Game Theory Based Competitive Pricing in Next Generation Networks

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Abstract – **In this paper we research possibilities of using game theoretic approaches for computing competitive prices in next generation networks. Possible applications and comparison of various game theory models, such as Cournot, Bertrand and Stackelberg game models are presented. We emphasize the advantages and importance of game theory based competitive pricing.**

Keywords – **Next generation networks, Competitive pricing, Stackelberg model, Bertrand duopoly, Cournot duopoly.**

I. Introduction

High speed Internet access and advances in network technologies provide incentives for service providers (SPs) to become more efficient and competitive. Many authors explore the possibilities of applying game theory for solving challenges in next generation networks (NGN) [1-4]. Game theory encompasses a set of mathematical tools adressing complex interactions among rational players, which can be used to explain the operation of various complex telecommunication systems. Among wide range of problems in telecommunications covered by game theory, pricing and competition issues in NGN environment are very important ones. Moreover, the aim of achieving efficient pricing policy can be obtained using the appropriate mechanisms of game theory.

In this paper we discuss the possible applications of game theory models for competitive pricing in NGN. We focus our research on several particular models: Stackelberg model with price and quantity leadership, Bertrand model and Cournot model. We analyzed all the models for the same case of two competing SPs in NGN market.

This paper is organized in the following way. After the brief overview of pricing issues in NGN given in Section 2, game theory based competitive models are discussed in Section 3. In Section 4 examples of possible applications of those models for competitive pricing NGN services are explained. Concluding remarks are given in Section 5.

II. Pricing in Next Generation Networks

Pricing greatly affects the usage of services and the resources consumption. Likewise, competition can be highly influenced by the architecture of a network and the ability of players to control scarce resources in access network.

NGN should be designed to provide an open competition environment for SPs and to provide new possibilities for providers and users to exchange economic signals on fast time scales. A wide range of different pricing schemes is likely to be applied for competitive pricing in NGN [1, 4] and is expected that competition will force service providers to rapidly create and deploy different pricing concepts.

It is required that pricing in NGN enables both off-line and on-line charging, open mechanisms for charging and billing management, various charging and billing policies (e.g. fixed rate charging and usage based per-session charging and billing). It is also expected that accounting functions support services with multicast functionality and to enable all possible types of accounting arrangements, including transfer of billing information between SPs and e-commerce arrangements.

III. Game Theory Based Competitive Pricing Models

A. Basic game theory components and assumptions

Game theory is a field of applied mathematics that describes and analyzes interactive decision making situations and consists of a set of analytical tools that predict the outcome of complex interactions among rational players [5, 6]. Basic components of a game are players, the possible strategies of the players and consequences of the chosen strategies. The players are decision makers and their strategy choices result in a consequence or outcome. They try to ensure the best possible consequence according to their preferences. The preferences of a player can be expressed either with a utility function, which maps every consequence to a real number, or with preference relations, which define the ranking of the consequences.

The most fundamental assumption in game theory is rationality. It is assumed that rational players try to maximize their payoff. If the game is not deterministic, the players maximize their expected payoff. It is also assumed that the players know the rules of the game well.

In game theory, a solution of a game is a set of the possible outcomes. A game describes what strategies the players can take and what the consequences of the strategies are. The solution of a game is a description of outcomes that may emerge in the game if the players act rationally and intelligently. Generally, a solution is an outcome from which no player wants to deviate unilaterally.

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In telecommunications game theory can be applied for solving a wide range of problems such as congesting control, resource allocation, routing, QoS provisioning, network security, sharing radio-communication spectrum and pricing telecommunication services. Various game models are proposed for pricing telecommunication networks. Some of the most appropriate game models for solving problems of pricing telecommunication services are Nash bargaining game, Stackelberg leader-follower game, Bertrand game and Cournot game.

B. The Nash equilibrium concept

Nash equilibrium is the well known concept for determining solutions in game theory. It implies that each player chooses the best strategy analyzing all possible strategies of all other players in the game.

