Path Control with Input Saturation for Reversing a Car-Like Mobile Robot

Plamen P. Petrov[1] and Lubomir M. Dimitrov[2]

Abstract – **This paper proposes a path control with input saturation for backward driving of a car-like mobile robot. The control law is constructed using high-gain design techniques and involves error coordinates and invariant properties with respect to the vehicle speed. The stability of the closed-loop system is analyzed using Lyapunov stability theory. Simulation results illustrate the effectiveness of the proposed controller.**

Keywords – **Car-like mobile robot, Path following, Backward driving, High-gain control, Input saturation**

I. Introduction

In this paper, we consider the path following problem for backward driving of a car-like mobile robot with input saturation. In recent years, significant advances have been made in designing feedback controllers for nonholonomic mobile robots. While there has been significant amount of work on controlling the motion of mobile robots without bound of the control inputs [1, 2, 3], there has been much less work on mobile robot motion control with input saturations [4].

In this paper, we propose a path following controller with input saturation for a car-like mobile robot during backward driving. The paper is organised as follows: In Section II, the kinematic model of the car-like mobile robot is presented and the control problem is formulated. In Section III, the control law is designed. Simulation results are presented in Section IV. Section V concludes the paper.

II. Problem formulation

A. Mobile Robot Kinematic Model

A plan view of the mobile robot is given in Fig. 1. The mobile robot has three non-deformable wheels. The wheels are assumed to roll on a horizontal plane without slipping. The longitudinal base *PS* of the vehicle is denoted by l. To describe the position and orientation of the robot in the plane, we assign the following coordinate frames: Px_Py_P located at the center of the rear wheel axle and stationary with respect to the robot body where the x_P axis is along the longitudinal base of the robot, and an inertial coordinate frame Fxy in the plane of motion. The coordinates of a reference point P placed at the center of the rear robot axle, with respect to *Fxy*, are denoted by (x_P, y_P). The angle θ is the orientation angle of the robot with respect to the frame *Fxy*. The angle α, is the front wheel steering angle. The steering angle is measured with respect to the robot body.

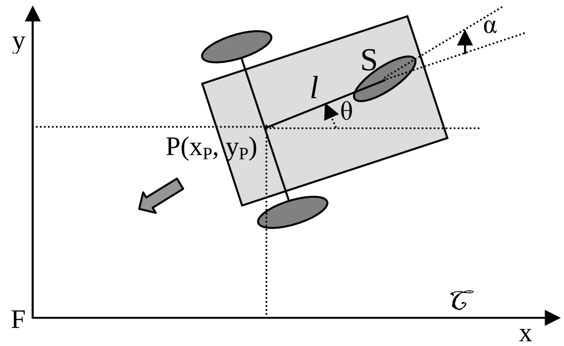

Fig. 1. A plan view of the car-like mobile robot

If the rotation of the wheels with respect to their proper axes is ignored, the mobile robot configuration can be described by four generalized coordinates $q = [x_P, y_P, \theta, \alpha]^T \in \Re^4$. The kinematic model of the robot in the plane can be described by the following system of nonlinear differential equations

$$\begin{aligned} \dot{x}_P &= v_P \cos\theta \\ \dot{y}_P &= v_P \sin\theta \\ \dot{\theta} &= \frac{v_P}{l} \tan\alpha \end{aligned} \qquad (1)$$

where v_p is the velocity of point P. The front-wheel steering angle α is the control input of the system and is a subject of physical limitations $|\alpha| \le \alpha_{\max}$.

