Sliding Hyperplane Design for Linear Systems with Unmatched External Disturbance Vector

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***Abstract* – A class of linear systems subjected to an unmatched external disturbance is considered within the framework of sliding mode control (SMC) theory. The paper offers a sliding hyperplane design method to minimize the effects of the unmatched disturbance upon the SM dynamics. The optimization criterion is minimization of the steady state vector norm. The suggested approach has been demonstrated on a numerical example.**

***Keywords* –Sliding mode control, Matching conditions, Unmatched disturbances, Hyperplane design.**

I. Introduction

The most prestigious property of the variable structure control systems (VSCSs) in sliding mode (SM) [1] is their invariance to parameter and exogenous disturbances, under so called matching conditions [2]. Physical meaning of these conditions is that disturbance acts through the control channel, i.e. the control is able to change coincidentally with the disturbance. Although there are many examples of systems where this requirement is fulfilled, there are still systems that structurally do not meet these conditions. While VSCSs in SM are completely insensitive to the matched uncertainties, on the other hand the SM dynamics, which is prescribed by the choice of the sliding manifold, is vulnerable to the unmatched uncertainties. In some cases sliding motion along certain manifolds may result in severe dynamics deterioration under action of unmatched uncertainties.

Several approaches are present in dealing with unmatched uncertainties, within the context of SM control (SMC): a continuous nonlinear control strategy [3], dynamical approach by means of pseudo-control inputs introduction into the reduced order system [4]-[6], a new invariance condition [7] in terms of linear matrix inequalities (LMI). All mentioned methods consider unmatched uncertainties having parametric nature, which diminish as system states approach origin. Therefore asymptotic stability can be ensured.

When it comes to the unmatched uncertainties that besides parametric contain external disturbances as well, asymptotic stability can not be attained. Although it is possible to establish a SM in such systems, the unmatched part of an external disturbance has impact on SM dynamics, forcing the system trajectory not to converge to the origin but to wander in its neighborhood along the sliding manifold. One way of addressing this problem may be to construct a sliding manifold that reduces system sensitivity in SM upon unmatched disturbances. A sliding manifold design that minimizes equivalent perturbation is suggested in [8], whereas another sliding manifold selection is proposed in [9], using the invariant ellipsoid method.

As in [8] and [9], this paper also searches for an adequate sliding hyperplane selection in a class of linear systems with scalar control and bounded unmatched external disturbance vector, which minimizes in some sense the impact of the disturbance onto the SM motion. The chosen optimization criterion here is minimization of the steady state vector norm. A systematic procedure for the sliding hyperplane design is developed, guaranteeing minimal static error. The proposed design method has been investigated on a numerical example.

II. SM in Case of Unmatched Disturbances

Consider a class of linear systems with scalar control that can be represented by the following state space model

$$\dot{\mathbf{x}}(t) = \mathbf{A}x(t) + \mathbf{b}u(t) + \mathbf{d}(t)\,, \tag{1}$$

where $\mathbf{x} \in \Re^{n\times 1}$ is the state vector, $u \in \Re$ is the control signal, $\mathbf{A} \in \Re^{n\times n}$ is the system matrix, $\mathbf{b} \in \Re^{n\times 1}$ is the control input vector and $\mathbf{d} \in \Re^{n\times 1}$ is a bounded external disturbance vector, $\|\mathbf{d}(t)\| \le M < \infty$, $\forall t > 0$. It is assumed that the pair $(\mathbf{A},\mathbf{b})$ is controllable and the matching condition [2]

$$\mathbf{d}(t) = \mathbf{b}\tilde{d}(t)\,, \tag{2}$$

is not fulfilled. This means that there does not exist a function $\tilde{d}(t)$ satisfying (2). Let a sliding hyperplane $\sigma(\mathbf{x}) = 0$ be defined by the switching function

$$\sigma(\mathbf{x}) = \mathbf{c}\mathbf{x}, \quad \mathbf{c} = [c_1, c_2, \ldots, c_n]\,, \tag{3}$$

which includes the state space origin as the equilibrium point.

