# Analysis of LCC resonant DC-DC converter with capacity filter in the load circuit

Aleksandar S. Vuchev<sup>1</sup>, Nikolay D. Bankov<sup>2</sup> and Georgi P. Terziyski<sup>3</sup>

*Abstract* – A study of LCC resonant DC-DC converter with capacity filter in the load circuit is accomplished. To this end, a harmonic analysis is used. Expressions for determination of main quantities of the converter are obtained. The areas of commutation of the power switches are determined at both zero current and zero voltage.

*Keywords* – LCC resonant DC-DC converter.

# I. INTRODUCTION

Resonant converters are widely used in the building of power supply devices. It is largely due to their peculiar capacity for soft commutation of the power devices. Converters with second order resonant circuits are the most widespread among them and their resonant circuit comprises only an inductor and a capacitor. When their operating frequency is lower than the resonant one, the controllable switches commutate at zero current, i.e., Zero Current Switching (ZCS) occurs. Conversely, when the operating frequency is higher than the resonant one, this results in Zero Voltage Switching (ZVS). For obtaining better power features it is preferred to have the operating frequency higher than the resonant one.

Despite the great number of advantages, the converters with second order resonant circuits also reveal a considerable drawback – their disability to work in the whole range from a no-load to a short circuit state retaining at the same time the conditions of soft commutation of the controllable switches. This limitation can be overcome by the use of a higher order resonant circuit. Very good results are obtained if just one more reactive element – an inductor or a capacitor – is added to the circuit [1].

LCC converters have been lately used in building arc welders [2], power supplies of fluorescent lamps [3], lasers, etc. In this case, the load circuit behaves like a rectifier with a capacitive filter with respect to the resonant inverter.

Very often in theoretical studies of resonant converters harmonic analysis is applied. Besides, to obtain results with

<sup>1</sup>Alexandar S. Vuchev is with the Faculty of Electrical Engineering and Electronic, 26 Maritza st., 4002 Plovdiv, Bulgaria, E-mail: <u>avuchev@yahoo.com</u>

<sup>2</sup>Nikolay D. Bankov is with the Faculty of Electrical Engineering and Electronic, 26 Maritza st., 4002 Plovdiv, Bulgaria, E-mail: <u>nikolay\_bankov@yahoo.com</u>

<sup>3</sup>Georgi P. Terziyski is with the Faculty of Electrical Engineering and Electronic, 26 Maritza st., 4002 Plovdiv, Bulgaria, E-mail: <u>g\_terziyski@yahoo.com</u> reasonable accuracy, the influence of the first harmonics of the currents and voltages are only taken into account [4] - i.e. the "first harmonic analysis" method is typically used.

The purpose of this paper is studying an LCC resonant DC -DC converter with a capacitive filter in the load circuit by the first harmonic analysis. As a result of the analysis dependencies of basic quantities are to be obtained by which the limits of the converter efficiency while retaining soft commutation conditions of the controllable switches are to be defined.

### II. ANALYSIS OF THE CONVERTER

### A. General Assumptions

The circuit diagram of the converter under consideration is shown in Figure 1. It comprises an inverter (controllable switches  $S_1 \div S_4$  with reverse diodes  $D_1 \div D_4$ ), a resonant circuit



Fig.1. LCC resonant DC-DC converter with capacity load filter

(*L*, *C*<sub>1</sub> and *C*<sub>2</sub>), an uncontrollable *DC*-*DC* converter ( $D_5 \div D_8$ ), a capacitive filter (*C<sub>F</sub>*), and a load resistor (*R*<sub>0</sub>).

For the purposes of the analysis it is assumed that all elements in the circuit diagram are ideal (no losses in them), the power devices switch instantly and the pulsations of the power supply voltage  $U_d$  and the output voltage  $U_0$  are negligible. It is also assumed that, according to the chosen analytical method, the first harmonics of the currents and voltages only act in the circuit under consideration.

In converters with third order resonant circuits two resonant frequencies are observed [5]. For the considered converter, they are defined as follows:

$$\omega_0 = 1/\sqrt{LC_1}$$
  $\omega_{01} = 1/\sqrt{LC_1C_2/(C_1+C_2)}$ . (1)

The lower resonant frequency ( $\omega_0$  here) is chosen to be the basic one.

In this study it is assumed that the operating frequency of the converter is higher than the basic resonant frequency  $\omega_0$ .

