Determining Bit Error Rate in Digital Optical Transmission Network Using the Q-Factor

Kiril K. Koitchev[1], Krasen K. Angelov[2], Stanimir S. Sadinov[3]

Abstract – **Knowledge of the ratio of the signal power to the noise power (signal-to-noise ratio – SNR) is important because it is directly related to the bit error ratio (BER) in digital communication systems, and the BER is a major indicator of the quality of the overall system. The SNR ratio related to the optical communications has its own specifics.**

The purpose of this paper is to show the relationship between the electrical and optical signal-to-noise ratio introducing the Q-factor. Although some of the principles can be applied to the existing coaxial cables, the scope of the paper is limited to binary digital signals over optical fiber. Application of codecs based on Read-Solomon codes will be considered as an example, where using this error correction method can reduce SNR in the receiver keeping the same value of BER.

Keywords – **Bit Error Rate, Q-factor, Optical Signal-to-Noise Ratio, Probability Density Function, block codes**

I. INTRODUCTION

So far *BER* estimation of characteristics in digital transmission systems, that use quadrature modulations, is important for studying especially in recording *SNR*. This ratio if related to optical communications is relatively sparsely represented and has its specific features [1].

The aim of this paper is to disclose the relationship between electrical and optical signal-to-noise ratio *SNR* by means of the relating parameter Q-factor. Although some of the principles could be applied in the coaxial cables, the particular case of this study will regard the transfer of binary digital signal along an optical fiber because in this way both bit and symbol rates will be equivalent. Application of codecs based on Read-Solomon codes will be considered as an example, since by this method of error correction it is possible to lower *SNR* on the receiving side with unchanged *BER*.

II. CALCULATION OF ERROR PROBABILITY

As is known the power of the optical signal is presented as the product of electric and magnetic field vectors:

$$P_o(t)=\left|\vec{E}(t)\times\vec{H}(t)\right|=\left|\frac{\vec{E}(t)}{\eta}\right|\cdot\left|\vec{E}(t)\right|=\left|\frac{\vec{E}(t)}{\eta}\right|^2. \qquad (1)$$

In practice direct measuring of optical power is rarely the case. The usual practice involves the use of PIN photodiode whose current is proportional to the optical power [5].

A point of interest is the studying of photo-receiver's performance. By synchronizing with the incoming optical signal the photo-receiver performs periodic processing of the received signal at the optimum moment. It registers the optical signal intensity and finally decides, on the basis of a certain threshold value, what type of signal is received: 0 or 1. Quality of the digital communication system performance could be evaluated by way of the so called eye-diagram (Fig. 1).

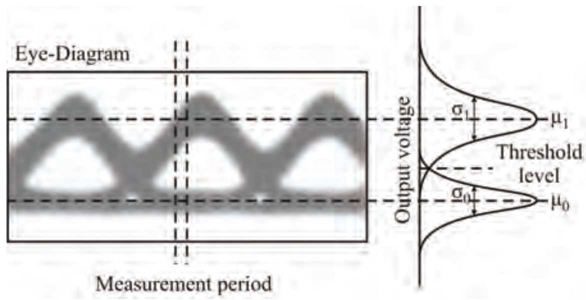

Fig. 1. Typical appearance of eye-diagram while using a code of the type RZ (return to zero) in optical signal receiving

Let us assume that in the optical receiver the converted voltage *v(t)* is compared with one definite value γ, which is referred to as threshold level. When *v(t)* is greater than γ this means that a binary "1" has been sent. When *v(t)* is less than γ – a binary "0" has been sent [1,2].

In presence of white Gaussian noise (AWGN), it is possible to calculate the statistical error probability. The probable density of *v(t)* in presence of AWGN can be mathematically expressed using Gaussian probability density function (*PDF*) [9,10,11]:

$$PROB\left[v(t),\sigma_x\right]=\frac{1}{\sqrt{2\pi\sigma_x^2}}e^{-\frac{1}{2}\left(\frac{v(t)-v(s)}{\sigma_x}\right)^2}, \qquad (2)$$

where *v(s)* is the mean value of the density function of the receiver, *v(t)* is the moment value of the receiver's voltage, and σ is the standard noise aberration. The graphic interpretation of Eq. 2 is shown in Fig. 2.

