Digital FIR Filter with Improved Selectivity

Peter Apostolov[1]

***Abstract* – This paper describes a theory and method for designing the finite impulse response (FIR) filters. The theory is based on the approximation of ideal low pass filter transfer function. An appropriate basis function that leads to obtaining FIR filters with improved selectivity has been developed. The method has been carried out using Remez' algorithm and an optimal polynomial that approximates ideal low pass filter transfer function with high precision, has been obtained. Furthermore, an analysis of FIR filters' parameters as well as comparison with those of Parks McClellan's method has also been carried out. In addition, an example for linear phase filter design, along with calculation minimization and a result's analysis, has also been presented. With the proposed method, FIR falters with better selectivity than those of Parks McClellan's can be obtained.**

***Keywords* – FIR digital filters, Frequency response, Polynomial approximation.**

I. Introduction

The digitalization of modern electronic equipment requires the use of filters with linear phase responses. The digital FIR filters have such properties. Their synthesis is accomplished by appropriate approximation of ideal low pass filter transfer function like this.

$$H_d\left(e^{j\omega}\right)=\begin{cases}1e^{j\omega}, & 0\le\omega\le\omega_t\\ 0 & ,\ \omega_t<\omega\le\pi\end{cases} \qquad (1)$$

where $\omega_t = 2\pi f_t$ is filter's normalized angular transition frequency. The most popular methods for synthesis are by using the window functions: Rectangular, Hamming, Kaiser [3], Hausdorff [1] etc. The best selective properties have Parks-McClellan's digital FIR filters [5]. They can be obtained by equiripple approximation (Fig.1) of ideal low pass filter transfer function with basis function $\cos x$. This article presents a new method for FIR filter design, which has better selectivity than Parks-McClellan's.

II. Background

The method's theoretical base is the Alternation Theorem:

- if a function $f(x)$ is defined and continuous in closed definition area, it can be approximated by trigonometric polynomial $P_m(x)$ by power *m*, with basis function $\cos x$;
- the polynomial is the unique and best approximation, if the error function $w(x)=f(x)-P_m(x)$ has at least $m+2$ extremes in the definition area;
- all extremes are alternative and their modules are equal to positive number ε.

[1] Assoc. Prof. Peter S. Apostolov, PhD, Institute for Special Technical Equipment, e-mail: p_apostolov@abv.bg

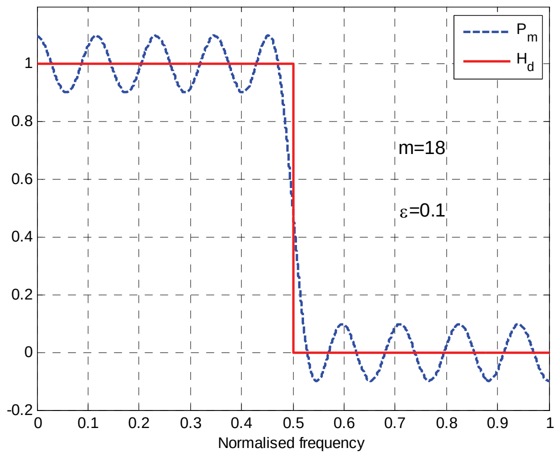

Fig.1. Parks-McClellan's approximation, *m*=18, $\varepsilon = 0.1$

The method's idea is an approximation with polynomial, which posses the following properties:

- even power *m*;
- accordance with Alternation Theorem;
- polynomial's oscillations are getting dense around transition frequency f_t.

Compressing of oscillations is obtained, when the argument of basis function is "modulated" as shown

$$y(\omega)=\cos\left\{\frac{m\pi}{2}\left[\tanh\left(\beta\omega_n-\frac{\beta}{2}\right)+1\right]\right\};\ \omega_n\in[0,1],\ (2)$$

where β is a parameter, which changes the power of function's oscillations compressing, $\omega_n = 2\omega/\omega_{sampl}$ is normalized frequency, $\omega\in\left[0,\omega_{sampl}/2\right]$ is current frequency and ω_{sampl} is sampling frequency.

On Fig. 2 the function's graph for $m=10$, for parameter's values $\beta=8$ and $\beta=15$ is shown.