In accordance with the above, by analogy with the analyses of converters wit second ordre resonant circuit, wave impedance and frequency distraction are determined:

$$\rho_0 = \sqrt{\frac{L}{C_1}} \qquad \nu = \frac{\omega_s}{\omega_0} \,, \tag{2}$$

where the operating frequency is denoted by  $\omega_s$ .

Using (2), for the impedances of the inductor and the capacitors of the resonant circuit it is obtained:

$$Z_L = j \nu \rho_0 \qquad Z_{C1} = -j \frac{\rho_0}{\nu} \qquad Z_{C2} = \frac{Z_{C1}}{a},$$
 (3)

where  $a = C_2 / C_1$  is the ratio between the capacitances of the two capacitors.

### B. Output Characteristic

In accordance with the assumptions made, the operation of the converter in the steady-state could be illustrated by the vector diagrams shown in Fig.2. The voltage vectors are shown by a continuous thin line while the current vectors are presented by a continuous thick line. By  $U_{ab}^{\&}$  the vector of the first harmonic of the inverter output voltage  $u_{ab}$  (between points *a* and *b*) is denoted, by  $U_{C2}^{\&}$  - the voltage  $u_{C2}$  over the capacitor  $C_2$ , and by  $U_{LC1}^{\&}$  - the voltage  $u_{LC1}$  over the inductor *L* and capacitor  $C_1$  (between points *a* and *c*). By  $f_{11}^{\&}$ ,  $f_{22}^{\&}$  and  $f_{33}^{\&}$  the vectors of the currents  $i_1$ ,  $i_2$ , and  $i_3$  are denoted, as shown in Figure 1. Angle  $\varphi$  indicates the phase shift of the current  $i_1$  with respect to the voltage  $u_{ab}$ , and angle  $\alpha$  – the phase shift between the voltages  $u_{C2}$  and  $u_{ab}$ .



Fig.2. Vector diagrams of the voltages and currents

To better visualize the analysis results in Fig.2 additional constructions are made (shown by means of a dotted line).

Using vector diagrams a system of equations describing the processes in the resonant circuit can be worked out:

After solution of the system an equation is obtained for the output circuit of the resonant inverter:

$$\boldsymbol{U}_{ab}^{\boldsymbol{k}} = \frac{\boldsymbol{U}_{C2}^{\boldsymbol{k}} (\boldsymbol{Z}_{L} + \boldsymbol{Z}_{C1} + \boldsymbol{Z}_{C2}) + (\boldsymbol{Z}_{L} \boldsymbol{Z}_{C2} + \boldsymbol{Z}_{C1} \boldsymbol{Z}_{C2}) \boldsymbol{I}_{2}^{\boldsymbol{k}}}{\boldsymbol{Z}_{C2}}.$$
 (5)

The operation of the power switches  $(S_1 \div S_4)$  and their antiparallel diodes  $(D_1 \div D_4)$  results in the rectangular shape of the voltage  $u_{ab}$ , its amplitude being equal to the supply voltage  $U_d$ . When the rectifier operates in a continuous current mode, the voltage  $u_{C2}$  also has a rectangular shape and its amplitude is equal to the output voltage  $U_0$ . Then the modules of the vectors of the first harmonics of  $u_{ab}$ ,  $u_{C2}$  and  $i_2$  are defined as:

$$U_{ab} = \frac{\sqrt{8}}{\pi} U_d \qquad U_{C2} = \frac{\sqrt{8}}{\pi} U_0 \qquad I_2 = \frac{\pi}{\sqrt{8}} I_0 \,. \tag{6}$$

Substitution of (3) and (6) in (5) and certain formal transformations lead to:

$$\frac{8}{\pi^2}U_d^2 = \left[\frac{\sqrt{8}(1+a-a\nu^2)}{\pi}\right]^2 U_0^2 + \left[\frac{\pi(1-\nu^2)}{\sqrt{8\nu}}\rho_0\right]^2 I_0^2.$$
 (7)

For obtaining summarized results all quantities are normalized as follows: voltages with respect to  $U_d$ ; and currents with respect to  $U_d/\rho_0$ . Thus, from equation (7) the expression of the normalized output characteristic of the converter is obtained:

$$1 = (1 + a - av^{2})^{2}U_{0}^{\prime 2} + \frac{\pi^{4}(1 - v^{2})^{2}}{64v^{2}}I_{0}^{\prime 2}.$$
 (8)

The above expression is an equation of an ellipse with semi-axes defined by the voltage of a no-load and the current of a short circuit:

$$U'_{0\max} = \frac{1}{1 + a - av^2} \qquad I'_{0\max} = \frac{8v}{\pi^2(v^2 - 1)}.$$
 (9)

Equation (8) shows that under certain conditions it is possible for the converter to behave like an ideal source of current. For this purpose it is necessary for the frequency distraction to obtain a limiting value:

$$v_{bound} = \sqrt{\frac{a+1}{a}} \,. \tag{10}$$

Then the output current is determined by the expression:

$$I'_{0bound} = \frac{8}{\pi^2} \sqrt{a(a+1)} \,. \tag{11}$$

The above equation shows that this value of the current depends only on the parameter a.

