$ and

$h_{22}(\theta(t))=\frac{a_{21}-\frac{\sin(\theta(t))}{\theta(t)}}{a_{21}-a_{22}}$, where $a_{21}=1$ and $a_{22}=\sin(d)/d$.

Region 3: $\theta(t) \le -d$

$\sin\theta(t) = \sum_{i=1}^{2} h_{3i}(\theta(t)) a_{3i}$, where $h_{31}(\theta(t)) = \dfrac{\sin(\theta(t)) - a_{32}}{a_{31} - a_{32}}$ and $h_{32}(\theta(t)) = \dfrac{a_{31} - \sin(\theta(t))}{a_{31} - a_{32}}$, where $a_{31} = -1$ and $a_{32} = \sin(-179^0) \approx -0.01$.

By aggregating the above results, according the relation (1), we construct the following switched fuzzy model:

Region Rule 1 : **IF** $\theta(t) \ge d$,
THEN
Local Plant Rule 1 : **IF** $\theta(t)$ **e** $h_{11}(\theta(t))$,
THEN $\dot{x}(t) = A_{11}x(t) + B_{11}u(t)$
Local Plant Rule 2 : **IF** $\theta(t)$ **e** $h_{12}(\theta(t))$,
THEN $\dot{x}(t) = A_{12}x(t) + B_{12}u(t)$

Region Rule 2 : **IF** - $d < \theta(t) < d$,
THEN
Local Plant Rule 1 : **IF** $\theta(t)$ **e** $h_{21}(\theta(t))$,
THEN $\dot{x}(t) = A_{21}x(t) + B_{21}u(t)$
Local Plant Rule 2 : **IF** $\theta(t)$ **e** $h_{22}(\theta(t))$,
THEN $\dot{x}(t) = A_{22}x(t) + B_{22}u(t)$

Region Rule 3 : **IF** $\theta(t) \le -d$,
THEN
Local Plant Rule 1 : **IF** $\theta(t)$ **e** $h_{31}(\theta(t))$,
THEN $\dot{x}(t) = A_{31}x(t) + B_{31}u(t)$
Local Plant Rule 2 : **IF** $\theta(t)$ **e** $h_{32}(\theta(t))$,
THEN $\dot{x}(t) = A_{32}x(t) + B_{32}u(t)$ (4)

where $u(t) = [f_1(t), f_2(t)]^T$ and $x(t) = [x_1(t) \;\; x_2(t) \;\; x_3(t) \;\; x_4(t)]$,

$$A_{11} = A_{12} = A_{31} = A_{32} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0 & 0 & \frac{C}{M}a_{21} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 0 & 0 & 0 & \frac{C}{M}a_{22} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} \frac{1}{M}a_{11} & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} \frac{1}{M}a_{12} & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix},$$

$$B_{21} = B_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix}, \quad B_{31} = \begin{bmatrix} \frac{1}{M}a_{31} & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix}, \quad B_{32} = \begin{bmatrix} \frac{1}{M}a_{32} & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix}.$$

B. Controller Design via Switched PDS

Region Rule 1 : **IF** $\theta(t) \ge d$,
THEN
Local Control Rule 1 : **IF** $\theta(t)$ **is** $h_{11}(\theta(t))$,
THEN $u(t) = -F_{11}x(t)$
Local Control Rule 2 : **IF** $\theta(t)$ **is** $h_{12}(\theta(t))$,
THEN $u(t) = -F_{12}x(t)$

Region Rule 2 : **IF** $-d < \theta(t) < d$,
THEN
Local Control Rule 1 : **IF** $\theta(t)$ **is** $h_{21}(\theta(t))$,
THEN $u(t) = -F_{21}x(t)$
Local Control Rule 2 : **IF** $\theta(t)$ **is** $h_{22}(\theta(t))$,
THEN $u(t) = -F_{22}x(t)$

Region Rule 3 : **IF** $\theta(t) \le -d$,
THEN
Local Control Rule 1 : **IF** $\theta(t)$ **is** $h_{31}(\theta(t))$,
THEN $u(t) = -F_{31}x(t)$
Local Control Rule 2 : **IF** $\theta(t)$ **is** $h_{32}(\theta(t))$,
THEN $u(t) = -F_{32}x(t)$ (5)

The switching fuzzy controller of PDC type for the switched fuzzy model (4) can be designed accoding the relation (2), having the form (5). From (5), it is obvious that the design process depends on the local feedback gains F_{il}, which can be obtained if there is a fisible solution to the LMI stability conditions given in [9], [10].

