# Optimal Distribution of PV Modules According to Dynamic Optimization Methods 

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#### Abstract

The article examines the possibility of an optimal location of photovoltaic modules of a photovoltaic power plant depending on the characteristics of the terrain. It is accented the possibility for determination of the maximum filling of a given territory, using various modules and inverters, but with different strings and that the number of modules in them. It is proposed to use the algorithm for dynamic optimization, which resizes a photovoltaic area. The target function is the maximum filling of the terrain, and with it a maximum installed capacity.


Keywords - PV system, dynamic optimization

## I. Introduction

One of the main stages in the design of photovoltaic plants is the placement of photovoltaic modules in the chosen field. Typically, the specialized software [5], [6] offers calculating the inverter, depending on the required electrical parameters of the construction system. Some of the application software offer desktop environments to determine the location of the photovoltaic modules, analysis of shading, etc. However, the designer is responsible for the desired location according to terrain's characteristics. Usual practice is to experiment with different possibilities. This approach is laborious and timeconsuming and the decision may not be optimal.
The paper examines the possibility to solve above described task by using validated algorithms for dynamic optimization. The resolve of the problem would increase the effectiveness of the design work and to improve the optimal use of terrain.
Use of dynamic optimization according to [1] requires:

- Optimal substructures of the decision - the decision of optimization task can be found as a function of optimal solutions of the subtasks.
- Overlapping subtasks - subtasks is calculated only once, thereby limiting the actual number of subtasks.

Obviously the application of dynamic programming problem in question is subject to analysis.

## II. ANALYSIS

The location of the modules over the terrain can be done differently. The decision of the task concerning the optimal
placement of photovoltaic modules on the selected area must consider:

- Determination of the terrain - it may be with an irregular geometric shape and / or divided into individual parcels. This complicates finding the optimal placement of the strings in order to best fill. In addition the requirement for insulation distances, paths, cable paths and others.
- Calculating the inverter - the ability to use several inverters with different parameters, which defines a different configuration of the strings. Thus needed optimal configuration for optimal filling can be selected.
- Defining the necessary photovoltaic modules - they may have different electrical and geometrical characteristics and they are grouped to a separate inverter. Thus the length of the strings can be changed. The ability to work with different constructions is particularly important when the area is with irregularly shaped parcels.
- Defining the parameters of the strings - the inverter allows a certain tolerance of the input variables, as they can define strings with different parameters.

Those features require reporting of possible combinations when working with different inverters and permitted parameters of strings.

Fig. 1 shows the design of a photovoltaic power plant, which uses the field of irregular shape. In order to optimal fill a territory it is parceled of the parts, in which the inverters and the strings with different parameters can be used. It is assumed that the parts are not predetermined and can be defined by the designer. In this sense, work environment can be considered as a variable.
Table 1 contains the parameters of used inverters. The number of PV in the string is determined by the tolerance in the $U_{D C}$, which allows working with different lengths. Thus a set of configurations applicable to different parts of the terrain is received.

An exemplary realization is made with Polycrystalline Framework modules. Sizing the model system was made according to validated literature sources [2], [3], [4] and aims to propose an experimental study of the applicability of the proposed calculation algorithms. Calculations are in the following sequence:

[^0]

Fig. 1. PV plant's project
A1 $\div$ A7 - Area of the whole terrain, divided into separate parcels
Minimum nominal (working) voltage:

$$
\begin{equation*}
\mathrm{U}_{\min (\tau T)}=\mathrm{U}_{\mathrm{mpp}(\mathrm{stc})}+\left(\mathrm{T}_{\mathrm{u}} \cdot\left(\tau_{T}-\tau_{S T C}\right)\right) \tag{1}
\end{equation*}
$$