From (8), the dependence of the output voltage on the output current in relative units is easily obtained:

$$U'_{0} = \sqrt{\frac{\nu^{2} - \left[\left(\pi^{2} / 8\right)\left(\nu^{2} - 1\right)\right]^{2} I'_{0}^{2}}{\left(\nu + a\nu - a\nu^{3}\right)^{2}}} .$$
 (12)

In accordance with the above equation, a family of output characteristics obtained at a = I and at different values of the frequency detuning  $v \ge I$  is presented in Fig.3. With the exception of the boundary characteristic obtained at  $v = v_{bound}$ , all the other characteristics are arcs from ellipses. It can be seen that the characteristics obtained at  $v > v_{bound}$  are concentric to each other and for them the no-load voltage and the short circuit current decrease with the increase of the operating frequency. The characteristics obtained at  $v < v_{bound}$  intersect. In this case with the increase of the operating frequency the short circuit current decreases, but the no-load voltage increases.



Fig.3. Output characteristics of the converter

Fig.3 shows that in the plan of the output characteristics an operating area in which  $U'_0>1$  exist. Therefore, under certain conditions the output voltage is higher than the supplied one. This effect is observed for all characteristics obtained at  $1 < v \le v_{bound}$ . The output voltage can be higher than the supplied one at  $v > v_{bound}$ .

The normalized dependence of the output power alteration on the output current is obtained from equation (12):

$$P_0' = I_0' U_0' = I_0' \sqrt{\frac{\nu^2 - \left[ \left( \pi^2 / 8 \right) \left( \nu^2 - 1 \right) \right]^2 I_0'^2}{\left( \nu + a \nu - a \nu^3 \right)^2}} .$$
(13)

The study of the above expression shows the maximum value of the power:

$$P'_{0\max} = \sqrt{\frac{16\nu^2}{\pi^4 (\nu^2 - 1)^2 (1 + a - a\nu^2)^2}}, \qquad (14)$$

which could be obtained when:

$$U'_{0(P\max)} = 1/\left[\sqrt{2}(1+a-av^2)\right]$$
  

$$I'_{0(P\max)} = \sqrt{32}v/\left[\pi^2(v^2-1)\right]$$
(15)

A family of dependencies of the output power on the output current obtained when a=1 and with different values of the frequency distraction  $v \ge 1$  is presented on figure 4.



Fig.4. Dependencies of output power  $P_0$  on the output current  $I_0$ 

It is obvious that the characteristics have a clearly expressed maximum, the position of which shifts towards the beginning of the coordinates with the increase of the frequency distraction.

### C. Conditions of soft commutation

As mentioned earlier, an important feature of the resonant converter operation is the mechanism of commutation of the inverter power switches. This is related, on the one hand, to the way of controlling the power switches, and on the other hand, to the type of auxiliary commutation circuits. The required commutation mechanism can be determined based on the phase shift  $\varphi$  between the inverter voltage  $u_{ab}$  and the inverter current (in this case  $i_l$ ). When the voltage is ahead of the current the controllable switches commutate at zero voltage, i.e., zero voltage switching (ZVS) occurs. Otherwise, zero current switching takes place (ZCS). The angle  $\varphi$  can be determined based on the vector diagrams in Fig.2. Since the current  $i_1$  and the voltage  $u_{LC1}$  are shifted with respect to each other at an angle of  $90^{\circ}$ , the segment bd is a height in the triangle abc. Its length is easily expressed by the lengths of the sides of  $\Delta abc$ . Then the same angle  $\varphi$  can be expressed by the lengths of the sides in the rectangular triangle abd. Before that, however, the length of the side ac has to be found (voltage  $u_{LC1}$ ). This is possible by applying the cosine theorem. To find the phase shift  $\alpha$  between the vectors of the voltages  $u_{ab}$  and  $u_{C2}$  equation (5) is used. Substitution of (3) and (6) leads to:

$$\alpha = \arctan\left[\frac{\pi^2}{8} \frac{I_0'(1-\nu^2)}{U_0'(-a\nu^3 + a\nu + \nu)}\right].$$
 (16)