In two players game, the couple of strategies (s_1*, s_2*) represents a Nash equilibrium if s_1* is the best strategy for the first player when the other player uses strategy s_2* and if at the same time s_2* is the other player's best strategic choice for s_1*. Mathematically expressed, the couple of strategies (s_1*, s_2*) represents a Nash equilibrium under following conditions:

$$U_1(s_1*, s_2*) \geq U_1(s_1, s_2*) \text{ for all } s_1 \subset S_1 \text{ and} \quad (1)$$

$$U_2(s_1*, s_2*) \geq U_2(s_1*, s_2) \text{ for all } s_2 \subset S_2. \quad (2)$$

Nash equilibrium doesn't have to be represented by a single best strategy for each player in a game. It can be represented by a set of strategies for each player, such that none is interested in choosing a strategy from any other set that is different from the Nash equilibrium.

Subgame perfect Nash equilibrium is defined as a solution such that players' strategies form Nash equilibrium in every game that is a part of the original game.

Nash equilibrium doesn't exist in every game. An opposite case is a game with several Nash equilibrium points. In both cases the single optimal solution should be chosen. The most commonly used criteria for finding the optimal solution are Pareto efficiency and social optimality.

The solution of the game is Pareto efficient if unilaterally deviating from that solution can not lead to higher payoff for any player in the game without reducing payoff of at least one of the other players in the game. The goal of applied game theory is to form a game with Pareto efficient outcome.

The solution of the game should also satisfy the criterion of social optimality. In more complex games with a great number of players optimal solution from individual players' point is not necessarily the optimal solution from point of system in which the game is implemented. The optimal solution from system's standpoint is actually the socially optimal solution. It can be determined using appropriate optimization techniques. In order to match the optimal solution in terms of individual players with socially optimal solution, pricing concepts with main objectives of system's revenue optimization and encouraging the efficient use of resources are widely used in computer and telecommunications networks. The solution that combines the goals of individual players and the system can be determined by means of appropriate pricing scheme. Pricing mechanism is considered to be incentive if it accomplishes both objectives. For that purpose usually dynamic or hybrid pricing schemes are considered. In those schemes users are charged in accordance to actual use of resources.

C. Stackelberg model

Stackelberg game is two-level optimization model in which at least one player is defined as the leader who chooses a strategy before other players defined as followers. Stackelberg model is an example of a dynamic or sequential game of perfect information. It is a game in which each player knows both the pay-off structures and the history of the game and can observe the actions of others before deciding upon his own optimal response [7].

In Stackelberg game there is a certain order in the decision-making process. Followers' decisions about their strategic choices are based on the strategy that was previously chosen by leader. Stackelberg game can be played with either price leadership or quantity leadership. They can provide a good base for defining prices in NGN especially in case of network congestions.

Regardless of whether the model is played with price leadership or quantity leadership, the problem of finding the optimal strategy for a leader has to be solved, considering that followers as rational players tend to optimize their utility functions with given leader's action.

In Stackelberg game interaction between leader and followers tend to be dynamic [8, 9]. The leader may choose a strategy with aim of maximizing his revenue assuming that the followers will choose their strategies to maximize their own utility functions i.e., best answers. The solution obtained in this way is called the Stackelberg equilibrium and can be analyzed by a backward induction method, which firstly considers the best answers of followers. The best responses of the followers in this game can be obtained as follows: with a given leader's action, total users' demand can be determined based on followers' utility functions. Then these best responses are used to calculate the leader's revenue and the leader chooses a strategy that maximizes his revenue. The equilibrium is achieved at the point of intersection of all the best responses.

D. Bertrand model

Bertrand model is a static or simultaneous game of complete information. In such a game players simultaneously choose their strategies and each player's pay-off structure is common knowledge during the game.

In Bertrand duopoly two players compete in terms of the price they charge users, rather than quantity levels. Thus, in Bertrand duopoly, the strategic variable is the price charged in the market. Players simultaneously decide their pricing structures and market forces then decide about demand share for each player.

