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Performance Analysis of SC Diversity System with Different Correlation Models in α-μ fading Environment

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Abstract – **In this paper, we present the performance analysis of selection combining (SC) diversity system operating over the channel subjected to *α-μ* fading and cochannel interference. Assuming the short distance between the received antennas, we consider the existed correlation between received signal envelopes. Constant correlation and exponential correlation models are analysed for proposed channel and system model and brief analytical and numerical results are given for both cases. Outage probability, as an important system's performance measure, is the main focus of the paper.**

Keywords – **fading environment, outage probability, selection diversity**

I. INTRODUCTION

Many urban mobile communication systems are subjected to fading, caused by multipath propagation due to reflections, refractions and scattering by buildings and other large obstucles [1]. As a result, the receiver sees the superposition of multiple copies of the transmitted signal, each traversing a different path. Several statistical models have been used in the literature to describe the fading envelope of the received signal [2]-[4]. For example, the Rayleigh and Rician distributions are used to characterize the envelope of faded signals over small geographical areas or short term fades while the log-normal distribution is used when much wider geographical areas are involved. Nakagami-*m* distribution is frequently used fading model distribution because of its wide range of applicability for many physical propagation channels than other distributions. Further, more general fading distribution is the α-μ distribution [5]. It includes all other distributions: the Gamma (and its discrete versions Erlang and central Chi-squared), Nakagami-*m* (and its discrete version Chi), exponential, Weibull, one-sided Gaussian and Rayleigh.

There are many proposed techniques to reduce bad fading influences. Space diversity techniques are extensively used to ameliorate signal propagation in fading channels and to improve the system performance gain. Maximal-ratio combining (MRC), equal-gain combining (EGC), selection combining (SC), switch and stay combining (SSC), or a combination of MRC and SC called generalized selection combining (GSC), are several principal types of diversity combining techniques that can be generally performed [6]. SC space-diversity technique is with lower complexity nature opposed to MRC and EGC combining techniques which require the amount of the channel-state information of transmitted signal.

In general, the SC combiner selects the branch with the highest signal-to-noise ratio (SNR), actually the branch with the strongest signal assuming equal noise power among the branches [3]. Also, it has been proposed the selection of the branch with the highest signal plus noise [7]. In environments where the level of the cochannel interference is sufficiently high as compared to the thermal noise (interference-limited systems), SC picks the highest signal-to-interference ratio (SIR; SIR-based selection diversity) [8].

The fading among the channels is correlated due to insufficient antenna spacing, which is a real scenario in practical diversity systems, resulting in a degradation of the diversity gain [1]. Therefore, it is important to understand how the correlation between received signals affects the system performance. Several correlation models have been proposed in the literature for the performance analysis of various wireless systems, corresponding to specific modulation, detection, and diversity combining schemes. Though useful in mathematics and some situations in engineering, the assumption of constant or exponential correlation generally matches the practical environment in mobile communications. When there is an antenna array with a totally symmetrical triangular configuration, we have a constant correlation. On the contrary, the assumption of exponential correlation is somewhat close to the situation in a linear array, but it requires equispaced diversity antennas.

II. CHANNEL AND SYSTEM MODEL

The desired signal received by the *i*-th antenna can be written as [9]:

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$$D_i(t) = R_i(t) e^{j\phi_i(t)} e^{j[2\pi f_c t + \Phi(t)]}, i = 1,2 \quad (1)$$

where f_c is carrier frequency, $\Phi(t)$ desired information signal, $\phi_i(t)$ the random phase uniformly distributed and $R_i(t)$ the α-μ distributed random amplitude process given by [5]:

$$f_{R_i}(t) = \frac{\alpha_i \mu_i^{\mu_i} t^{\alpha_i \mu_i - 1}}{\hat{t}^{\alpha_i \mu_i} \Gamma(\mu_i)} \exp\left(-\mu_i \frac{t^{\alpha_i}}{\hat{t}_i^{\alpha_i}}\right), t \geq 0 \quad (2)$$

where $\Gamma(.)$ is the Gamma function [10, eq. (8.31)], with $\hat{t} = \sqrt[\alpha_i]{E(R_i^{\alpha_i})}$ being the α -root mean value, μ is the inverse of normalized variance of R^α i.e. $\mu = E^2(R^\alpha)/V(R^\alpha)$, $\mu > 0$, and $E(\cdot)$ and $V(\cdot)$ are, respectively, the expectation and variance operators.

