Hybrid ARQ Schemes Using Diagonally Interleaved Turbo Product Codes

Nikolay Kostov[1], Slava Yordanova[2], Ginka Marinova[3]

Abstract: **In this paper new hybrid automatic repeat request (HARQ) schemes using interleaved two-dimensional single parity check turbo product codes are proposed. Throughput and bit error rate of the proposed schemes are studied through simulations and are compared to the conventional forward error correction schemes. It is shown that using these HARQ schemes a significant energy gain over the corresponding forward error correction schemes is obtained.**

Keywords: **Hybrid automatic repeat request, interleaving, turbo product codes, simulation.**

I. INTRODUCTION

Automatic repeat request (ARQ) schemes are used to achieve near error-free transmission and to reduce link margins when the channel characteristics are poorly predictable [1]. A hybrid ARQ (HARQ) scheme uses a forward error correction (FEC) code in conjunction with a retransmission scheme (ARQ). Typically a cyclic redundancy check (CRC) code is used for frame error detection and this is an example of the so-called two-code approach since two different error control schemes are used for HARQ purpose [2]. Another HARQ method is the so-called one-code approach since only one error control code is used for both error correction and error detection [3]. A class of interleaved single parity check turbo product codes was introduced in [4], [5]. Typically a pseudorandom interleaver is used with the examined turbo product codes.

In this paper, new HARQ schemes based on interleaved two-dimensional single parity check turbo product codes (2D-ITPC) are proposed. The above mentioned one-code approach is used along with a diagonal interleaver. As it will be shown, these HARQ schemes produce a significant energy gain over the corresponding forward error correction schemes.

II. SYSTEM MODEL

First consider the encoding/decoding processes of the conventional interleaved 2D-ITPC [4], [5]. In case of parallel concatenation, the data frame to be transmitted is first encoded by a single 2D single parity check product code.

Then it is interleaved end re-encoded by the same 2D single parity check product code. In case of serial concatenation, the data frame to be transmitted is first encoded by a 2D $(n-1, n-2)$ single parity check product code. Further the data and parity bits are interleaved end re-encoded using a 2D $(n, n-1)$ 2D single parity check product code. Thus, the code rate of this serially concatenated 2D-ITPC is

$$R_c = \left[\frac{n-2}{n}\right]^2 \tag{1}$$

while the data block length and the interleaver size are $(n-2)^2$ and $(n-1)^2$, respectively. In both cases the overall interleaved product code is composed of the original data bits and all parity bits from the single parity check codes. There is no noticeable performance difference between these serial and parallel concatenated 2D-ITPC. Therefore, in this paper, only serially concatenated shames will be examined.

In order to improve the performance of the considered HARQ schemes a diagonal interleaver is proposed. The interleaver is of size N ($N = m^2$) and has the following property: any two or more bits in the initial non interleaved block, located in the same raw or column, do not lie in a raw or column in the interleaved block.

Let us now consider binary phase shift keying (BPSK) transmission of the coded data via an additive white Gaussian noise (AWGN) channel. For an AWGN channel model the input/output relationship can be expressed as

$$r_i = \sqrt{E_s}\,(2b_i - 1) + n_i \ , \tag{2}$$

where r_i is the received symbol, E_s is the energy per code symbol, $(2b_i - 1)$ is the binary phase shift keying modulated code symbol and n_i is a zero-mean Gaussian variable with variance $\sigma^2 = N_0/2$.

The decoding process is the following. The original (non interleaved) noisy frame is first decoded using a soft-input/soft-output (SISO) decoding method [6] for a single decoding cycle. Then the interleaved data frame is also SISO decoded for a single decoding cycle and so on. The two constituent decoders exchange the so-called extrinsic information (the error correction term gained from the decoding) at each full iteration and the decoding iterations are executed until a predetermined stopping criteria is satisfied.

A simplified diagram of the turbo decoder is shown in Fig.1.

[1] Nikolay T. Kostov is with the Technical University of Varna, Department of Radio engineering, Studentska Street 1, Varna 9010, Bulgaria, E-mail: n_kostov@mail.bg

[2] Slava M. Yordanova is with the Technical University of Varna, Department of Computer Science and Technologies, Studentska Street 1, Varna 9010, Bulgaria, E-mail:slava_y@abv.bg

[3] Ginka K. Marinova is with the Technical University of Varna, Department of Computer Science and Technologies, Studentska Street 1, Varna 9010, Bulgaria, E-mail: gin_kaleva@abv.bg

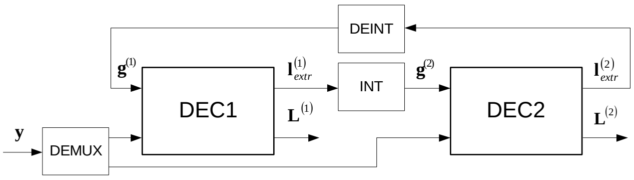

Fig.1. Turbo decoder block diagram.