B. Control Problem

The path following geometry used in this paper is represented in Fig. 1. Consider a car-like mobile robot moving backward on a flat surface. We assume that the path $\mathcal{C}$ is a straight line which for simplicity coincides with the *Fx* axis of the inertial frame *Fxy*. We assume that v_P together with its derivative are bounded and also, that the following inequalities hold: $0 < v_{Pmin} \le |v_P(t)| \le v_{Pmax}$. In this case, using the parameterization (y_P, θ) and given a path $\mathcal{C}$, the path

[1]Plamen P. Petrov is with the Faculty of Mechanical Engineering, Technical University of Sofia, 8, Kl. Ohridski str., 1797 Sofia, Bulgaria, E-mail: ppetrov@tu-sofia.bg

[2]Lubomir M. Dimitrov is with the Faculty of Mechanical Engineering, Technical University of Sofia, 8, Kl. Ohridski str., 1797 Sofia, Bulgaria, E-mail: lubomir_dimitrov@tu-sofia.bg

following problem consists of finding a feedback control law for the system which consists of the second and third equation of (1) system with control input α subject to physical limitations $|\alpha| \le \alpha_{\max}$, such that the state vector $[y_P, \theta]^T$ tends to $[0, 0]^T$, as $t \to \infty$.

III. Feedback control design

In this Section, we present a path following controller for backward driving of the robot given by the second and third equations of (1) with input saturation for the steering angle α. The control objective is to regulate the state vector $[y, \theta]^T$ to zero. Since $v_P(t)$ is assumed to be strictly negative ($v_P(t) = -|v_P(t)| < 0$), to obtain a time-invariant system, the differentiation with respect to time is replaced by differentiation with respect to s ($ds = v_P dt$), where s is the real path length drown by the reference point P. In that way, we express the vehicle's equations of motion in terms of s and denote the derivation with respect to s by " ′ ". Next, we proceed with the following change of input

$$u = \frac{\tan\alpha}{l} \tag{2}$$

In that way, the system can be written in the form

$$\begin{aligned} y_P' &= -\sin\theta \\ \theta' &= -u \end{aligned} . \tag{3}$$

For the first equation of (3), the control

$$\xi(y_P) = \theta = y_P \tag{4}$$

achieves local exponential stability of $y_P = 0$. For the (y_P, θ) system (3), we propose the following feedback control

$$u = L sat\left(\frac{\lambda}{L}\right) \tag{5}$$

where

$$\lambda = ka(\theta - \xi), \quad k = cte > 0, \quad a = cte > 0 \tag{6}$$

and $sat(.)$ is a saturation function

$$|u(t)| \le L, \qquad L = cte > 0$$

$$sat\left(\frac{\lambda}{L}\right) = \begin{cases} -1 & for \quad \frac{\lambda}{L} < -1 \\ \frac{\lambda}{L} & for \quad \left|\frac{\lambda}{L}\right| \le 1 \\ 1 & for \quad \frac{\lambda}{L} > 1 \end{cases} . \tag{7}$$

The control (6) achieves semi-global stabilization of (3). The semi-global stabilization [5] means that for any compact neighborhood Γ of (y_P, θ) = (0, 0), there exists k^* such that for all $k \ge k^*$, the region of attraction contains Γ. Indeed, we introduce the following change of coordinates

$$\eta = \theta - \xi \ . \tag{8}$$

The system (3) can be rewritten in the new coordinates as

$$\begin{aligned} y_P' &= -\sin(y_P + \eta) \\ \eta' &= -ka\eta - L sat\left(\frac{\lambda}{L}\right) + \lambda + \sin(\eta + y_P) \end{aligned} . \tag{9}$$

For the system (9) we consider the following Lyapunov function

$$V = \frac{1}{2} y_P^2 + \frac{1}{2}\eta^2 . \tag{10}$$

Using (9), for the derivative of (10), we obtain

$$\begin{aligned} V' = &-y_P^2 \frac{\sin(\eta + y_P)}{\eta + y_P} - ka\eta^2 - y_P \eta \frac{\sin(\eta + y_P)}{\eta + y_P} \\ &+ \eta\left[\lambda - L sat\left(\frac{\lambda}{L}\right) + \sin(\eta + y_P)\right] \end{aligned} . \tag{11}$$

Denoting

$$\sigma = \lambda - L sat\left(\frac{\lambda}{L}\right), \tag{12}$$

and using the fact that

$$\forall \ |\lambda| \le L(1+\delta) \ \rightarrow \ |\sigma(\lambda)| \le \frac{\delta}{1+\delta}|\lambda| \tag{13}$$

we have

$$V' \le -y_P^2 \frac{\sin(\eta + y_P)}{\eta + y_P} - \eta^2\left[ka - \frac{\delta}{1+\delta}ka - \frac{\sin(\eta + y_P)}{\eta + y_P}\right]. \tag{14}$$

Since

$$0 < \frac{\sin(\eta + y_P)}{\eta + y_P} \le 1 \tag{15}$$

by choosing

$$k \ge k^* = \frac{1+\delta}{a} \tag{16}$$

we have $V' < 0$.