The resultant interfering signal received by the i-th antenna is:

$$C_i(t) = r_i(t) e^{j\theta_i(t)} e^{j[2\pi f_c t + \psi(t)]}, i = 1,2 \quad (3)$$

where $r_i(t)$ is also α-μ distributed random amplitude process, $\theta_i(t)$ is the random phase, and $\psi(t)$ is the information signal. This model refers to the case of a single cochannel interferer.

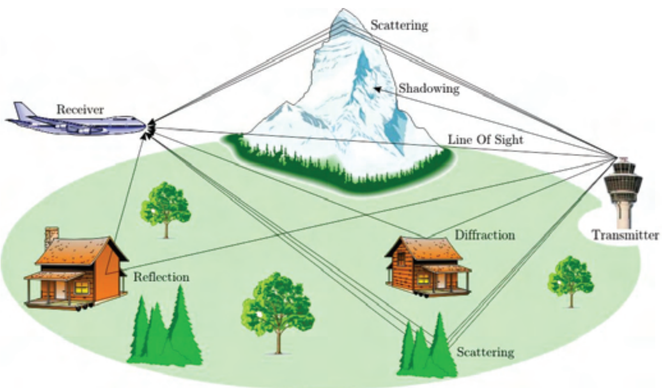

Fig. 1. Fading propagation environment

The performance of the SC can be carried out by considering the effect of only the strongest interferer, assuming that the remaining interferers are combined and considered as uncorrelated interference between antennas, as in [11]. Furthermore, $R_i(t)$, $r_i(t)$, $\phi_i(t)$, and $\theta_i(t)$ are assumed to be mutually independent and the effect of the interference on system performance is higher than that of the thermal noise (interference-limited environment) [9]. Now, due to a scenario with closely placed diversity antennas, both desired and interfering signal envelopes experience correlative α–μ fading.

There is a need to derive the joint statistics for two α-μ variables. We are relaying on some results which are already available in the literature for the constant correlation model of Nakagami-*m* distribution [12, eq. (10)] and exponential correlation model of Nakagami-*m* distribution [13, eq. (3)]. We also consider the proposed relations between α-μ and Nakagami-*m* variables [5, eqs. (19),(31)].

So, by setting $\Sigma_{i,j} \equiv 1$ for $i = j$ and $\Sigma_{i,j} \equiv \rho$ for $i \neq j$ in correlation matrix [14], where ρ denotes the power correlation coefficient defined as $cov(R_i^2, R_j^2)/(var(R_i^2)var(R_j^2))^{1/2}$ and after the standard statistical procedure of transformation of variates and some mathematical manipulations, joint probability density functions (joint pdfs) for both desired and interfering signal envelopes can be expressed as:

$$\begin{aligned} p_{R_1,R_2}(R_1,R_2) = & \frac{\left(1-\sqrt{\rho_d}\right)^{\mu_d}}{\Gamma(\mu_d)} \sum_{k_1,k_2=0}^{\infty} \frac{\Gamma(\mu_d + k_1 + k_2)\rho_d^{\frac{k_1+k_2}{2}}}{\left(1-\sqrt{\rho_d}\right)^{2\mu_d+k_1+.k_2}} \\ & \times \mu_d^{2\mu_d+k_1+.k_2} \left(\frac{1}{1+\sqrt{\rho_d}}\right)^{\mu_d+k_1+k_2} \\ & \times \prod_{i=1}^{2} \frac{\alpha_i R_i^{\alpha_i(\mu_d+k_i)-1}}{\Gamma(\mu_d+k_i) k_i! \hat{R}_i^{\alpha_i(\mu_d+k_i)}} \exp\left(-\frac{\mu_d R_i^{\alpha_i}}{\hat{R}_i^{\alpha_i}(1-\sqrt{\rho_d})}\right) \end{aligned} \quad (4)$$