The approximation's polynomial is accomplished by Remez' algorithm. It comprises iteration computing of system of $m+2$ linear equations. The obtained $m+2$ solutions are the polynomial's coefficients and the approximation error ε. The polynomial is obtained as follows

$$P_m(\omega)=\sum_{k=1}^{m+1}b_k\cos\left\{\frac{\pi(k-1)}{2}\left[\tanh\left(\beta\omega_n-\frac{\beta}{2}\right)+1\right]\right\}.\ (3)$$

On Fig.3 an approximation with the polynomial for $m=4$, $\beta=10$ and $\varepsilon=0.1$ is shown

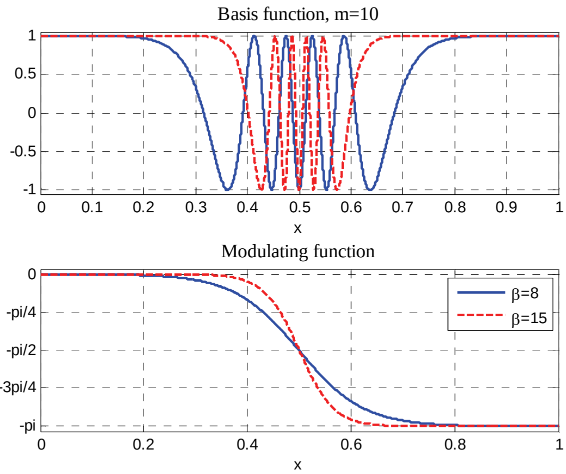

Fig.2. Basis function, with argument's modulation, $m=10$

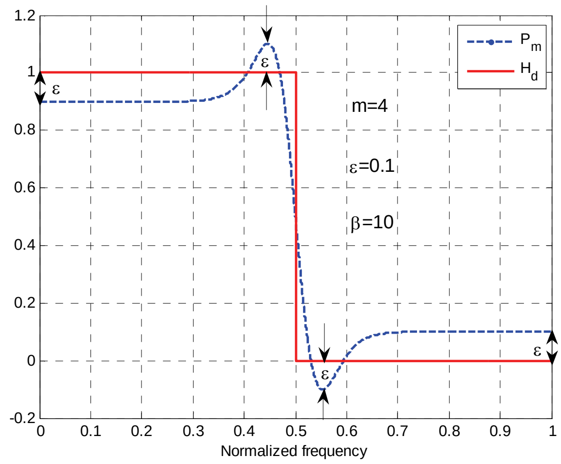

Fig.3. Approximation by offered method

On Fig.4 an approximation by offered method and Parks-McClellan's is shown. From the comparison can be seen, that with the offered method (by equal values of ε) the approximation is made by polynomial of less power.

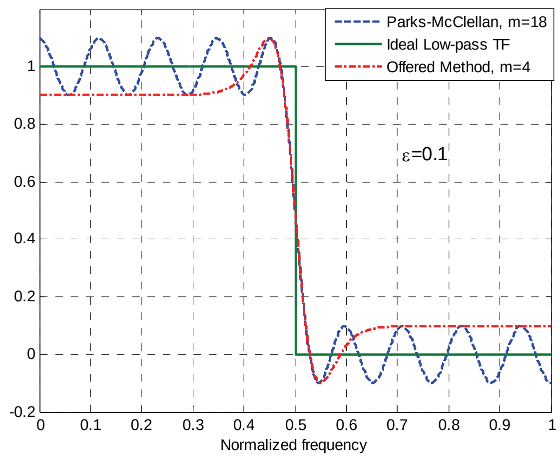

Fig.4. Approximations' comparison

The digital filter's coefficients are derived from the polynomial's coefficients in the following sequence

$$h_n = \frac{b_{m+1}}{2}, \frac{b_m}{2}, \dots, \frac{b_2}{2}, b_1, \frac{b_2}{2}, \dots, \frac{b_m}{2}, \frac{b_{m+1}}{2}, \qquad (4)$$

where $n=2m+1$ is the filter's length. The filter's transfer function is like this:

$$H(\omega) = \sum_{k=1}^{n} h_k \exp\left\{-j(n-k)\frac{\pi}{2}\left[\tanh\left(\beta\omega_n - \frac{\beta}{2}\right)+1\right]\right\} = $$

$$= \sum_{k=1}^{n} h_k \exp\left[-j(n-k)\varphi(\omega)\right] \qquad (5)$$

III. Filter's Implementation

The implementation will be illustrated with an example for FIR filter design. Filter's specification: power of the polynomial $m=4$ (filter length $n=9$); transition frequency $f_t = 600$ Hz; transition band $\Delta f_t = 60$ Hz, sampling frequency $f_{sampl} = 8000$ Hz; parameter $\beta = 131$. After Remez' algorithm approximation, an optimal polynomial has been obtained with coefficients: $b_1 = 0.5$; $b_2 = 0.5655$; $b_3 = 0$; $b_4 = -0.0656$; $b_5 = 0$ and approximation error $\varepsilon = 0.0001$.