In Bertrand duopoly model, stability depends on whether or not services or products sold by competing firms are identical. In case of identical services/products a Nash equilibrium exists only if both competing firms charge the same prices and make normal profits [7]. The situation where players (i.e.firms) make a competitive outcome without any collusion to increase profits above the normal, is known as Bertrand paradox. One way to overcome the Bertrand paradox is to have firms sell distinguishable services/products. In that case, Nash equilibrium with different equilibrium prices can be obtained [7, 10].

E. Cournot model

Like Bertrand duopoly, Cournot duopoly is a static game, but one in which two players compete in terms of quantity levels, rather than the price they charge users. In Cournot model players choose their strategies independently and simultaneously and each player announces as his strategic choice the quantities of services that he intends to supply.

Cournot duopoly describes how two players selling the same service/product can settle on their respective output levels so as to maximise their own profits. With aim of determining Nash equilibrium, a reaction function has to be defined for each player. Reaction function of a player in Cournot game is a curve that shows his every optimal quantity level for every possible quantity level of the other player. Prices adjust in response to the aggregate supply, so that all the quantity can be sold, and each player obtains a proportional amount of outlay.

IV. Possible Applications of Game Theory Based Competitive Pricing Models

Here we consider possible applications of game theoretic approaches described in previous Section on determining competitive prices of NGN services. We give examples of Stackelberg, Bertrand and Cournot games:

1. Stackelberg duopoly model with price leadership,
2. Stackelberg duopoly model with quantity leadership,
3. Bertrand duopoly model and
4. Cournot duopoly model.

Players in games are two SPs and assumptions are as follows:

- set of possible prices is $[0, 1]$,
- quantity level in function of service price is: $x_i(M_i) = 1 - M_i$ and
- marginal cost of provided services in function of quantity level is: $c_i(x_i) = x_i^2$.

Stackelberg model can be convenient for determining competitive prices at the telecommunication market where some SPs have a competitive advantage over the others (because of technological, historical or legal reasons, or just because their entry was not possible at an earlier stage). SPs with a competitive advantage will act as leaders in Stackelberg game and therefore they will be able to choose their strategies before other SPs who will act as followers.

In Stackelberg game with price leadership, a leader (or leaders) proposes the service price. Based on that price followers decide about their prices. In this paper we explain an example of Stackelberg game with price leadership with two SPs: SP_1 is the leader who commits to price M_1 and SP_2 is the follower. SP_2 will take the leader's price as given, decreasing it by an infinitesimal amount and choosing his quantity level x_2, to maximize $M_2\ x_2 - x_2^2$, giving $x_2 = M_2/2$. Nash equilibrium is obtained for $M_1 = M_2 = 8/15$, $x_1 = 3/15$ and $x_2 = 4/15$. This result confirms the fact that in price leadership model a second mover has advantage over the leader. In this game leader obtains less profit than its follower. That is the reason Stackelberg game with price leadership is less frequently used compare to quantity leadership model.

In Stackelberg game with quantity leadership, a leader (or leaders) proposes the service quantity level. After that, followers decide about quantity level they will provide to their users. Here, we use the same example as for the previously explained price leadership model with two SPs. Initial assumption is: SP_1 commits to supply a quantity x_1. After SP_2 observe this, he chooses to supply x_2 with aim of maximizing $x_2(1-x_1-x_2)$, i.e. $x_2(x_1) = 1/2(1-x_1)$. Hence, SP_1 should choose x_1 to maximize $x_1(1 - x_1 - x_2(x_1))$, which gives $x_1=1/2$. Thus Nash equilibrium in this game is: $(x_1, x_2)=(1/2,1/4)$. This result indicates that the leader, i.e. SP_1 has advantage over the follower, i.e. SP_2, which holds in all Stackelberg games with quantity leadership.