Then the final expression for the angle  $\varphi$  has the form:

$$\varphi = \arctan\left(\frac{8v^2 a + \pi^2 W_0' \sin \alpha}{\pi^2 W_0' \cos \alpha}\right). \tag{17}$$

Fig.5 presents dependencies of the angle  $\varphi$  on the output current  $I_0$ , obtained when a=1 for different values of the frequency distraction  $\nu \ge 1$ . It can be seen that when  $\nu > \nu_{bound}$ the angle  $\varphi$  always has a positive value. Consequently, the inverter voltage  $u_{ab}$  is ahead of the inverter current  $i_1$  and the controllable switches commute at zero voltage. When  $\nu < \nu_{bound}$ , with the increase in the output current  $I_0$  from 0 to  $I_{0max}$  the angle  $\varphi$  changes from  $-90^\circ$  дo  $+90^\circ$ . It means that whereas at heavy loads the controllable switches commute at zero voltage, at small loads switching is accomplished at zero current.



Fig.5. Dependencies of phase-shift angle  $\varphi$  on the output current  $I_0$ 

During the process of operation while retaining the conditions for soft commutation, the commutation mechanisms of the controllable switches cannot be easily changed. Therefore, if it is necessary to use ZCS, the range of load should be limited to the modes, close to a no-load, and the frequency distraction should not exceed  $v_{bound}$ . When the control devices have been chosen to switch at zero voltage the converter can operate in the whole range from a no-load to a short circuit. To maintain the conditions for soft commutation, however, the frequency distraction in the area of small loads should be higher than  $v_{bound}$ .

## **III.** CONCLUSION

A study of LCC resonant DC-DC converter with capacitive filter in the load circuit is accomplished by means of harmonic analysis, taking into account the influence of the firs harmonic only. It is assumed in the process of consideration that the converter operates with a higher frequency than the basic resonant frequency of the resonant circuit.

As a result of the analysis an expression for the output characteristic of the converter is obtained. Dependencies of characteristic angles of the phase shift between main quantities for the converter circuit under consideration are determined. Based on the obtained results the type of the output characteristics of the LCC converters is defined. It is shown that under certain conditions these converters behave like an ideal current source without using any additional resources (e.g., as with converters with second order resonant circuits, where current feedback is used). This mode is possible at a precisely defined frequency distraction that depends only on the values of the capacitors in the resonant circuit. It is established that the output voltage of the converter may have a higher value than the supply voltage. Some of the conditions under which this effect is observed are discussed.

It is established from the dependencies of the angle of phase shift between the voltage and the current of the inverter what the commutation mechanisms of the controllable converter device should be. Although the operating frequency is higher than the basic resonant frequency of the resonant circuit, from small loads the switches are supposed to commutate at zero current, i.e., zero current switching occurs. This effect is observed at frequency distraction lower than that at which the converter behaves like a current source. In the rest of the cases, the controllable devices should switch at zero voltage. The transition from ZVS to ZCS is mainly due to the change of the load and can be avoided by changing the operating frequency.

The study results can be used in designing LCC converters used as power supplies of electric arc welders, fluorescent lamps, lasers, etc.

# REFERENCES

- Batarseh I., "Resonant Converter Topologies with Three and Four Energy Storage Elements", IEEE Transactions on Power Electronics, Vol. 9, No. 1, January 1994 pp. 64-73.
- [2] Malesani, L., L. Mattavelli, L. Rossetto, R. Tenti, W. Marin, A. Pollmann, "Electronic Welder With High-Frequency Resonant Inverter", Industry Applications, IEEE Transactions, Vol. 31, Iss. 2, March/April 1995, pp. 273 279.
- [3] Alexandrov F.I., "Resonant Inverter Including Feed Back Circuit Having Phase Compensator and Controller", United States Patent, P.N. 7,045,966 B2, 16 May 2006.
- [4] De Simone S., C. Adragna, C. Spini, G. Gatavari, "Design-Oriented Steady State Analysis of LLC Resonant Converters Based on FHA", IEEE International Symposium on Power Electronics, Electrical Drives, Automation and Motion, Speedam 2006 pp. S41-16 – S41-23.
- [5] Haung G., A.J. Zhang, Y. Gu, "LLC Series Resonant DC-to-DC Converter", United States Patent, P.N. 6,344,979 B1, 5 February 2002.