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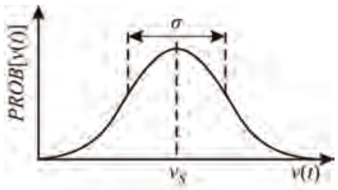

Fig. 2. Gaussian probability density function

If we assume that *v(s)* could take one of the two voltage levels designated as v_L and v_H then the probability for wrong decision making in the receiver is:

$$P[\varepsilon]=P[v(t)>\gamma\,|\,v_S=v_L]P[v_S=v_L]+P[v(t)<\gamma\,|\,v_S=v_H],\quad(3)$$

where $P[\varepsilon]$ is the error probability, $P[x,y]$ is the conditional probability of *x* for assigned value of *y*. Suppose that transmission and receiving probability is *50%* then $P[v_S=v_L] = P[v_S=v_H] = 0{,}5$. In this case Eq. 3 is reduced to:

$$\begin{aligned}P[\varepsilon]&=P[v(t)>\gamma\,|\,v_S=v_L]\times 0.5+P[v(t)<\gamma\,|\,v_S=v_H]\times 0.5=\\&=\frac{1}{2}\int_{-\infty}^{\gamma}PROB[v(t),\sigma_L]dt+\frac{1}{2}\int_{\gamma}^{\infty}PROB[v(t),\sigma_H]dt\end{aligned}\quad ,(4)$$

where $PROB[v(t),\sigma_x]$ is determined by Eq. 2. This interpretation is graphically presented in Fig. 3.

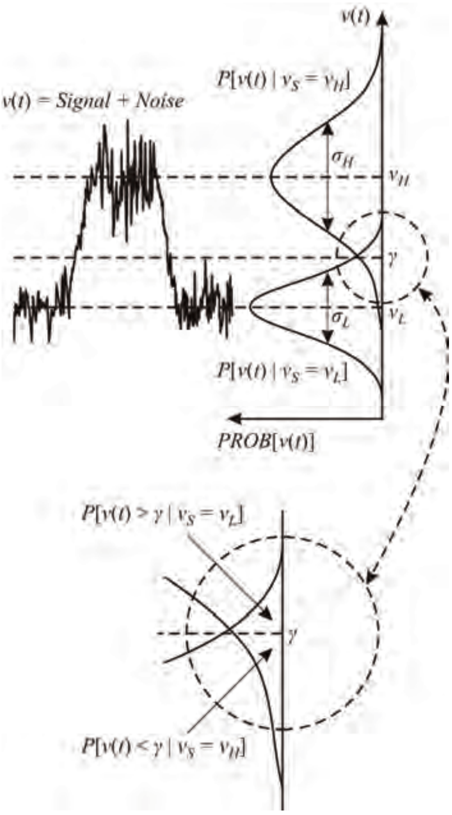

Fig. 3. Error probability in binary signal

Considering Fig. 3 and Eqs. 3 and 4 it is possible to reach the conclusion that the error probability is the area of *γ* beneath the graphs of the probability density functions. This area determines the magnitude of *BER* which depends on two factors:

– on the standard noise aberrations (σ_L and σ_H);
– on the difference between voltages v_L and v_H.

When $\sigma_L = \sigma_H$, then $\gamma=(v_H-v_L)/2$, i.e. γ is the average value between the high and low level. In most cases $\sigma_L \neq \sigma_H$. Then the optimum value of *BER* will be larger or less than the value of $(v_H-v_L)/2$.

To solve Eq. 4 it is necessary to calculate the result of integrating Gaussian distribution *PDF* $(PROB[v(t),\sigma_x])$ which is defined through Eq. 2. This is possible by employing numerical methods. For this purpose Eq. 2 can be solved by setting it into standard form and applying $z=(x-\mu)/\sigma$ where $x=v(t)$ and $\mu=v_S$.