$\tau_{\mathrm{T}}-$ temperature for calculations (module's temperature);
$\tau_{\text {stc }}$ - temperature of the module under standard testing conditions $\left(25^{\circ} \mathrm{C}\right)$
$\mathbf{U}_{\text {mpp(stc) }}$ - voltage at maximum power for standard testing conditions;
$\mathbf{T}_{\mathbf{u}}$ - temperature coefficient of the voltage;
For chosen modules at $\mathbf{U}_{\mathbf{m p p}(\mathrm{stc})}=35,5 \mathrm{~V}, \mathbf{T}_{\mathbf{u}}=-0,1665 \mathrm{~V} / \mathrm{C}^{0}$ and maximum temperature $\tau_{\mathbf{r}}=70^{\circ} \mathrm{C}$, the minimum voltage is:

$$
\mathrm{U}_{\min (70)}=35,5+(-0,1665 \cdot(70-25))=28,0075 \mathrm{~V}
$$

The maximum nominal voltage:

$$
\begin{equation*}
\mathrm{U}_{\max (\tau T)}=\mathrm{U}_{\mathrm{mpp}(\mathrm{stc})}+\left(\mathrm{T}_{\mathrm{u}} \cdot\left(\tau_{T}-\tau_{S T C}\right)\right) \tag{2}
\end{equation*}
$$

For chosen modules at minimum temperature $\tau_{\mathrm{r}}=-15^{\circ} \mathrm{C}$ :

$$
\mathrm{U}_{\min (-15)}=35,5+(-0,1665 \cdot(-15-25))=42,16 \mathrm{~V}
$$

The voltage of the open circuit:

$$
\begin{equation*}
\mathrm{U}_{0 \max (\tau T)}=\mathrm{U}_{0(\mathrm{stc})}+\left(\mathrm{T}_{\mathrm{u}} \cdot\left(\tau_{T}-\tau_{S T C}\right)\right) \tag{3}
\end{equation*}
$$

TABLE I
Parameters of the inverters Solarmax

| Инвертор | $\mathrm{P}_{\mathrm{DC} \text { max }}$ [KW] | $\mathrm{U}_{\mathrm{DC}}$ | $\begin{gathered} \hline \mathrm{U}_{\mathrm{DC} \max } \\ {[\mathrm{~V}]} \\ \hline \end{gathered}$ | $\begin{aligned} & \hline \hline \mathrm{I}_{\mathrm{DC}} \\ & \text { [A] } \\ & \hline \end{aligned}$ | $\mathrm{P}_{\mathrm{ACnom}}$ <br> [KW] | $\left[\begin{array}{l} \mathrm{P}_{\mathrm{ACmax}} \\ {[\mathrm{KW}]} \end{array}\right.$ | $\begin{aligned} & \hline \mathrm{I}_{\mathrm{AC}} \\ & {[\mathrm{~A}]} \\ & \hline \end{aligned}$ | $\mathrm{U}_{\mathrm{AC}}$, $\mathrm{f}[\mathrm{Hz}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|c\|} \hline \text { SolarMax } \\ 20 \mathrm{C} \\ \hline \end{array}$ | 24 | $\begin{gathered} 430 \\ \div \\ 800 \end{gathered}$ | 900 | $0 \div 48$ | 20 | 22 | $0 \div 31$ | $\left\lvert\, \begin{gathered} 3 \mathrm{X} \\ 400 \\ 50 \mathrm{~Hz} \end{gathered}\right.$ |
| SolarMax 35C | 45 |  |  | $0 \div 78$ | 35 | 38,5 | $0 \div 45$ |  |
| $\left\lvert\, \begin{array}{\|c\|} \hline \text { SolarMax } \\ 50 \mathrm{C} \end{array}\right.$ | 66 |  |  | $0 \div 120$ | 50 | 55 | $0 \div 77$ |  |
| $\left\lvert\, \begin{array}{\|c\|} \hline \text { SolarMax } \\ 80 \mathrm{C} \end{array}\right.$ | 105 |  |  | $0 \div 180$ | 80 | 88 | $0 \div 122$ |  |
| $\begin{array}{\|c\|} \hline \text { SolarMax } \\ 20 \mathrm{~S} \\ \hline \end{array}$ | 24 | $\left\|\begin{array}{c} 400 \\ \div \\ 800 \end{array}\right\|$ | 900 | $0 \div 48$ | 20 | 22 | $0 \div 31$ | $\begin{aligned} & 3 X \\ & 400 \end{aligned}$ |
| $\begin{array}{\|c\|} \hline \text { SolarMax } \\ 35 S \\ \hline \end{array}$ | 45 |  |  | $0 \div 78$ | 35 | 38,5 | $0 \div 54$ |  |