IV. Simulation results

Simulation results using MATLAB were performed to illustrate the effectiveness of the proposed controller. A straight line reference path which coincides with the Fx axis of an inertial frame was chosen for the simulations. The longitudinal base of the robot was *1m*, and velocity of the mobile robot was chosen to be *1m/s*. The bound for the front wheel steering angle was $|\alpha| \leq \alpha_{max} = 0.785 rad$ (*45 deg*). The controller gains were $k = 1$, $a = 1$. Initial conditions was chosen to be $y_P(0) = 1.5m$; $\theta(0) = -0.5rad$, $\alpha(0) = 0rad$. Evolution of the state coordinates $[y_P, \theta]^T$ with respect to the variable $s = \int_0^t |v_P| d\tau$ is depicted in Fig. 2.

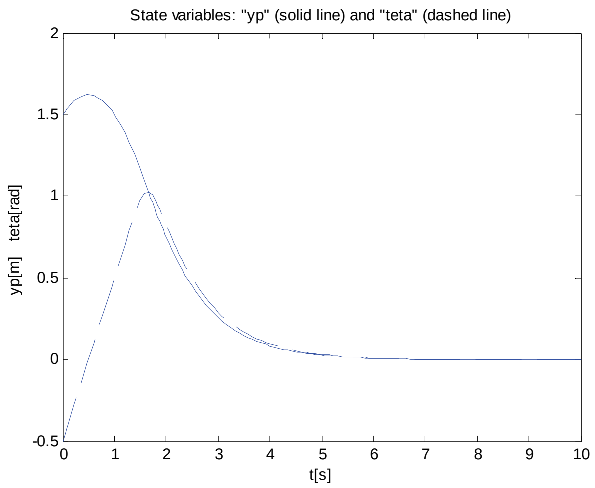

Fig. 2. Evolution in time of the state coordinates (y_P, θ)

Evolution of the front wheel steering angle α is depicted in Fig. 3.

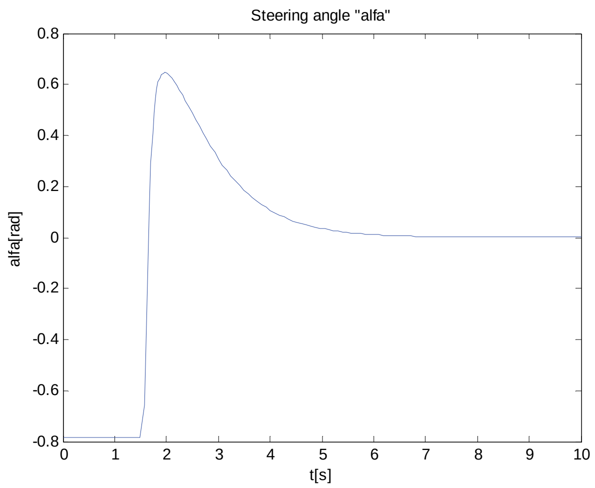

Fig. 3. Evolution in time of the front wheel steering angle α

The simulation results confirm the effectiveness of the proposed controller.

V. Conclusion

In this paper, a path control with input saturation for backward driving of a car-like mobile robot is proposed. The control law is constructed using high-gain design techniques and involves error coordinates and invariant properties with respect to the vehicle speed. The stability of the closed-loop system is analyzed using Lyapunov stability theory. Simulation results illustrate the effectiveness of the proposed controller. Our future work will consist in dynamic extension of the proposed controller in the presence of uncertainties in the model of the mobile robot.

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