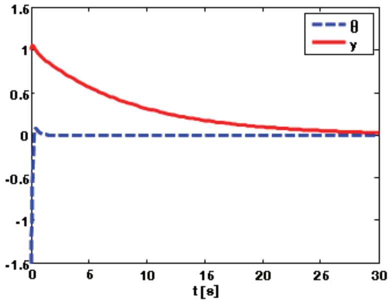

Fig. 5. Values for θ and y, using the control signals, shown on Fig. 6, when the LMI stability conditions from [9], [10] are used. Initial conditions are $x(0) = [0 \;\; 1 \;\; 0 \;\; -1.5]^T$ and (M=0.1, $\phi = \pi/4$, I=0.5, l=0.1, C=0.5, $d = \pi/50$).

From Fig.5 it is clear that the control purpose ($\lim_{t\to\infty} y(t) = 0$ and $\lim_{t\to\infty} \theta(t) = 0$) is achievable. Showing this it is obvious that the use of switched fuzzy controllers is worthwhile in comparison of ordinary T-S fuzzy controllers.

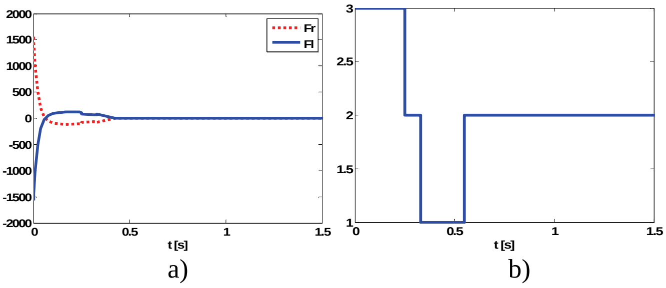

Fig. 6. a) Control inputs $f_R(t)$ and $f_L(t)$; b).Region selection.

However, if we view the control effort, i.e. the values of $f_R(t)$ and $f_L(t)$, (see fig. 6-a), it is obvious that they have very high amplitudes. The respective switching signal that selects the appropriate region during the control period is shown on Fig. 6-b.

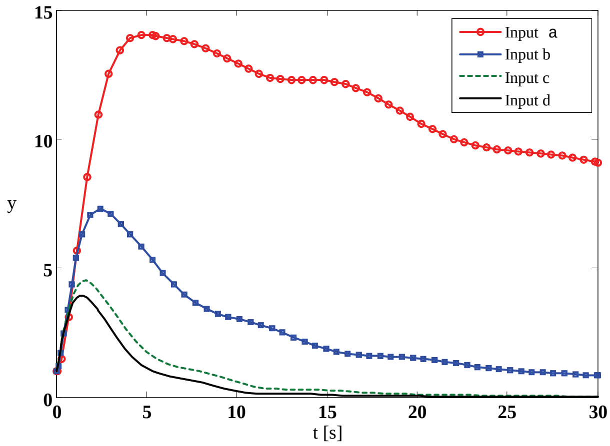

Fig. 7. Values for y, using the control signals, shown on Fig. 9.

For constraining the control inputs $f_R(t)$ and $f_L(t)$, we can use the LMI conditions given in [11], [12]. Please note that those LMI conditions depend on the initial conditions, so whenever we would like to change the initial conditions it would be necessary to recalculate the feedback control gains.