TABLE II
CALCULATING VOLTAGES OF POSSIBLE STRINGS.

|  | Limit <br> according <br> Table 1 | One <br> module | In string |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathbf{1 7}$ |  |  |  |
| $\mathbf{U}_{\text {min mpp }}$ [V] | 430 |  | 420,1 | 448,1 | 476,1 |  |
| $\mathbf{U}_{\text {max mpp }}$ [V] | 800 | 42,2 | 632,4 | 674,6 | 716,7 |  |
| $\mathbf{U}_{\text {max oc }}$ [V] | 900 | 51,7 | 774,9 | 826,6 | 878,2 |  |
| Applicable <br> inverters <br> (Table 1) |  | SolarMax <br> 20S | all | all |  |  |

Table III
MPOWER AND PERMITED NOMBERS OF THE STRINGS OF A INVERTER

| Number <br> of <br> modules | $\mathrm{P}_{\mathrm{DC}}$ [KW] AT NUMBER OF STRINGS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 15 | 8,4 | 12,6 | 16,8 | 21 | 25,2 | 29,4 | 33,6 | 37,8 | 42 |  | 46,2 | 50,4 | 54,6 | 58,8 | 63 | 67,2 | 71,4 | 75,6 | 79,8 | 84 |
| 16 | 8,96 | 13,44 | 17,92 | 22,4 | 26,9 | 31,4 | 38,8 | 40,3 | 44,8 |  | 49,3 | 53,8 | 58,2 | 62,7 | 67,2 | 71,7 | 76,2 | 80,6 | 85,1 | 89,6 |
| 17 | 9,52 | 14,28 | 19,04 | 23,8 | 28,5 | 33,3 | 38,1 | 42,8 | 47,6 |  | 52,4 | 57,1 | 61,9 | 66,6 | 71,4 | 76,2 | 80,9 | 85,7 | 90,4 | 95,2 |
| Inverter | SolarMax20C |  |  |  |  |  |  |  |  |  |  |  | - |  |  |  |  |  |  |  |
|  | SolarMax35C |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | SolarMax50C |  |  |  |  |  |  |  |  |  |  |  |  |  | - |  |  |  |  |  |
|  | SolarMax80C |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | SolarMax20S |  |  |  | - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | SolarMax35S |  |  |  |  |  |  |  | - |  |  |  |  |  |  |  |  |  |  |  |

$U_{0(\text { stc) })}$ - idling voltage.
For chosen modules $U_{0(s t c)}=45 \mathrm{~V}$ :

$$
\mathrm{U}_{\min (-15)}=45+(-0,1665 \cdot(-15-25))=51,66 \mathrm{~V}
$$

Maximum nominal current

$$
\begin{equation*}
I_{\max (\tau T)}=I_{\mathrm{mpp}(\mathrm{stc})}+\left(\mathrm{T}_{\mathrm{I}} \cdot\left(\tau_{T}-\tau_{\text {STC }}\right)\right) \tag{4}
\end{equation*}
$$

$\mathbf{I}_{\mathbf{m p p}(\text { stc) }}$ - nominal current at standard testing conditions;
$\mathrm{T}_{\mathrm{I}}$ - temperature coefficient of the current;
For chosen modules $\mathbf{I}_{\text {mpp(stc) }}=7,89 \mathrm{~A}, \mathrm{~T}_{\mathrm{I}}=0,00501 \mathrm{~A} / \mathrm{C}^{0}$ :

$$
I_{\max (70)}=7,89+(0,00501 \cdot(70-25))=8,115 A
$$

Geometrical parameters of selected modules are: $1.9 \times 1 \mathrm{~m}$. Parameters of possible strings (with 15,16 and 17 modules) are shown in Table 2. Whereas catalog PV power P (stc) $=$ $0,28 \mathrm{~kW}$ and the resulting current of $8,1 \mathrm{~A}$, the power of the three configurations of strings can be determined: P (15 modules) $=4,2 \mathrm{~kW}, \mathrm{P}$ ( 16 modules ) $=4,48 \mathrm{~kW}, \mathrm{P}$ (17 modules) $=4,76 \mathrm{~kW}$. The allowable number of strings to each inverter is shown in the table 3. It is obtained: SolarMax20S up to 5 strings, String SolarMax35S up to 9, etc.