For the sake of facilitation let us apply this substitution for the error function taking into account Eqs. 2 and 4,

$$\int_{\gamma}^{\infty}PROB[x,\sigma]dx=\int_{\gamma}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx\ ,$$

and by applying $z=(x-\mu)/\sigma$ (where $x=z\sigma-\mu$ and $dx=\sigma\,dz$) we obtain:

$$\frac{1}{\sqrt{2\pi}\sigma}.\int_{x=z\sigma+\mu=\gamma}^{\infty}e^{\left(\frac{-z^2}{2}\right)}\sigma\,dz\ .$$

This result defines the error function $E_r(z)$

$$E_z=\frac{1}{2\pi}\int_{z=\gamma}^{\infty}e^{\left(\frac{-z^2}{2}\right)}dz\ .\quad(5)$$

In this way Eq. 4 can be represent as

$$P[\varepsilon]=\frac{1}{2}E_r\left[\frac{v_H-\gamma}{\sigma_H}\right]+\frac{1}{2}E_r\left[\frac{\gamma-v_L}{\sigma_L}\right].\quad(6)$$

III. Estimating the Q- factor by means of the error function

The argument in the error function from Eq. 6 is the square root of the signal divided by the square root of the signal level (see Eq. 11). This is equivalent to the optical signal-to-noise ratio and could be written as follows:

$$P[\varepsilon]=\frac{1}{2}E_r[SNR_{OH}]+\frac{1}{2}E_r[SNR_{OL}],\quad(7)$$

where SNR_{OH} and SNR_{OL} are optical signal-to-noise ratios for the high and low level respectively.

Optimum initial level γ_{opt} is determined by this initial level which defines the least bit error probability. In addition when determining optimum initial level it is important if high signal levels are transmitted as low level [3,4]. This means that γ_{opt} is obtained when $SNR_{OH}=SNR_{OL}$ which results in the following representation of Q-factor:

$$Q \equiv \frac{v_H - \gamma_{opt}}{\sigma_H} = \frac{\gamma_{opt} - v_L}{\sigma_L}. \quad (8)$$

If Eq. 8 is substituted in Eq. 6 taking in consideration that $\gamma = \gamma_{opt}$

$$P[\varepsilon] = \frac{1}{2} E_r[Q] + \frac{1}{2} E_r[Q] = E_r[Q]. \quad (9)$$

By taking the value of γ_{opt} from Eq. 8 we obtain:

$$\gamma_{opt} = \frac{v_H \sigma_L + v_L \sigma_H}{\sigma_L + \sigma_H}. \quad (10)$$

Considering Eq. 10 from Eq. 8 we obatin:

$$Q = \frac{v_H - v_L}{\sigma_L + \sigma_H}. \quad (11)$$

If both sides of Eq. 11 are multiplied by resistance or impedance then the expression will be converted into equivalent expressions which contain current or optical power:

$$Q = \frac{i_H - i_L}{\sigma_L + \sigma_H} = \frac{P_{OH} - P_{OL}}{\sigma_L + \sigma_H}. \quad (12)$$

Finally, by substituting Eq. 11 in Eq. 9 the outcome will be:

$$P[\varepsilon] = E_r[Q] = E_r\left[\frac{v_H - v_L}{\sigma_L + \sigma_H}\right]. \quad (13)$$

On the other hand *BER* could be logically represented as a function of Q-factor and by using error function.

If the necessary *BER* ratio is assigned, it is possible to obtain the corresponding value for the Q-factor by using the following expression:

$$BER(Q) = \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}. \quad (14)$$

For example, for *BER* = $1,0^{-11}$ it is necessary to ensure a value of *Q = 16,53*; for *BER* = $4,15.10^{-6}$ is obtained *Q = 13,00*.

IV. Results

The FEC combined with the use of appropriate code will strongly raise the quality of performance of the communication line and, in particular, allows keeping the same value of *BER* at lower value of the *SNR* ratio, i.e longer regeneration section.

The error correction device is usually a basic element of the modern fiber optic data transmission systems [7,8,12]. There are various methods of implementation of FEC-codecs. For example, the application of network standards Ethernet or Gigabit Ethernet in optical interface features increased bit rate by 25% and the concerned encoding circuits are referred to as 4B/5B and 8B/10B. The 10 Gigabit Ethernet standard uses two types of encoding: 64B/66B for transmission along single mode fiber and 8B/10B for transmission along multimode fiber.

In optical communication lines which contain linear optical amplifiers, the codecs based on Reed-Solomon block codes of RS(n,k) type with *s*-bit symbols and in particular RS(255,251), RS(255,239) and RS(255,223) codes with single-byte (*s = 8*) symbols are widely distributed [3,4].