The realization is done using a method, known as "frequency sampling filter" [2], [4]. It is based on FFT in 2^N discretes, where N is an integer positive number. A value of $N=10$ is set, i.e. 1024-FFT will be chosen. The following procedure must be carried out:

1. The minimal frequency step is calculated

$$df = \frac{f_{sampl}}{2^N} = 7.8125 \qquad (6)$$

A massive, containing 512 frequencies is created

$$f_j = df : df : f_{sampl}/2; \; j = 1 \div 2^N/2. \qquad (7)$$

2. A 512 values of polynomial are calculated

$$\theta_j = \sum_{k}^{m+1} b_k \cos\left\{(k-1)\frac{\pi}{2}\left[\tanh\left(\beta\left(\frac{2f_j}{f_{sampl}}\right)^X - \frac{\beta}{2}\right)+1\right]\right\}, \qquad (8)$$

where

$$X = \log_2^{-1}\left(\frac{f_{sampl}}{2f_t}\right) = 0.36537. \qquad (9)$$

θ_j is so called filter's "mask". It is shown on Fig.5.

3. After analog to digital conversion, the signal is applied to 2^N buffer.

4. A 1024-FFT is being performed. As a result 2^N complex frequencies F_i are obtained.

5. The first 512 frequencies are multiplied by the filter's mask θ_j, which corresponds to convolution in frequency domain. A spectrum with $2^N/2 = 512$ frequencies is obtained.

$$S_j = F_j \otimes \theta_j; \; j = 1 \div 2^N/2. \qquad (10)$$

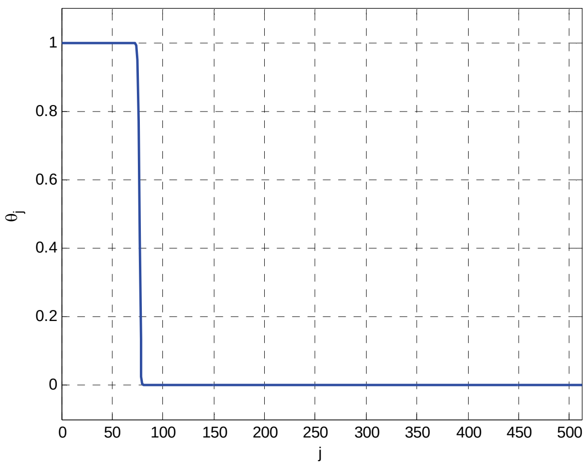

Fig.5. Filter's mask

6. The spectrum is added with $2^N/2 = 512$ zeros

$$S_i = [S_j, zeros(1,512)];\ j = 1 \div 2^N/2;\ i = 1 \div 2^N. \quad (11)$$

7. A 1024-IFFT is performed. As a result 2^N complex numbers are obtained, which real parts are the filtered signal.

8. The next 2^N discretes are taken from the input buffer and the procedure returns to p.4.

On Fig.6 the filter's structure is shown.

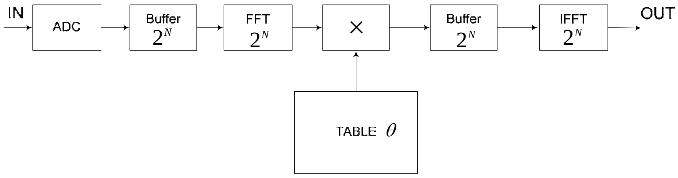

Fig.6. Filter's structure

The filter's pass-band ripple is

$$DA = 20\lg\left(1 - |\varepsilon|\right) = -8.6155\text{e-}4\ \text{dB}, \quad (12)$$

and stop band attenuation

$$DS = 10\lg|\varepsilon| = -40.0356\ \text{dB}. \quad (13)$$

On Fig.7 the filter's magnitude response is shown.