The task can be reduced to the following:
A peak installed power of a photovoltaic power plant being built on a terrain with an irregular shape. In order to optimal sizing, the terrain is divided into individual parcels, which have capacity ( M ) of a number of modules ( N ), respectively certain maximum power. It is necessary to find the optimum amount of strings that can be placed on the terrain, taking into account the data from Table 1, 2,3. Put parameters are strings, numbers (m) and total power (r) (Table 3). Thus the optimal combination of inverters, working with allowable number of strings and which can fill most the terrain, need to be determined.

The task is reduced to finding the maximum amount [1]:

$$
\begin{equation*}
\sum_{i=1}^{N} x_{i} c_{i} \tag{5}
\end{equation*}
$$

At the limiting conditions:

$$
\begin{align*}
& \sum_{i=1}^{N} x_{i} m_{i} \leq M  \tag{6}\\
& c_{i}>0, m_{i}>0, x_{i} \in\{0,1\}, i=1,2 \ldots, n
\end{align*}
$$

The limitations over $\boldsymbol{c}_{\boldsymbol{i}}$ and $\boldsymbol{m}_{\boldsymbol{i}}$ are connected to the condition of the task; $\boldsymbol{x}_{\boldsymbol{i}}$ is the restriction relates to the sought decision.
It is necessary to define a recurrent target function, giving the decision to parcel with a capacity i.

$$
F(i)=\left\{\begin{array}{lr}
0 & \mathrm{i}=0  \tag{7}\\
\max \left[c_{j}+F\left(i-m_{j}\right)\right] & \mathrm{i}>0 \\
\mathrm{j}=1,2, \ldots, \mathrm{~N} ; \mathrm{m}_{\mathrm{j}} \leq i &
\end{array}\right.
$$

Iterative solution is obtained by successively calculating the target function. A realization is shown in listing 1. In the case of proposed PV system is sized using the "task of the backpack [1]:
Listing 1:
void opt_photovoltaic (void) \{
for ( $\mathrm{i}=1$; $\mathrm{i}<=\mathrm{M} ; \mathrm{i}++$ ) \{
/* Find the maximum value of the target function M - Displacement parcel */
$\operatorname{maxV}=\operatorname{maxI}=0$;
/*maxV - maximum achieved value */
/*maxI - index at which it is achieved */
for ( $\mathrm{j}=1 ; \mathrm{j}<=\mathrm{N} ; \mathrm{j}++$ ) \{ /*N - strings */
if (m[j] <= i \&\& !(set[i - m[j]][j >> 3] \& (1 << (j \& 7))))
$/ * m[]$-array of number of modules in the string according to Table 3
*/
if $(c[j]+\operatorname{Ph}[i-m[j]]>\operatorname{maxV})\{$
/*Ph - target function */
$\operatorname{maxV}=c[j]+\mathrm{Ph}[\mathrm{i}-\mathrm{m}[\mathrm{j}]] ;$
maxI $=\mathrm{j}$;
\}
if $(\operatorname{maxI}>0)\{/ *$ Check for lower index $m$ */

```
    Ph[i] = maxV;
    memcpy(set[i], set[i - m[maxI]], (N >> 3) + 1);
/*set[] - array with elements satisfying the target function */
    set[i][maxI >> 3] |= 1 << (maxI & 7);
}
    if (Ph[i] < Ph[i - 1]) {
/* The plot fits all strings */
    Ph[i] = Ph[i - 1];
    memcpy(set[i], set[i 1],(N >> 3) + 1);
    }
}
}
```

