Mathematical Model for Water Use and Transfer Between Hydro Power Plants

Mile Spirovski and Arsen Arsenov

Abstract – **In this paper optaining the basic principles for maintance continuity and amouth of movement, there are elaborated some methods for hydrograph transfer analysis throught accumulations and canals. Additional hardship creates the numerous restrictions who has to gratify the power plants like one complete unit. Like: minimal and maximal licenced expiration throught capturation constructions care for the capacity of the addictive pressure tube and drainage narrow passage, allowed velocity quickness of increase drain throught tubes, changes of the accomulations rates satisfying the given hydrograpys who with is define direct water take off from the analysed accumulation.**

Keywords – **Hydropower plant, hydrographs, equation of…**

I. INTRODUCTION

Model of a liquid product, or that use to be called river model, is made to be able to explain the strait between two hydropower plants. There are two ways of transfer of hydrographs: Transfer of hydrograph in accumulation or reservoir and transfer of hydrograph by canal. Transfer of hydrograph by accumulation is analyze with a target to be made suffocated of the point of the hydrograph that he is subject since he gets in the accumulation. For that analyze are needed incoming data's and that hydrograph of the entrance in the accumulation and characteristic of accumulation (volume) and plants for evacuation of water from the accumulation.

Transfer of hydrograph by canal is important for fortify influence of the canal over the spade of the hydrograph ant time of travel. For that target there is a need of incoming data's for incoming hydrograph in the canal characteristic canal itself.

II. TRANSFER OF HYDROGRAPHS BY CANALS

When the hydrograph that is given travels by canal his characteristic are changing like result of the resistance of the movement that is made by the canal and his volume characteristic. In this case absence of important income through length of the canal top flow generally is suffocated. This phenomenon is called diffusion (mutually mix of fluids) and it happens in natural canals. Time difference between top (spade) of hydrograph on the entrance in the canal and on the exit of it is measure for the time of travel of the hydrograph by canal.

Under west entrance hydrograph and canal, target of the travel of the hydrograph by the canal is to be deciding the shape of the exit hydrograph (chronological phenomenon of the movement of the exit of the canal). Typical view of transferable hydrograph is given on picture 1.

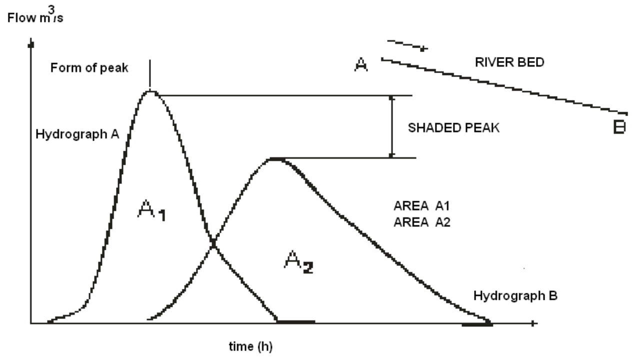

Fig. 1. Effect off transfer of hydrographs

III. SAINT VENAN - EQUATIONS

With a help of SAINT VENAN – equations, that are called and primary equations, it's describe stretch of given water wave in open canal. Water waves can be classified as: dynamical waves, gravitation waves, diffusion waves and kinematical waves in a relation how big is the number of element that are taken in the structure of the model that descries the movements of the wave. Dynamical wave, for example, it takes in consideration all the members in the structure of Saint Venan – equation for maintain of the content of movement. Under viewing of gravitation wave is underplay the effect of bow of the trough and the effect of rubbing (friction) that is develop between the water and the walls of the river. In this case the article $g\left(S_0 - S_f\right)$, the other articles of the equation for maintains of the quantity of movement are taken in consideration.

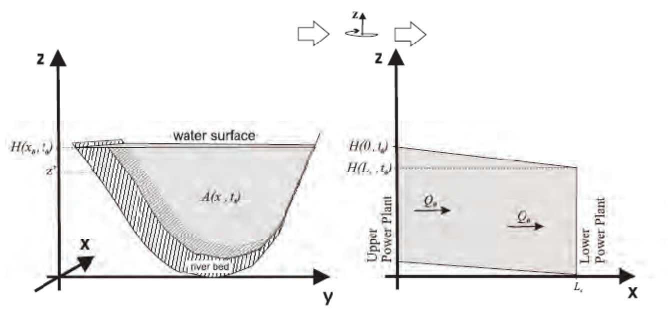

Fig. 2. Flow of water through the river

[1]Mile Spirovski Faculty of Technical Sciences, I.L.Ribar bb, 7000 Bitola, Macedonia, E-mail: mile.spirovski@uklo.edu.mk,and Arsen Arsenov FEIT Skopje, E-mail: aarsenov@feit.ukim.edu.mk