SM reaching and existence condition $\sigma\dot{\sigma} \le -\eta|\sigma|$, $\eta > 0$ can be attained by an appropriate discontinuous control

$$u = -(\mathbf{cb})^{-1}(\mathbf{cAx} + \alpha\,\mathrm{sgn}\,\sigma) \tag{4}$$

if the switching gain overcomes disturbance $\alpha > \|\mathbf{c}\|M$.

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Hence, SM along $\sigma(\mathbf{x})=0$ will be attained in a finite time. System dynamics in ideal SM is obtained by transformation of (1) into the regular form by coordinate change $\mathbf{x}=\mathbf{P}_1\bar{\mathbf{x}}$, where the transformation matrix is given by [1]

$$\mathbf{P}_1=\mathbf{M}_c\begin{bmatrix} a_2 & a_3 & \ldots & a_n & 1 \\ a_3 & a_4 & \ldots & 1 & \\ \vdots & \vdots & \ddots & & \\ a_n & 1 & & \mathbf{0} & \\ 1 & & & & \end{bmatrix}. \tag{5}$$

$\mathbf{M}_c$ is the controllability matrix and a_i, $i=1,\ldots,n$ are the coefficients of the characteristic polynomial $\det(s\mathbf{I}-\mathbf{A})=s^n+a_ns^{n-1}+\ldots+a_2s+a_1$, where s is a complex variable. The resulting regular form is obtained as

$$\begin{aligned}
&\dot{\bar{\mathbf{x}}}(t)=\bar{\mathbf{A}}\bar{\mathbf{x}}(t)+\bar{\mathbf{b}}u(t)+\bar{\mathbf{d}}(t),\ \bar{\mathbf{x}}=\begin{bmatrix}\bar{\mathbf{x}}_r & \bar{x}_n\end{bmatrix}^{\mathrm{T}},\\
&\bar{\mathbf{d}}=\mathbf{P}_1^{-1}\mathbf{d}=\begin{bmatrix}\bar{\mathbf{d}}_r\\ \bar{d}_n\end{bmatrix},\ \bar{\mathbf{A}}=\mathbf{P}_1^{-1}\mathbf{A}\mathbf{P}_1=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{a}_{12}\\ \mathbf{a}_{21} & a_{22}\end{bmatrix},\ \bar{\mathbf{b}}=\mathbf{P}_1^{-1}\mathbf{b}=\begin{bmatrix}\mathbf{0}\\ 1\end{bmatrix}\\
&\bar{\mathbf{x}}_r=[\bar{x}_1\ \ \bar{x}_2\ \ \ldots\ \ \bar{x}_{n-1}]^{\mathrm{T}},\ \bar{\mathbf{d}}_r=[\bar{d}_1\ \ \bar{d}_2\ \ \ldots\ \ \bar{d}_{n-1}]^{\mathrm{T}},\\
&\mathbf{A}_{11}=\begin{bmatrix}0 & 1 & 0 & \ldots & 0\\ 0 & 0 & 1 & \ldots & 0\\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \ldots & 1\\ 0 & 0 & 0 & \ldots & 0\end{bmatrix}_{n-1\times n-1},\ \mathbf{a}_{12}=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix},\\
&\mathbf{a}_{21}=\begin{bmatrix}-a_1 & -a_2 & \ldots & -a_{n-2} & -a_{n-1}\end{bmatrix},\ a_{22}=-a_n.
\end{aligned} \tag{6}$$

The switching function may be rewritten using the regular form as $\sigma=\mathbf{c}\mathbf{x}=\mathbf{c}\mathbf{P}_1\bar{\mathbf{x}}=\bar{\mathbf{c}}\bar{\mathbf{x}},\ \bar{\mathbf{c}}=\mathbf{c}\mathbf{P}_1,\ \bar{\mathbf{c}}=\begin{bmatrix}\bar{\mathbf{c}}_0 & \vdots & 1\end{bmatrix}$, with vector $\bar{\mathbf{c}}_0=\begin{bmatrix}\bar{c}_1 & \bar{c}_2 & \ldots & \bar{c}_{n-1}\end{bmatrix}$. In SM $\sigma(\mathbf{x})=\bar{\mathbf{c}}_0\bar{\mathbf{x}}_r+\bar{x}_n=0$, which gives $\bar{x}_n=-\bar{\mathbf{c}}_0\bar{\mathbf{x}}_r$, meaning that the SM dynamics is described by a reduced order system