The use of recursive function is proposed in the listing 2 [1]: Listing 2:
void Photovoltaic (unsigned k)\{ /* Recursive functions */ unsigned i , bestI, fnBest, fnCur;
/* Initialization of variables */
/ * Calculates the largest value */
for (bestI = fnBest $=0, \mathrm{i}=1 ; \mathrm{i}<=\mathrm{N} ; \mathrm{i}++$ ) $\{$
if ( $k>=m[i]$ ) \{
if (if_not_calculated $==\mathrm{Ph}[\mathrm{k}-\mathrm{m}[\mathrm{i}] \mathrm{]})$ Photovoltaic(k -m[i]);
/*if_not_calculated - value -1 if the target function Ph is not calculated*/

```
        if (!set[k - m[i]][i])
```

        fnCur \(=c[i]+\operatorname{Ph}[k-m[i]] ;\)
        else
            fnCur \(=0\);
        if (fnCur > fnBest) \{
        bestI \(=\mathrm{i}\);
        fnBest = fnCur;
        \}
    \}
    \} /*registering the biggest value of the function*/
    Ph[k] = fnBest;
    if (bestI > 0) \{
    memcpy(set[k], set[k - m[bestI]], N);
    set[k][bestI] = 1;
    \}
    \}
void opt_photovoltaic (void) \{
/*Calculating the value of the function */
unsigned i, sumM; /* Initialization of variables */
memset(set, 0, sizeof(set));
/* Initialization of the multitudes of possible configurations */
for (i=0; i<=M; i++)
/* Initialization of the values of the target function */
Fn[i]= if_not_calculated;
for(sumM=m[1], i=2; i<=N; i++)
/* Check for all configurations */
sumM += m[i];
if ( $\mathrm{M}>=$ sumM) \{
printf("ln The field can accommodate all configurations ");
return;
\}
else \{
Photovoltaic (M);
/* computation of recursive function */
printf("\n configuration number:\n");
for ( $\mathrm{i}=1 ; \mathrm{i}<=\mathrm{N} ; \mathrm{i}++$ )
if (set[M][i])
printf("\%5u", i);
printf("\n Maximum value: \%u", Ph[M]);
\}
\}

Using the proposed algorithm gives the following results:

- parcel A1 (Fig. 1) - SolarMax80S inverter, PV system 20 string about 17 modules;
- A2 - SolarMax35S, 9 string of 16 modules;
- A3 - SolarMax35S, 9 string of 16 modules, SolarMax20, 5 string about 17 modules;
- A4 - SolarMax35S, 9 string about 17 modules;
- A5 - SolarMax35S, 9 string about 17 modules;
- A6 - SolarMax20S, 5 string about 17 modules, SolarMax20, 5 string of 16 modules;
- A7 - SolarMax50S, 14 string of 15 modules, SolarMax35, 8 string about 17 modules;
For given example at this distribution optimal fill the field in figure 1 is obtained. As seen from the decision in parcels A3, A6, A7 two different inverter are used with different numbers of strings and components in them. This was imposed by the typical geometric features of the parcel. Thus, the maximum duty cycle and the maximum installed capacity is determined by the necessary combination of strings. There may be other solutions that do not meet the target function:
- In A3 parcel instead of two inverter SolarMax35S (9 string of 16 modules) and SolarMax20 (5 string about 17 modules), using one SolarMax50;
- In A6 parcel instead of two inverter SolarMax20S (5 string about 17 modules) and SolarMax20 (5 string of 16 modules), using one SolarMax35;
- In the Land A7 - instead of two inverter SolarMax50S (14 string of 15 modules) and SolarMax35 (8 string about 17 modules) using one SolarMax80;


## III. Conclusions

Dynamic optimization methods are applicable in the design work of photovoltaic plants. The algorithms in the proposed listings provide a decision on the location of the modules for optimal use of terrain. Thus the eligible configurations inverter-number of strings- number of modules that they are determined. The proposed approach allows, by decision of the task set out to determine the maximum possible installed capacity of a given territory. Multivariate decision, limited by the target function, creates options for the type of each element with additional requirements and patterns.

## References

[1] Nakov P., P. Dobrikov. "Programming =++ Algorithms" Sofia 2003.
[2] Christopher L, Photovoltaics. Design and installation manual. Solr Energy international 2004.
[3] Messenger R. Photovoltaic Systems Engineering. Washingon DC. 2003
[4] Brooks Engineering, Advanced PV System. Design and Installation. 2008
[5] http://www.pvsyst.com/
[6] http://solarmax.com/


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