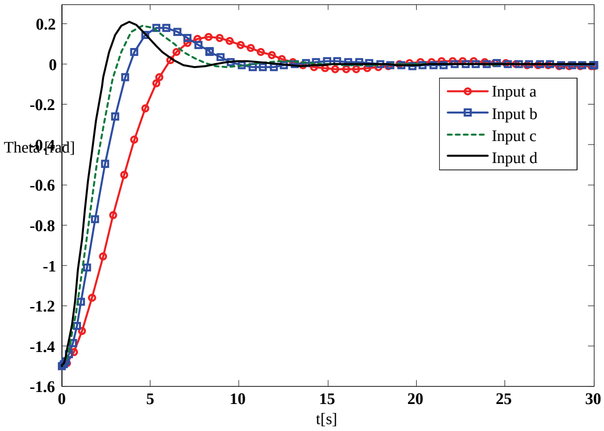

Fig. 8. Values for θ, using the control signals, shown on Fig. 9.

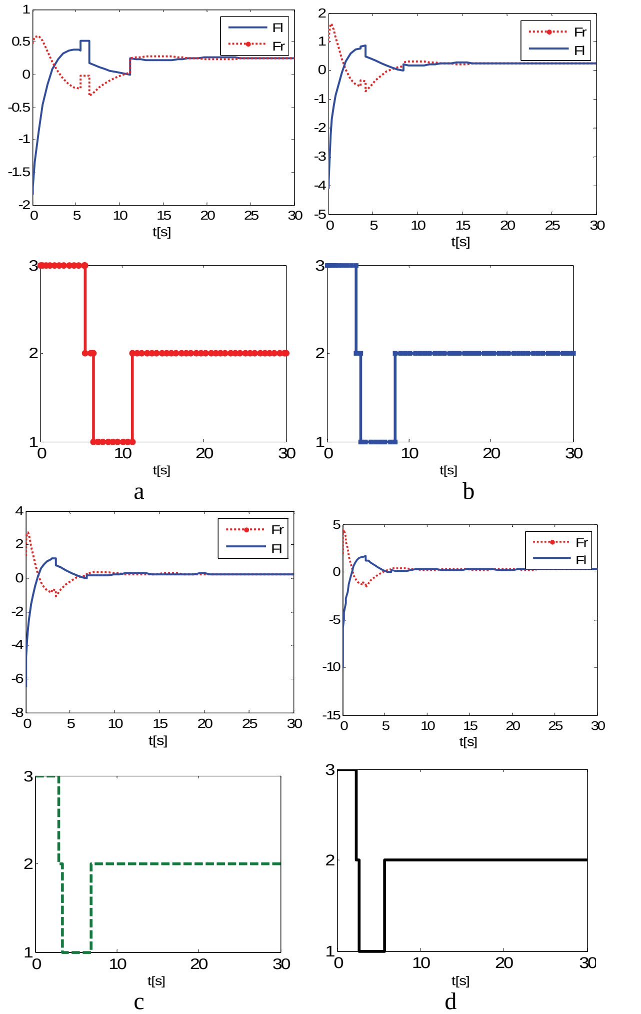

Fig. 9. Control inputs $f_R(t)$ and $f_L(t)$ and the appropriate switching signal, when the joined LMI stability conditions, along with the LMI conditions for the constraints on the control input are used. ($x(0)=[0 \quad 1 \quad 0 \quad -1.5]^T$, M=0.1, $\phi=\pi/4$, I=0.5, l=0.1, C=0.5, $d=\pi/50$).

Fig. 7 and Fig. 8 show the values of the control variables for the four different constraint inputs, given on Fig. 9-a,b,c,d. It is obvious that as much as the control inputs are constraint, as hard as the control purpose is achieved.

V. Conclusion

We have showed that the proposed mechanism for modeling and simulation of the performance of switched fuzzy systems is applicable for building and simulating the switched fuzzy control of the hovercraft vehicle, whereas we expect that these concepts can be used for building and simulating the switched fuzzy control of other nonlinear systems. Also, with the derived simulation results that show the feasibleness of the use of switched fuzzy controllers for the systems that are not stabilizible with any continuous feedback control laws, we showed that these types of controllers can be used in solving many control problems that are not solvable with other types of controllers.

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