$$\begin{aligned}\dot{\bar{\mathbf{x}}}_r&=(\mathbf{A}_{11}-\mathbf{a}_{12}\bar{\mathbf{c}}_0)\bar{\mathbf{x}}_r+\bar{\mathbf{d}}_r=\mathbf{A}_r\bar{\mathbf{x}}_r+\bar{\mathbf{d}}_r,\\ \bar{x}_n&=-\bar{\mathbf{c}}_0\bar{\mathbf{x}}_r.\end{aligned} \tag{7}$$

For the controllable pair $(\mathbf{A},\mathbf{b})$, the pair $(\mathbf{A}_{11},\mathbf{a}_{12})$ is also controllable and by selection of vector $\bar{\mathbf{c}}_0$ the eigenvalues of the system matrix $\mathbf{A}_r=(\mathbf{A}_{11}-\mathbf{a}_{12}\bar{\mathbf{c}}_0)$ can be adjusted. Characteristic equation of the system in SM is given by

$$\det(s\mathbf{I}-\mathbf{A}_r)=s^{n-1}+\sum\nolimits_{i=1}^{n-1}\bar{c}_is^{i-1}=0. \tag{8}$$

In systems where matching condition (2) is fulfilled, $\bar{\mathbf{d}}_r=\mathbf{0}$ and the system is invariant to disturbances in SM, confirmed by (7). Its dynamics (8) exclusively depends on the sliding hyperplane parameters. System (7) becomes autonomous and the origin is equilibrium point. In an unmatched case, $\bar{\mathbf{d}}_r\neq\mathbf{0}$ so the SM dynamics (7) is affected by a disturbance. A new equilibrium point on the sliding hyperplane is formed depending on the disturbances, since the SM existence conditions are satisfied by the control signal. Steady state of the system (7) can be easily evaluated using the following relations

$$\begin{aligned}\bar{\mathbf{x}}_r(\infty)&=\lim_{s\to 0}\{s(s\mathbf{I}-\mathbf{A}_r)^{-1}[\bar{\mathbf{x}}_r(0)+\bar{\mathbf{d}}_r(s)]\},\\ \bar{x}_n(\infty)&=-\bar{\mathbf{c}}_0\bar{\mathbf{x}}_r(\infty),\end{aligned} \tag{9}$$

where $\bullet(\infty)=\lim_{t\to\infty}\bullet(t)$. Since the sliding hyperplane parameters ensures stable eigenvalues of $\mathbf{A}_r$, the system motion caused only by initial condition $\bar{\mathbf{x}}_r(0)$ will converge into the origin. Therefore, (9) may be expressed as

$$\begin{aligned}&\bar{\mathbf{x}}_r(\infty)=\Phi_{r0}(\bar{\mathbf{c}}_0)\bar{\mathbf{d}}_r(\infty),\\ &\bar{x}_n(\infty)=-\bar{\mathbf{c}}_0\bar{\mathbf{x}}_r(\infty),\\ &\Phi_{r0}(\bar{\mathbf{c}}_0)=\lim_{s\to 0}(s\mathbf{I}-\mathbf{A}_r)^{-1}=\begin{bmatrix}\frac{\bar{c}_2}{\bar{c}_1} & \frac{\bar{c}_3}{\bar{c}_1} & \ldots & \frac{\bar{c}_{n-1}}{\bar{c}_1} & \frac{1}{\bar{c}_1}\\ -1 & 0 & \ldots & 0 & 0\\ 0 & -1 & \ldots & 0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ 0 & 0 & \ldots & -1 & 0\end{bmatrix}\end{aligned} \tag{10}$$

Steady state (10) will exist if $\bar{\mathbf{d}}_r(\infty)$ exists. Based on (10), using (6) and (7), the transformed steady state can be calculated as

$$\bar{\mathbf{x}}(\infty)=[a(\bar{\mathbf{d}}_r(\infty),\bar{\mathbf{c}}_0)\ \vdots\ -\bar{\mathbf{d}}_r(\infty)]^{\mathrm{T}}, \tag{11}$$

$$a(\bar{\mathbf{d}}_r(\infty),\bar{\mathbf{c}}_0)=[\bar{d}_{n-1}(\infty)+\sum_{i=1}^{n-2}\bar{d}_i(\infty)\bar{c}_{i+1}]/\bar{c}_1, \tag{12}$$

showing that it depends on the disturbance steady state and the sliding hyperplane coefficients. The only way to intentionally affect the steady state error is by means of sliding hyperplane selection. Hence, apart from defining the SM dynamics, hyperplane design also has impact on the system sensitivity to unmatched disturbances.