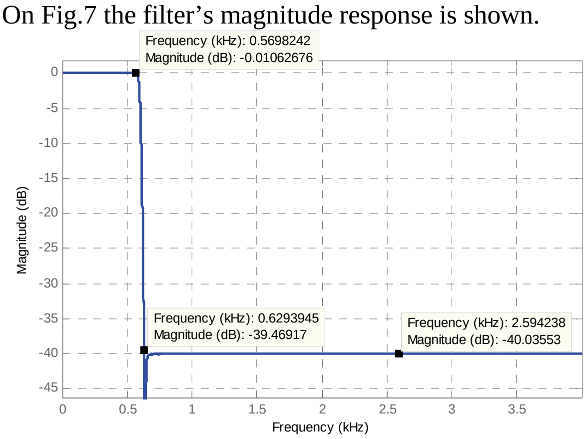

Fig.7. Magnitude response

The use of FFT leads to decreasing the filter's frequencies responses recreation accuracy. This inaccuracy comprises increasing of the pass-band ripple. The obtained value of $DA = -0.01627$ dB is reasonable from a practical point of view.

On Fig.8 the impulse response of the designed filter is shown. From the figure it can be seen, that the filter has symmetrical impulse response. This is a response of linear-phase filter.

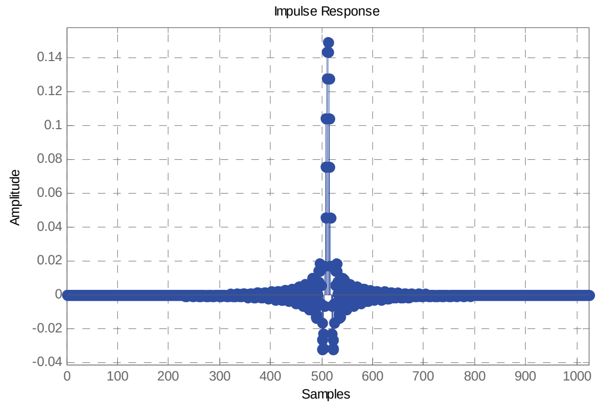

Fig.8. Impulse Response

On the next figure the phase response is shown.

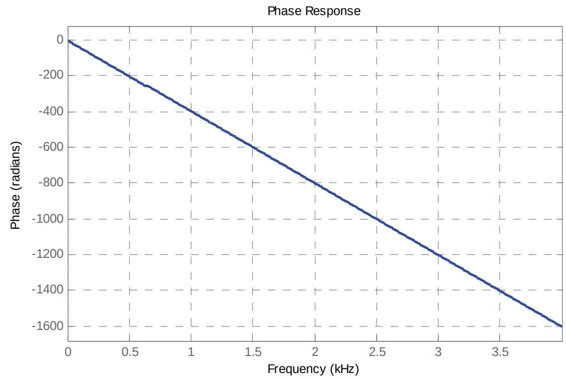

Fig.9. Phase response

The filter's group delay time (GDT) is the product of the phase response. Due to the phase response linearity the, GDT is constant – Fig.10.

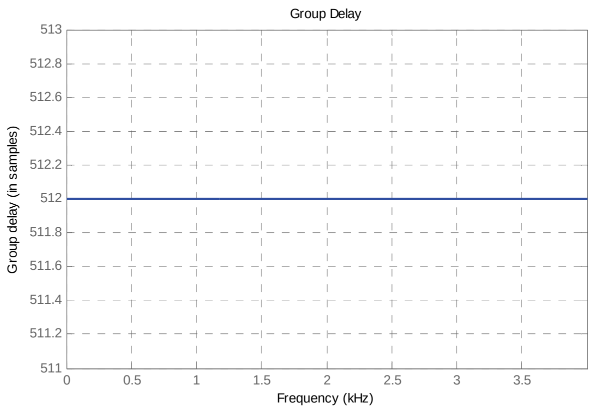

Fig.10. GDT

The GDT value corresponds to the digital processing of the input buffer numbers and can be defined by the relation

$$GDT = 2^{N-1}\ \text{samples/sec.} \quad (14)$$

IV. Calculation's Minimization

From the above mentioned it can be seen, that the main calculations are the convolution (10). To get one input buffer, the $2^N/2 = 512$ multiplications of complex with real numbers must be done, which are $2^N = 1024$ multiplications. From Fig.5 is seen, that the filter's mask contains 512 numbers, the greatest part of them, are equal either to $1-\varepsilon$ (in the pass-band), or to ε (in the stop-band). That are numbers very close to 1 or 0, because $\varepsilon = 0.0001$. The calculations can be significantly reduced if the filter's mask is divided in three zones:

1. pass-band – the numbers are set to 1;
2. transition band – the calculated mask values are accepted;
3. stop-band – the numbers are set to 0.