Kinematical wave, again, it takes in consideration only the effects from existing of the bow river trough and the effects of the rubbing that are develop between the water and the walls of the river trough. In general form, view of Saint Venan – equations is next:

- Equation for maintence of continuity:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=q \qquad (1)$$

As we can presumption that step distance is infinitely small. Farther the change of the volume can be shown over first assumption of the change of the section for pass $A(x_0,t_0)\delta x$ over the time δt:

- Equation for maintains of the quantity of movement or equation for maintains of impulse

$$\frac{\partial Q}{\partial t}+\frac{\partial\left(Q^2/A\right)}{\partial x}+gA\left(\frac{\partial y}{\partial x}-S_0\right)+gAS_f=0 \qquad (2)$$

Where x is lengthwise distance under the length of the canal in m, t is time in s, A is surface lengthways section of the trough m^2; y is level of the surface of the water in the canal in m, S_0 bow on the bottom of the canal (negative); q is lengthwise income of water (continual income of water under unit length of the canal – from side) in (m^3/s)/m; S_f is so called bow of rubbing that can be define with a help of Manning's equation:

$$S_f=\frac{n^2Q^2}{A^2R^{4/3}}, \qquad (3)$$

and g is earth acceleration, in ms^{-2}.

In addition, R presents hydraulic radius, Q is expiration in m^3/s; n is so called coefficient of roughness or Manning's coefficient.

Equations (1) and (2) can be meet and in the next form:

Equation for maitains of coninuity:

$$B\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=q \qquad (4)$$

Equation for maintains of the quantity of movment:

$$\frac{\partial v}{\partial t}+\alpha\cdot v\frac{\partial v}{\partial x}+g\frac{\partial h}{\partial x}-g\left(S_0-S_f\right)=0 \qquad (5)$$

Where B is width of the mirror-surface of the water in the canal in m, and α is a coefficient of distribute of speed by given transversely section. Then with h is shown the depth of the water in the canal in m, and the rest of the marks have already the same mining.

IV. Primary equations of continuity

Calculation methods connect with the transfer of hydrographs by canal and accumulations are based on principles for maintains of mass (equation for continuity) and maintains of the equational impulse in relation with the saving of the dynamic equality.

Equation of continuity it shows meaning of saving of the mass that means that the change of the accumulation is equal of the income of the water in her minus flowing.

$$I-O=\frac{dS}{dt} \qquad (6)$$

Where t is time in seconds, and S accumulation volume in m^3, volume contain between two sections of the canal. Then, I is income in m^3/s, he is constitute from incoming income and every one additional that enters in the canal between consider two sections, O is a flowing in m^3/s, in which is exit flow and every loss of water (for example infiltration over the bottom of the canal, deduction of the water in the canal for other needs and other types of deduction).

Equation for maintains of the quantity of movement, also, called dynamic equation, is actually same with the Second Newton low under which the change of the quantity of the movement is sum of all powers that act to the view volume. That is to say,

$$\frac{d(mv)}{dt}=\sum F \qquad (7)$$

Where with m is the mass in kg, v is speed in m/s, of movement of the water volume between two sections in the canal. The sum of the powers F in self considers the pressure, rubbing and gravitation conditions.

The transfer of mass includes in self solution that consider and the equation of continiuty and the equation of moments while are made surtin simplifaction.

V. Methods for transport of water by acumulations and canals

For assignment of the movment of the water canals and waterflow when there is a word for short time intervals (daily or shorter) are more methodes, and that:

- Straddle-Stagger
- Tatum
- Muskingum
- Modified Puls,
- Modified Puls in a fuction of income,
- Working R&D,
- SSARR Time-of-storage and
- Direct import of the coefficient of transport.

Part of these methods will be present in this paper.

- Modified Plus Method

This method uses the next form of equation of continuity:

$$\left(\frac{S_2}{\Delta t}+\frac{O_2}{2}\right)=\left(\frac{S_1}{\Delta t}+\frac{O_1}{2}\right)-O_1+\left(\frac{I_1}{\Delta t}+\frac{I_2}{2}\right) \qquad (8)$$

From where is:

$$\frac{I_1+I_2}{2}-\frac{O_1+O_2}{2}=\frac{S_2-S_1}{2} \qquad (9)$$

or

$$\underbrace{I_1 + I_2 + \left(\frac{2S_1}{\Delta t} - O_1 \right)}_{poznata\ strana} = \underbrace{\frac{2S_2}{\Delta t} - O_2}_{treba\ da\ se\ opredli} \quad (10)$$

or generaly,

$$\underbrace{I_n + I_{n+1} + \left(\frac{2S_n}{\Delta t} - O_n \right)}_{poznata\ strana} = \underbrace{\frac{2S_{n+1}}{\Delta t} - O_{n+1}}_{treba\ da\ se\ opredli} \quad (11)$$

Where is known dependence $O_{n+1} = f(S_{n+1})$ in the table or analytical form.