Original steady state is obtained using coordinate transformation $\mathbf{x}(\infty)=\mathbf{P}_1\bar{\mathbf{x}}(\infty)$. If $\mathbf{P}_1$ is expressed in general form

$$\mathbf{P}_1=\begin{bmatrix}p_{11} & \ldots & p_{1n}\\ \vdots & & \\ p_{n1} & \ldots & p_{nn}\end{bmatrix}, \tag{13}$$

original steady state is obtained as

$$\mathbf{x}(\infty)=\begin{bmatrix}a(\bar{\mathbf{d}}_r(\infty),\bar{\mathbf{c}}_0)p_{11}-\sum_{i=1}^{n-1}\bar{d}_i(\infty)p_{1,(i+1)}\\ a(\bar{\mathbf{d}}_r(\infty),\bar{\mathbf{c}}_0)p_{21}-\sum_{i=1}^{n-1}\bar{d}_i(\infty)p_{2,(i+1)}\\ \vdots\\ a(\bar{\mathbf{d}}_r(\infty),\bar{\mathbf{c}}_0)p_{n1}-\sum_{i=1}^{n-1}\bar{d}_i(\infty)p_{n,(i+1)}\end{bmatrix}. \tag{14}$$

III. Sliding Hyperplane Design

Since SM dynamics is sensitive to unmatched disturbances, a sliding hyperplane design procedure will be derived that provides minimization of the steady state error for the considered class of linear systems with unmatched external disturbance. Euclid vector norm $\|\mathbf{x}\| = (\mathbf{x}^{\mathrm{T}}\mathbf{x})^{1/2}$ may serve as a measure of the steady state distance from the origin. Therefore, the norm of the steady state (14) is obtained as

$$\|\mathbf{x}(\infty)\|^2 = \sum_{k=1}^{n}(a(\overline{\mathbf{d}}_r(\infty),\overline{\mathbf{c}}_0)p_{k1} - \sum_{i=1}^{n-1}\overline{d}_i(\infty)p_{k,(i+1)})^2 . \quad (15)$$

The only adjustable parameter $a(\overline{\mathbf{d}}_r(\infty),\overline{\mathbf{c}}_0)$ will be determined according to the condition that the norm (15) has its minimum for $a = a_m$, i.e.

$$\frac{\partial\|\mathbf{x}(\infty)\|^2}{\partial a}\Big|_{a=a_m} = 0 . \quad (16)$$

By differentiation of (15) with respect to *a*, (16) transforms into the following equation

$$a_m\sum_{k=1}^{n}p_{k1}^2 - \sum_{k=1}^{n}p_{k1}\sum_{i=1}^{n-1}\overline{d}_i(\infty)p_{k,(i+1)} = 0 , \quad (17)$$

whose solution is

$$a_m = \left(\sum_{k=1}^{n}p_{k1}^2\right)^{-1}\sum_{k=1}^{n}p_{k1}\sum_{i=1}^{n-1}\overline{d}_i(\infty)p_{k,(i+1)} . \quad (18)$$

Now, vector $\overline{\mathbf{c}}_0$ should be calculated form (12) using the obtained solution a_m. (12) is a single equation with *n*-1 unknowns that cannot be uniquely solved. Solution can be reached only by reducing the number of degrees of freedom to one. By setting $\overline{\mathbf{c}}_0$, and consequently the vector $\mathbf{c}$, $n-1$ eigenvalues of the SM dynamics (8) can be independently chosen. Reduction of degrees of freedom can be done by imposing certain conditions to the eigenvalues. For example, if a single multiple eigenvalue $s = -s_p$, $s_p > 0$ of order $n-1$ is required, number of degrees of freedom is one. The desired SM characteristic equation can be expressed as

$$(s+s_p)^{n-1} = \sum_{i=0}^{n-1}\frac{(n-1)!}{i!(n-1-i)!}s^{n-1-i}s_p^i = 0 . \quad (19)$$