and

$$\begin{aligned} p_{r_1,r_2}(r_1,r_2) = & \frac{\left(1-\sqrt{\rho_c}\right)^{\mu_c}}{\Gamma(\mu_c)} \sum_{l_1,l_2=0}^{\infty} \frac{\Gamma(\mu_c + l_1 + l_2)\rho_c^{\frac{l_1+l_2}{2}}}{\left(1-\sqrt{\rho_c}\right)^{2\mu_c+l_1+l_2}} \\ & \times \mu_c^{2\mu_c+l_1+l_2} \left(\frac{1}{1+\sqrt{\rho_c}}\right)^{\mu_c+l_1+l_2} \\ & \times \prod_{i=1}^{2} \frac{\alpha_i r_i^{\alpha_i(\mu_c+l_i)-1}}{\Gamma(\mu_c+l_i) l_i! \hat{r}_i^{\alpha_i(\mu_c+l_i)}} \exp\left(-\frac{\mu_c r_i^{\alpha_i}}{\hat{r}_i^{\alpha_i}(1-\sqrt{\rho_c})}\right) \end{aligned} \quad (5)$$

where ρ_d and ρ_c are the correlation coefficients, and μ_d and μ_c are the fading severity parameters for the desired and interference signal, respectively. The average desired signal and interference powers at the i-th branch are denoted by $\hat{R}_i$ and $\hat{r}_i$, respectively.

Otherwise, by setting $\Sigma_{i,j} \equiv \rho^{|i-j|}$ in correlation matrix [14], for both desired signal and interference, the exponential correlation model can be obtained. So, the joint pdfs of desired and interfering signal envelopes for the exponential correlation system model can be expressed as:

$$\begin{aligned} p_{R_1,R_2}(R_1,R_2) = & \sum_{k=0}^{\infty} \alpha_1 \alpha_2 \frac{\rho_d^{2k}}{2^{2(\mu_d+k)} \Gamma(\mu_d) \Gamma(\mu_d+k) k!} \times \frac{1}{\left(1-\rho_d^2\right)^{\mu_d+2k}} \\ & \times \exp\left(-\frac{R_1^{\alpha_1} + R_2^{\alpha_2}}{2(1-\rho_d^2)}\right) \times R_1^{\frac{\alpha_1}{2}(\mu_d+1)-1} R_2^{\frac{\alpha_2}{2}(\mu_d+1)-1} \end{aligned} \quad (6)$$

and

$$p_{r_1,r_2}(r_1,r_2)=\sum_{l=0}^{\infty}\alpha_1\alpha_2\frac{\rho_c^{2l}}{2^{2(\mu_c+l)}\Gamma(\mu_c)\Gamma(\mu_c+l)l!}\times\frac{1}{\left(1-\rho_c^2\right)^{\mu_c+2l}}$$
$$\times\exp\left(-\frac{r_1^{\alpha_1}+r_2^{\alpha_2}}{2(1-\rho_c^2)}\right)\times r_1^{\frac{\alpha_1}{2}(\mu_c+l)-1}r_2^{\frac{\alpha_2}{2}(\mu_c+l)-1} \tag{7}$$

Instantaneous values of SIR at the two input branches can be defined as $\lambda_1=R_1^2/r_1^2$, $\lambda_2=R_2^2/r_2^2$. The selection combiner (Fig. 2.) chooses and outputs the branch with the largest SIR:

$$\lambda=\lambda_{out}=\max(\lambda_1,\lambda_2) \tag{8}$$

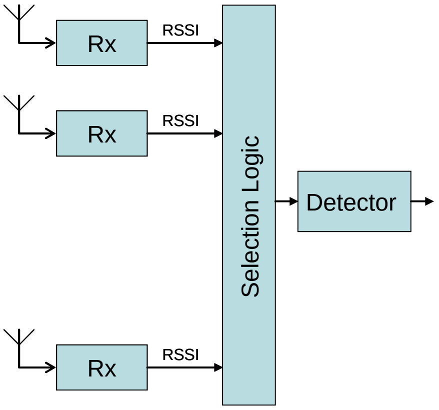

Fig. 2. SC receiver

Let $S_1=\hat{R}_1^2/\hat{r}_1^2$, $S_2=\hat{R}_2^2/\hat{r}_2^2$ be the average SIR's at the two input branches of the selection combiner. The joint pdf of instantaneous values of SIRs at the two input branches of SC, λ_1, λ_2 can be given by [15]:

$$p_{\lambda_1,\lambda_2}(t_1,t_2)=\frac{1}{4\sqrt{t_1t_2}}\int_0^{\infty}\int_0^{\infty}p_{R_1,R_2}\left(r_1\sqrt{t_1},r_2\sqrt{t_2}\right)p_{r_1,r_2}(r_1,r_2)\times r_1r_2\,dr_1dr_2 \tag{9}$$

For this case, the joint cumulative distribution function (joint cdf) of λ_i, $i=1,2$ can be written as [15]:

$$F_{\lambda_1,\lambda_2}(t_1,t_2)=\int_0^{t_1}\int_0^{t_2}p_{\lambda_1,\lambda_2}(x_1,x_2)\,dx_1dx_2 \tag{10}$$

Substituting the derived version of expression (9) in (10) and after two successive integrations, the joint cdf becomes:

$$F_\lambda(t)=\sum_{k_1,k_2=0}^{\infty}\sum_{l_1,l_2=0}^{\infty}G\times$$
$$\times\prod_{i=1}^{2}\mathrm{B}\left(\frac{\mu_d t^{\frac{\alpha_i}{2}}}{\mu_d t^{\frac{\alpha_i}{2}}+\mu_c\frac{\left(1-\sqrt{\rho_d}\right)}{\left(1-\sqrt{\rho_c}\right)}S_i^{\frac{\alpha_i}{2}}},\mu_d+k_i,\mu_c+l_i\right) \tag{11}$$

with

$$G=\frac{\left(1-\sqrt{\rho_d}\right)^{\mu_d}\left(1-\sqrt{\rho_c}\right)^{\mu_c}}{\Gamma(\mu_d)\Gamma(\mu_c)}\Gamma(\mu_d+k_1+k_2)\Gamma(\mu_c+l_1+l_2)\rho_d^{\frac{k_1+k_2}{2}}\rho_c^{\frac{l_1+l_2}{2}}$$
$$\times\left(\frac{1}{1+\sqrt{\rho_d}}\right)^{\mu_d+k_1+k_2}\left(\frac{1}{1+\sqrt{\rho_c}}\right)^{\mu_c+l_1+l_2}\times\prod_{i=1}^{2}\frac{\Gamma(\mu_d+\mu_c+k_i+l_i)}{\Gamma(\mu_d+k_i)\Gamma(\mu_c+l_i)k_i!l_i!} \tag{12}$$

for the constant correlation system model and

$$F_\lambda(t)=\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\left(\Gamma(\mu_d+\mu_c+k+l)\right)^2\rho_d^{2k}\rho_c^{2l}\left(1-\rho_d^2\right)^{\mu_d}\left(1-\rho_c^2\right)^{\mu_c}}{\Gamma(\mu_d)\Gamma(\mu_c)\Gamma(\mu_d+k)\Gamma(\mu_c+l)k!l!}$$
$$\times\prod_{i=1}^{2}\mathrm{B}\left(\frac{t^{\frac{\alpha_i}{2}}}{\frac{\left(1-\rho_d^2\right)}{\left(1-\rho_c^2\right)}+t^{\frac{\alpha_i}{2}}},\mu_d+k,\mu_c+l\right) \tag{13}$$

for the exponential correlation system model, with $\mathrm{B}(z,a,b)$ being the incomplete Beta function [10, eq. 8.391].