- WORKING R&D METHOD

This method uses nonlinear dependence of the volume from the flowing of the water, similarly like the method called Modified Plus Method. Also, this method allows use of accumulations with transversal section in a form of wedge, similarly like the method called Muskingum method. It shows that under linear dependencies volume in function of flowing from accumulation this method gives same result with Muskingum method. If it's work for transfer of hydrograph by accumulations that don't have a form of wedge (coefficient of Muskingum is $x = 0$), this method gives results same as they are with the Modified plus method. Working R&D method use volume of accumulation that is called "worked" volume, that is mention in the explanation of the next equations:

$$\frac{R_2}{\Delta t} = \frac{R_1}{\Delta t} + Q_h - \frac{D_1 - D_2}{2} \quad (12)$$

In adition are:

R_1- "worked" volume of end of a time interval;

R_2 -"worked" volume of the start of a time interval;

Q_h -everage income for time of one transfer interval;

D_1 -"worked" take over of the bigining of a time interval;

D_2 -"worked" take over of the end of a time interval, that is function of R_2 shown with dependence: worked volume in a function of flowing, $D_2 = f(R_2)$.

Flowing, again, is a function from "working" discharge and is define with a help of the next equation:

$$O_2 = D_2 - \frac{x}{1-x} \cdot (I_2 - I_1) \quad (13)$$

Where there,

x -coefficient of Muskingum, with out dimension;

O_2 -flowing of end of a interval;

I_2 -income of the end of a interval.

- METHOD OF COEFFICIENTS

All methods of coefficients flowings of the end of the canals conections are calculate like linear functions from flowings of water of the entrence in the canal, to wit:

$$O_n = C_1 \cdot I_n + C_2 \cdot I_{n-1} + C_3 \cdot I_{n-3} + C_4 \cdot I_{n-4} + ... \quad (14)$$

Where there are:

O_n -ordinate of hydrograph of flowing in time interval n

$I_n,\ I_{n-1},\ I_{n-2}...$ -ordinates of hydrograph of income in time intervals $n, n-1,\ n-2.....$ appositely

$C_1, C_2, C_3...$ -coefficients of movements, like coefficients of income.

- MUSKINGUM METHOD

Muskingum method can be understand like method of storing in canals elemental parts define like sum of padding in form of prisms and wedges. At the same time, connection of the exit padding with the entrance is linear. For defining of the coefficients $C_1, C_2, C_3...$, under this method are use recurrent equations.

This method ignors moments equations and is only firmly based by the equations for continuity. This method is especialy used when is worked for difuse waves. Under the transfer of given hydroraph his top is suficating like a result of difusion that comes with it that is caused by the efects of egsisting volume of water. In addition, is assumped that the volume of the water in the canal depend from the quantity of the water that enters in him and the quantity of water that exits from him and can be determine on the next modus:

$$Q = xI + (1-x)O \quad (15)$$

or

$$S = KQ \quad (16)$$

Where K is a time of travel in seconds between two sections of given canal, x is a coefficient without dimension, respectively load factor that value is between 0,0 and 0,5 with which is expressing of the enter canal and exit hydrograph from the canal by the volume of water contained in the canal. In addition, if $x = 0,5$ than the volume depend equally from the entrance and exit hydrograph.

$$I - O = \frac{dS}{\Delta t} \quad (17)$$

where I is entering, O- exit hydrograph, $\frac{dS}{\Delta t}$ change of volume, and its get (for example for the second time interval):

$$\frac{S_2 - S_1}{\Delta t} = \frac{\Delta S}{\Delta t} = K \cdot \frac{\Delta Q}{\Delta t} = \frac{Q_2 - Q_1}{\Delta t} = K \cdot \frac{(I_2 - I_1) + (1-x)(O_2 - O_1)}{\Delta t} \quad (18)$$

or with experiment of that is:

$$\frac{S_2 - S_1}{2} = \frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} \quad (19)$$

is get:

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = K \cdot \frac{(I_2 - I_1) + (1-x)(O_2 - O_1)}{\Delta t} \quad (20)$$

if the equation (20) is solowed under O_2, is get:

$$O_2 = \frac{\Delta t - 2Kx}{2K(1-x)+\Delta t} \cdot I_2 + \frac{\Delta t + 2Kx}{2K(1-x)+\Delta t} \cdot I_1 + \frac{2K(1-x)-\Delta t}{2K(1-x)+\Delta t} \cdot O_1 = \quad (21)$$
$$= C_1 \cdot I_2 + C_2 \cdot I_1 + C_3 \cdot O_1$$

where there are:

$$C_1 = \frac{\Delta t - 2xK}{2K(1-x)+\Delta t} \quad (22)$$

$$C_2 = \frac{\Delta t + 2Kx}{2K(1-x)+\Delta t} \quad (23)$$

$$C_3 = \frac{2K(1-x)-\Delta t}{2K(1-x)+\Delta t} \quad (24)$$

In addition, must to be sutisfed the nezt equation:

$$C_1 + C_2 + C_3 = 1 \cdot \quad (25)$$

Analogously, flowing in the third time interval bringup:

$$O_3 = \frac{\Delta t - 2Kx}{2K(1-x)+\Delta t} \cdot I_3 + \frac{\Delta t + 2Kx}{2K(1-x)+\Delta t} \cdot I_2 + \frac{2K(1-x)-\Delta t}{2K(1-x)+\Delta t} \cdot O_2 = (26)$$
$$= C_1 \cdot I_3 + C_2 \cdot I_2 + C_3 \cdot I_1 + C_4 \cdot O_1$$

or generally,

$$O_n = \frac{\Delta t - 2Kx}{2K(1-x)+\Delta t} \cdot I_n + \frac{\Delta t + 2Kx}{2K(1-x)+\Delta t} \cdot I_n + \frac{2K(1-x)-\Delta t}{2K(1-x)+\Delta t} \cdot O_{n-1} = (27)$$
$$= C_1 \cdot I_n + C_2 \cdot I_{n-1} + C_3 \cdot I_{n-2} + C_n \cdot I_1 + C_{n+1} \cdot O_1$$

where which are:

$$C_1 = \frac{\Delta t - 2xK}{2K(1-x)+\Delta t} \quad (28)$$

$$CC = \frac{(2K(1-x)+\Delta t) - 2\Delta t}{2K(1-x)+\Delta t} \quad (29)$$

$$C_2 = C_1 \cdot CC + \frac{\Delta t - 2Kx}{2K(1-x)+\Delta t} \quad (30)$$

$$C_i = C_{i-1} \cdot CC \quad (\text{за } 2 < i \le n+1) \quad (31)$$

Surely, the sum of coefficients $C_i \ (i = 1, n+1)$ is one, id est.:

$$\sum_{i=1}^{n+1} C_i = 1 \quad (32)$$

In the upper equations parts of the index have the next meaning:

Δt -value of time under transfer of water mass;

K -time of travel shown in hours;

x -coefficient pf transfer without dimension that value is between 0 and 0,5.

With a prpuse to be avoid apperance of negativ coefficients in the equations (18) and (21) till (30) the coefficient of Muskingum it has to be satisfy the3 next condition:

$$\frac{\Delta t}{2(1-x)} \le K \le \frac{\Delta t}{2x} \quad (33)$$

I has to be said that x weight factor of importing of water in the canal. If is x=0, income of water in the canal doesn't efects the change of the volume of the water in the canal (like the all tank is moved). If is, again, x=0,5 income will have maximal efect and at the same time there wont be sufication of the exit of the canal (only transfer based on K hours).

It has to be said that under use of this method critical phase is qualification of the values for K and x. This values has to be determine in precise way on the used data's for valued enter and exit hydrograph for given canal section. If that is impossible it is recommended to be taken for x values in the interval $0,2 < x < 0,5$.

After determinaton of K and x for given canal section, the procedure for calculation for getting of the exit hydrograph is next:

1. It discreet the enter hydrograph with time interval with width Δt;
2. The coefficients are calculated $C_i \ (i = 1, n+1)$;
3. It uses the equation of Muskingum for deciding of exit hydrograph on the end of the canal;
4. Step 3 is repeat until all of his intervals of the enter hydrograph are done.

VI. CONCLUSION

In this paper are shown models for transfer of water between two or more hydropower plants. It had to be said that one same plum hardly that the condition can be made respectively one owner of all hydropower plants. From hire and the need of inclusion of the method for distribution of hydrographs or use of water potential, with it is made ideal solution for use of waters (includinc all the restrictions). That means that the technic documentation, practical on ever level (only, notional proect even and fisibility study) has to be isolated with one of a kind mathematical model. While that can be formed "timetable" of the hydropower plants and with smalest consuption of the water potential to be made bigest production of electrical energy.

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