The log-likelihood ratio (LLR) L_i at the output of the SISO decoder can be represented as [5, 6]

$$L_i = y_i + g_i + l_i \,, \tag{3}$$

where y_i is the weighted channel observation, g_i is the *a priori* information and l_i is the so-called extrinsic information gained by the current stage of decoding. Both decoders (DEC1 and DEC2) in Fig.1 use the algorithm described in [6] form an estimate of the LLR of each bit encoded by the single parity check equations. The extrinsic information l_i of the *i*th bit associated with a single parity check equation is computed as [6]

$$l_i = \; (-1) \cdot (\min_{\substack{j=1,\dots,n \\ j \neq i}} \; | l_j |) \cdot \prod_{\substack{j=1 \\ j \neq i}}^{n} \operatorname{sign}\left(l_j \right). \tag{4}$$

According to (4) the magnitude of the extrinsic information for a particular code element is equal to the minimum magnitude of all of the other parity elements. The sign of the extrinsic information for a particular code element is equal to the sign of the element itself if the parity of the overall equation is satisfied, and opposite to the sign of the element if the overall parity fails. The final soft decision Λ_i of the *i*th data bit is given by

$$\Lambda_i = L_i^{ch} + L_i^1 + L_i^2 \,, \tag{5}$$

where L_i^{ch} is the noisy channel observation for the *i*th data bit and L_i^1, L_i^2 are the extrinsic information terms from the first and second two-dimensional decoder, respectively. The corresponding hard decision a_i of the *i*th data bit is

$$a_i = \begin{cases} 1, \; if \;\; \Lambda_i > 0 \\ 0, \; if \;\; \Lambda_\mathrm{i} < 0. \end{cases} \tag{6}$$

Now consider the retransmission algorithm associated with the proposed HARQ schemes. The algorithm is as follows:

1. Apply SISO iterative decoding (with a predetermined maximum number of iterations) on the received encoded frame, checking all parity equations of the component single parity check codes after each full iteration. If all parity equations are satisfied, (e.g., the stopping criterion is fulfilled) go to 3. Otherwise proceed to 2.
2. Request a retransmission of the decoded frame by sending a negative acknowledgement to the transmitter via the feedback channel. If the maximum number of retransmissions is reached, proceed to 3.
3. Output hard quantized data.

It can be observed that with this retransmission algorithm the frame error detection (and consequently the retransmission request) is based on the checks of all parity equations of the overall interleaved product code. Because of the interleaving, a significant reduction of undetected erroneous frames can be expected for the considered error control scheme as compared to the conventional two-dimensional product codes. Consequently, an efficient ARQ mode operation could be implemented for this interleaved turbo product codes. As performance results show further improvement is possible if the conventional pseudorandom interleaver is replaced with the proposed diagonal interleaver.

III. THE SIMULATION CODE

MATLAB-based Monte Carlo simulations were executed in order to estimate the performance of the proposed HARQ schemes. The simulations are based on BPSK signaling over the AWGN channel with at most two retransmissions allowed and an error-free feedback channel. Soft equal gain combining of the retransmitted frames was applied in the receiver.

Below are given the maim functions in the simulation, e.g. **encode_msg.m** (performing 2D single parity check encoding of data), **diag_perm.m** (performing a diagonal permutation) and **dec_2D** (performing soft decoding of a 2D data block).

```
function  PC = encode_msg(A)

M = length(A);
m = sqrt(M);
n = m + 1;
A = reshape(A,m,m)';
PC = zeros(n);
PC (1:m,1:m) = A;
  for k = 1:m  % row
      PC (k,n) = mod(sum(A(k,:)),2);
  end

  for k = 1:n  % column
      PC (n,k) = mod(sum(PC (1:m,k)),2);
  end

function f = diag_perm(mm)
 m=sqrt(mm); a= 1:mm;
b = reshape(a,m,m);
b= b'; c = [b b];
d = zeros(m,m);
e = zeros(m,m);
f = zeros(m,m);
  for i = 1:m
    e = c(:,i:i+m-1);
    d(i,:) = diag(e)';
  end
 f(1,:) = d(1,:);
  for i = 1:m-1
     f(i+1,1:m-i) = d(i+1,i+1:m);
    f(i+1,m-i+1:m) = d(i+1,1:i);
   end
```

For a data block of 25 elements with one dimensional indexing given below

```
 1   2   3   4   5
 6   7   8   9  10
11  12  13  14  15
16  17  18  19  20
21  22  23  24  25
```

the function **diag_perm.m** produces

```
 1   7  13  19  25
 8  14  20  21   2
15  16  22   3   9
17  23   4  10  11
24   5   6  12  18
```

```
function  [Le_hor, Le_ver] = dec_2D(Ch_LLR)

CodeLLR = Ch_LLR;
      for k = 1:n  % decode rows
         C10 = CodeLLR (k,:);
         for j = 1:n
            C1 = C10;  C1(j) = [];
             Le_hor(k,j) = prod (sign (C1))*min(abs(C1));
         end
      end
      CodeLLRtmp = CodeLLR + Le_hor;
      for k = 1:n  %decode columns
         C10 =  CodeLLRtmp (:,k);
         for j = 1:n
            C1 = C10;  C1(j) = [];
            Le_ver(j,k) = prod (sign (C1))*min(abs(C1));
         end
      end
```