Comparison of (19) with (8) gives the needed sliding hyperplane coefficients as functions of s_p

$$\overline{c}_i = \frac{(n-1)!}{(n-i)!(i-1)!}s_p^{n-i},\ i = 1,\Lambda\ ,n-1,\ \overline{c}_1 = s_p^{n-1} . \quad (20)$$

Using (20), the equation (12) becomes

$$a_m s_p^{n-1} - \sum_{i=1}^{n-2}\frac{(n-1)!}{(n-1-i)!(i-1)!}\overline{d}_i(\infty)s_p^{n-1-i} - \overline{d}_{n-1}(\infty) = 0 . \quad (21)$$

Recall that a_m, which minimize the norm (15), has been already determined using (18), hence (21) is a polynomial equation of order $n-1$ with respect to unknown s_p. This equation is now solvable and has $n-1$ solutions. The number of positive real solutions indicates the number of possible different stabile hyperplanes that minimizes the norm (15). Naturally, the largest positive real solution should be selected because it guaranties the fastest SM dynamics. For the selected s_p, the components of vector $\overline{\mathbf{c}}_0$ are calculated according to (20). Finally, the original vector $\mathbf{c}$ of the sliding hyperplane can be determined using the inverse transformation.

$$\mathbf{c} = [\overline{\mathbf{c}}_0 \ \vdots\ 1]\mathbf{P}_1^{-1} . \quad (22)$$

IV. A Numerical Example

Consider a stable controllable third order linear system whose model (1) is represented by

$$\mathbf{A} = \begin{bmatrix}-1 & 0 & 1\\ 1 & -2 & 1\\ 0 & 0 & -3\end{bmatrix},\ \mathbf{b} = \begin{bmatrix}0\\ 1\\ -1\end{bmatrix},\ \mathbf{d}(t) = \begin{bmatrix}2\cdot h(t-6)\\ -1\cdot h(t-5)\\ 0\end{bmatrix} . \quad (23)$$

Note that the disturbance vector is unmatched since condition (2) can never be reached. The transformation matrix and the regular form are defined by

$$\mathbf{P}_1 = \begin{bmatrix}-2 & -1 & 0\\ 1 & 3 & 1\\ -2 & -3 & -1\end{bmatrix},\ \overline{\mathbf{A}} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix},\ \overline{\mathbf{b}} = \begin{bmatrix}0\\ 0\\ 1\end{bmatrix},$$
$$\overline{\mathbf{d}}(t) = \begin{bmatrix}\overline{\mathbf{d}}_r(t)\\ \overline{d}_n\end{bmatrix} = \begin{bmatrix}1\cdot h(t-5)\\ -2\cdot h(t-6) - 2\cdot h(t-5)\\ 6\cdot h(t-6) + 4\cdot h(t-5)\end{bmatrix} . \quad (24)$$

Relation (18) gives $a_m = -0.11$ that according to (14) ensures the steady state $\mathbf{x}(\infty) = [1.22 \quad 0.89 \quad -0.78]^{\mathrm{T}}$, i.e. the norm $\|\mathbf{x}(\infty)\| = 1.7$. Two eigenvalues, defined with $s_{p1} = -19.8$ and $s_{p2} = 1.8$, are obtained by solving (21) for $a_m = -0.11$. Positive real value $s_p = 1.8$ is selected in sliding plane determination, which according to (20) and (22) produces $\mathbf{c} = [-0.63 \quad -0.03 \quad -1.03]$.

A very simple control law (4) is employed in the realization of SM. Switching gain $\alpha = 5$ provides reaching and existing conditions in the presence of the disturbance. The control signal is depicted in Fig. 1a. It can be noticed that the high-frequency switching component is dominant, which would inevitably induce unwanted chattering in a real system. However, the reason for the application of such control algorithm is that the attained SM as much as possible resembles an ideal SM, which is the assumption of the conducted analysis. Fig. 1b shows the switching function, whose annulment confirms occurrence and existence of SM, even after the action of the unmatched disturbance.