III. OUTAGE PROBABILITY

Outage probability is an important system's performance measure, used to control the cochannel interference level. The designers of wireless communications systems readjust the system's operating parameters considering outage probability, in order to achieve the quality of service (QoS) and grade-of-service (GoS) demands. In the interference-limited environment, outage probability P_{out} is defined as the probability which combined-SIR falls below a given outage threshold q, also known as a protection ratio. If q is the protection ratio, defined as the ratio of the desired signal power to the interference power at the output of the combiner, the outage probability can be expressed as:

$$P_{\text{out}}=\text{Probability}(\lambda<q)=\int_0^q p_\lambda(t)dt=F_\lambda(q) \tag{14}$$

The protection ratio q depends upon the used modulation technique as well as on the desired QoS.

Fig. 3 and Fig. 4 illustrate the influence of correlation coefficients ρ_d and ρ_c and different values of fading parameters μ_d and μ_c on P_{out}. It is shown that outage probability increases when correlation among branches increases, which means degradation in performance gain. Otherwise, when parameter μ_d and/or μ_c increase (fading severity decreases), the values of outage probability also decreases, which means better system performances.

Comparing the two correlation models, it is noticed that there is compromise between them. In some cases the constant correlation model gives better results, in the other the exponential correlation model, which depends on elected correlation coefficients and fading parameters.

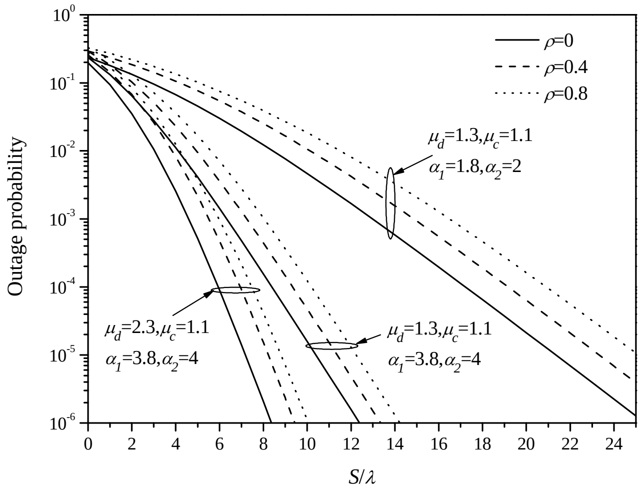

Fig. 3. Outage probability of constant correlation model versus S_1/q for different values of correlation coefficients and fading severity

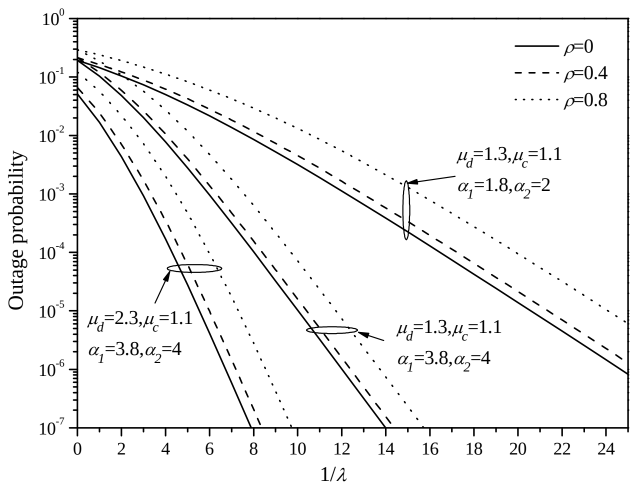

Fig. 4. Outage probability of exponential correlation model versus $1/q$ for different values of correlation coefficients and fading severity

IV. Conclusion

In this paper, the performance analysis of system with SC, based on SIR over correlated α–μ fading channels, was obtained. Constant correlation and exponential correlation models were observed for evaluating performances of proposed diversity system. The desired signal envelopes and interference were α–μ distributed. By considering α–μ distribution, we have included all important distributions which describe the statistics of radio signal through various environments. The analytical analysis for evaluating outage probability was presented. Using those results, the effects of various parameters such as the fading severity and level of correlation to the outage probability were observed.

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