IV. PERFORMANCE RESULTS

MATLAB-based Monte Carlo simulations were executed in order to estimate the performance of the proposed HARQ schemes. The simulations are based on BPSK signaling over the AWGN channel with at most two retransmissions allowed and an error-free feedback channel. Soft equal gain combining of the retransmitted frames was applied in the receiver.

Two basic HARQ schemes with the above mentioned diagonal interleaver were studied: HARQ1 with a parent code rate $R_c \approx 3/4$ and HARQ2 with a parent code rate $R_c \approx 4/5$. In this paper the throughput is defined to be the effective code rate R_{eff} of the corresponding HARQ schemes. Obviously $R_{eff} < R_c$ and $R_{eff} = R_c$ only in case of no retransmissions.

Throughput versus signal-to-noise ratio (SNR) performance of HARQ1 and HARQ2 schemes is shown in Fig.1. The required effective SNR's for the HARQ schemes, necessary for achieving typical bit error rates (BER's), are shown in Table 1 along with performance of the parent diagonally interleaved turbo product codes (DITPC).

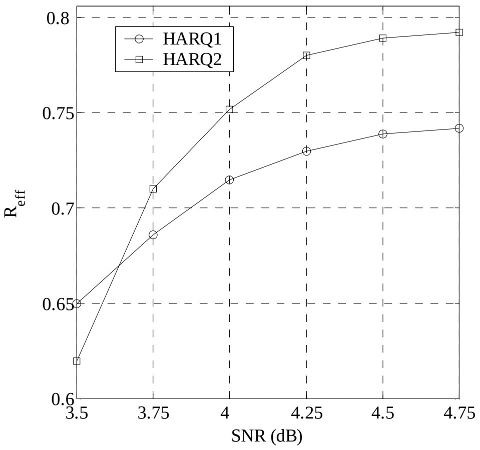

Fig.2. Throughput versus SNR performance of HARQ1 and HARQ2 schemes.

Table 1. Estimated HARQ schemes performance

	HARQ1	DITPC1 $R_c \approx 3/4$	HARQ2	DITPC2 $R_c \approx 4/5$
SNR$_{eff}$ (dB) for $\mathrm{BER} = 10^{-5}$	4.2	5.1	4.2	4.95
SNR$_{eff}$ (dB) for $\mathrm{BER} = 10^{-6}$	4.6	5.45	4.55	5.3

According to the data in Table 1, an energy gain of at least 0.75 dB is obtained with the considered HARQ schemes over the parent DITPC schemes. Further, an energy gain of approximately 0.25 dB is obtained when the considered diagonal interleaver is used instead of the conventional pseudorandom interleaver. At $\mathrm{BER} \le 10^{-5}$ the effective code rate R_{eff} of both HARQ schemes is close to code rate of the parent schemes (e.g., $R_{eff} \approx R_c$) because of the relatively small overall number of retransmissions.

V. CONCLUSION

New simple HARQ schemes based on interleaved two-dimensional single parity check product codes have been proposed in this paper. It is shown that energy gain of 0.75 dB or even higher is obtained over the parent forward error correcting schemes with practically no data rate reduction. Further, better performance results in terms of frame error rate and bit error rate were obtained when the conventional pseudorandom interleaver is replaced with the considered diagonal interleaver. Finally, it should be mentioned that for moderate-to-high SNR's the BER performance of the considered "one-code approach" HARQ schemes is dominated by the undetectable erroneous frames and no significant performance improvement can be expected with more than two retransmissions allowed.

REFERENCES

[1] S. Lin, D. Costello, M. Miller, “Automatic-repeat-request error control schemes”, IEEE Communications Magazine, No12, pp. 5-16, 1984.

[2] S. Lin, D. Costello, *Error Control Coding: Fundamentals and Applications,* Prentice-Hall Inc., N.J., 1983.

[3] H. Yamamoto, K. Itoh, “Viterbi decoding algorithm for convolutional codes with repeat request”, IEEE Transactions on Information Theory, No 5 pp. 540-547, 1980.

[4] D. Rankin, *Single parity check product codes and iterative decoding*, Ph.D. thesis, University of Canterbury, Christchurch, New Zealand, 2001.

[5] D. Rankin, T. Gulliver, D. Taylor, “Parallel and serial concatenated single parity check product codes”, EURASIP Journal of Applied Signal Processing, No 6, pp. 775-78, 2005.

[6] B. Sklar, ”A primer on turbo code concepts”, IEEE Communications Magazine 12, pp. 94-102, 1997.