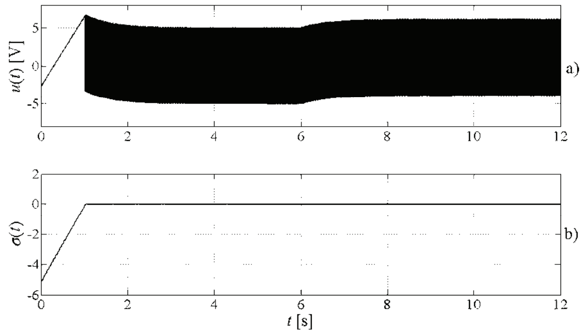

Fig. 1. a) Control signal; b) Switching function.

System trajectory in the phase space is depicted in Fig. 2. From the initial state $\mathbf{x}(0)=[5 \quad -3 \quad 2]^{\mathrm{T}}$ system trajectory reaches and slides along the plane into the origin. After the action of the unmatched disturbance, phase point is thrown out of the origin along the plane into a new equilibrium point. Time response of the state coordinates for this case is given in Fig. 3, by solid lines. Data analysis has confirmed the analytically predicted steady state and its norm.

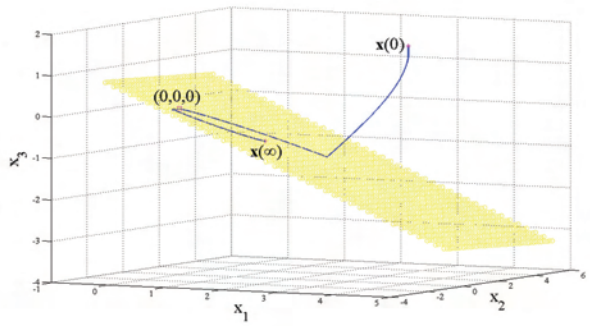

Fig. 2. System trajectory and the sliding plane

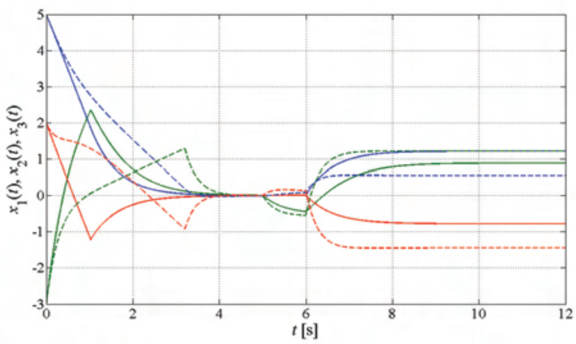

Fig. 3. State coordinates (solid lines for s_p=1.5, dashed lines for s_p=3).

In order to prove that the obtained norm is minimal, some other slope of the sliding plane is selected. Namely, the chosen new sliding plane ensures faster SM dynamics, given by $s_p=3$, which results in $\mathbf{c}=[-3 \quad -1 \quad -2]$. Recalculated steady state and its norm are respectively $\mathbf{x}(\infty)=[0.55 \quad 1.22 \quad -1.44]^{\mathrm{T}}$ and $\|\mathbf{x}(\infty)\|=1.97$, which are confirmed by the state coordinate response in Fig. 3, denoted by the dashed lines. The obtained norm is larger then the previous one. Unexpectedly, the faster SM dynamics of prescribed by the new plane, which better rejects matched disturbances, produces worse performance under action of the unmatched disturbances.

V. Conclusion

The paper studies the influence of unmatched disturbances upon SM motion and offers a sliding hyperplane design method that minimizes the system sensitivity against such disturbances, by minimization of the steady state vector norm. It is demonstrated that for a class of linear systems this optimization task can be explicitly solved by renouncing of a certain number of degrees of freedom in SM eigenvalues allocation.

The developed systematic sliding hyperplane design procedure has been demonstrated on an illustrative numerical example. The simulation results confirm the analytically predicted behavior, and the calculated minimum of the steady state norm is achieved. It is also shown that certain SM features in systems satisfying the matching conditions are not necessarily present when unmatched disturbances arise.

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