In communication lines, the so called “out-of-band” standard FEC ITU G.975 is the most widely used code which is based on Reed-Solomon code RS(255,239). FEC increases the bit rate from *9,95* to *10,66 Gbps* and allows decreasing of *BER* from 10^{-5} to 10^{-15} and improving the *SNR* with *6dB.*

It is a conventional practice to measure the efficiency of FEC correction according to *SNR* change; in other words how much does the error correction method allow to reduce *SNR* on the receiving side keeping the previous value of *BER*.

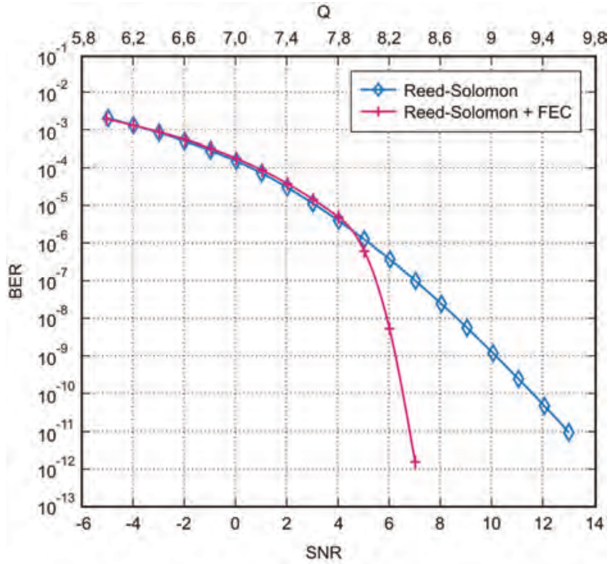

Fig. 4. *BER* parameters for BPSK modulation and addition of FEC to Reed-Solomon code

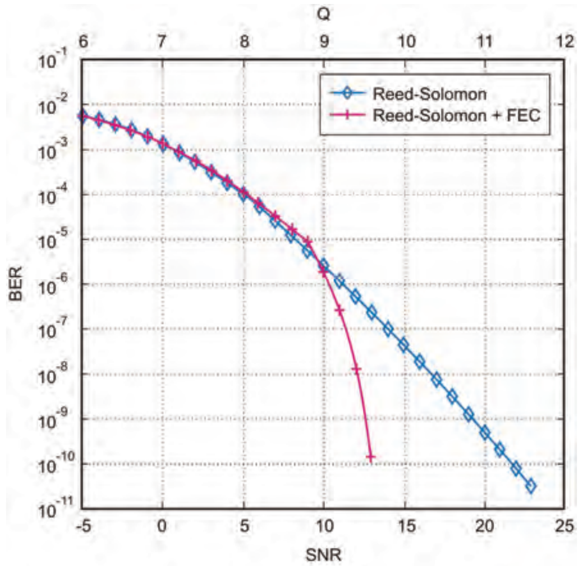

Fig. 5. *BER* parameters for QPSK modulation and addition of FEC to Reed-Solomon code

In Figs. 4 and 5 are presented the results from digital data transfer using Reed-Solomon encoding. It is evident that information transfer is improved when using FEC combined with the Reed-Solomon code while using QPSK and BPSK modulations.

Obtained results indicate that by selection of appropriate Q-factor line's length can be extended, keeping all previous parameters such as the input power, amplification coefficient and amplifiers' noise coefficient and the distance between two contiguous amplifiers.

V. CONCLUSION

The Q-factor estimated by Eqs. 8 and 11 represents optical signal-to-noise ratio for binary optical communication systems. In this way it incorporates individual *SNR* ratios related to high and low levels into a complete system of *SNR* ratios.

The Q-factor which is represented by Eq. 11 facilitates both measurement of *SNR* ratio and the theoretical calculation of *BER* in presence of noise in the system.

It is evident that, *BER* can be improved either by increasing the difference between high and low levels in the nominator or by reducing the noise in the denominator of the Q-factor equation.

In this way, using the Q-factor, it is possible to perform a simplified analysis of the communication system efficiency.

By using appropriate code and error correction it is possible to increase the digital data transfer efficiency along optical communication channels. This efficiency is determined by the margin of reduced *SNR* ratio while keeping the previous value of *BER*.

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