In our case it is appropriate for transition band that 25 numbers should be accepted, which have indexes from 66 to 90 (Fig.11)

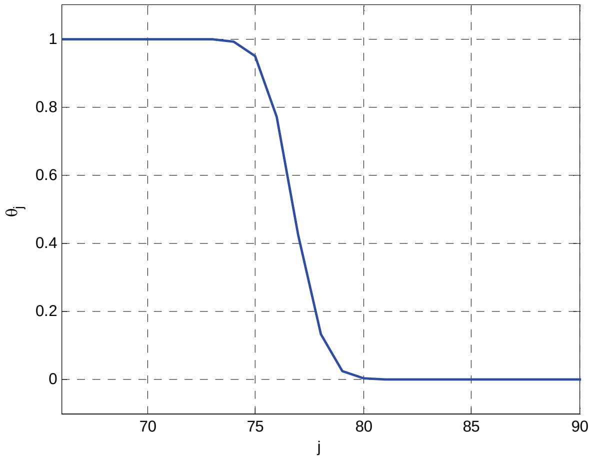

Fig.11. Filter's mask – transition band

So the filter's mask will consist of 65 ones, 25 numbers from the transition band and 422 zeroes. The first 65 complex frequencies from the input buffer go freely to the output (as if are multiplied by 1). The next 25 are multiplied with the numbers from the transition band, thereafter 422+512=934 zeroes are added without any calculation. So the calculations are reduced to 25 multiplications of complex with real number that means 50 multiplications.

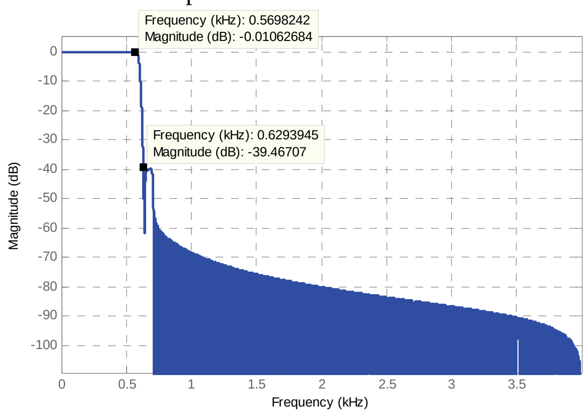

Fig.12. Magnitude response with reducing of the calculations

On Fig12 a magnitude response of a filter, realized by the offered method, is shown. It is seen, that the reducing of the calculations more than 20 times doesn't lead to Gibbs' effect and the filter remains optimal. From the comparison with Fig.7 is seen, that the magnitude response keeps its parameters in the pass-band and transition-band, the attenuation in the stop-band increases. That means the calculation's minimization can be applied successfully in practice.

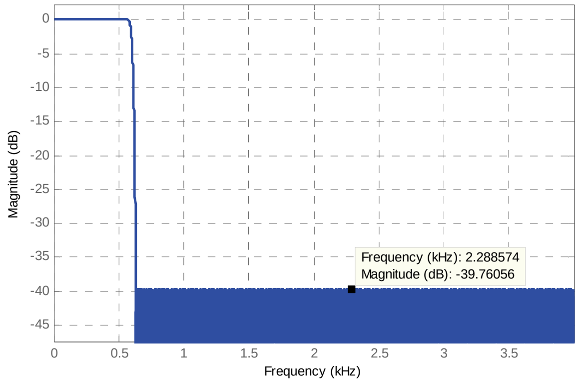

Fig.13. Parks-McClellan - Magnitude response, *n*=615

On Fig.13 a magnitude response of Parks-McClellan's filter, which has a Fig.7 fitter specification, is shown. It is seen, that the filter's length *n*=615 is over than 68 times higher.

V. Conclusion

Offered method's advantages are due to the following two circumstances:

- transfer function's coefficients are obtained from optimal approximation (Fig.3);
- every addend of transfer function (5) is multiplied with factor $\varphi(\omega) = \frac{\pi}{2}\left[\tanh\left(\beta\omega_n - \beta/2\right)+1\right]$, which is the phase response of the resonance circuit, whereas for the other FIR filters it is linear function $\varphi(\omega) = \exp(-j\omega)$.

The filters' selectivity depends on parameter β, which can grow unlimitedly. The filters don't change their length, when the ratio between transition and sampling frequency changes. They have linear phase response. They can be realized with minimal calculations.

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