In Stackelberg pricing problem with a provider as leader and users as followers there are two levels of decision making process: tariff setting by a provider, and then selection of the best alternatives by users. This game can be formed with a large number of followers, i.e. users and it can be based on both price and quantity leadership. For determining the Stackelberg equilibrium in such a game there is no unique procedure. Various authors propose different solutions [3, 8, 9, 11] based on Stackelberg model of interaction between leader and followers.

The solution of Bertrand game with two SPs and different marginal costs: $c_1<c_2$, which are known to both providers, can be defined by Nash equilibrium. Nash equilibrium in Bertrand duopoly is represented by the prices SPs charge their users for using the service: $M_2 \in (c_1, c_2]$ and $M_1 = M_2 - \varepsilon$ (M_1 and M_2 are the service prices provided by SP_1 and SP_2, respectively and ε is infinitesimally small positive value). This Nash equilibrium confirms the fact that if SP_1 charges his users with price M_1, such that $M_1> c_1$, SP_2 has no incentive to deviate from M_2, because any reduction in price below marginal cost would mean a loss for him. SP_1 maximizes his profit by taking M_1 only slightly lower than M_2. Hence, SP_1 will always win with a net benefit equal to approximately $M_2 - c_1$ per unit sold. Since none will offer the price below the marginal cost, for SP_2 the strategy $M_2=c_2$ dominates the strategy $M_2=M'_2$ for all $M'_2< c_2$, i.e. the first strategy is as good or better than the second, for all values of M_1. Thus, by imposing the constraint $M_2 \geq c_2$, we conclude that $(M_1, M_2) = (c_2 - \varepsilon, c_2) = (x_2^2 - \varepsilon, x_2^2)$ is the Nash equilibrium of this Bertrand game. Following the same assumptions as in the previously explained Stackelberg game models, Nash equilibrium in the Bertrand game can also be interpreted in terms of quantity levels: (x_1, x_2)=(1/4,1/4).

Generally, in Bertrand game with more than two players with equal and constant marginal cost, the optimal solution does not dependent on the number of players and corresponding price for each player is equal to marginal cost.

In Cournot model with two SPs offering the same service with total quantity level $x = x_1 + x_2$, where quantity level of SPi is x_i for i =1,2, the resulting price in the market will be $M(x)$. Each provider must choose an amount of output to be produced, and then, as a function of both choices, receive a payoff (that is his net benefit). SPi's net benefit can be written as: $\pi_i(x_1,x_2)=M(x_1+x_2)x_i-c_i(x_i)$, where $c_i(x_i)$ is his cost of providing the quantity x_i. The Nash equilibrium in this game is the pair of outputs (x_1^*, x_2^*) with the property that if SPi chooses x_i^*, then there is no incentive for SPj to choose other than x_j^*, where $i, j \in \{1,2\},\ i \neq j$.

To obtain reaction functions x_i (x_j) for each SPi, the first order conditions are defined:

$$\partial\pi_1(x_1, x_2)/\partial x_1 = M(x_1 + x_2) + M'(x_1 + x_2)x_1 - c'_1(x_1) = 0 \quad (3)$$

$$\partial\pi_2(x_1, x_2)/\partial x_2 = M(x_1 + x_2) + M'(x_1 + x_2)x_2 - c'_2(x_2) = 0 \quad (4)$$

The Nash equilibrium point can be found in the intersection of these curves. It can be shown that with the adoption of appropriate assumptions for the reaction curves (such as concavity), the optimal solution is always stable. This means that if the game is played in several rounds and if the players sequentially choose their strategies depending on the previous moves of other players, then their outputs converge to a Nash equilibrium point.

In our example of Cournot game x_i is chosen within the interval [0,1] and the inverse demand curve is: $M(x_1 + x_2) = 1 - (x_1 + x_2)$. Then: $x_i(x_j) = 1/2(1 - x_j)$ and the Nash equilibrium is: (x_1, x_2)=(1/3,1/3).

The solution of Cournot game compare to Stackelberg model with price leadership provides higher quantity level for both SPs. On the other hand, Stackelberg model with quantity leadership provides lower quantity level for SP_2, but higher quantity level for SP_1 compare to Cournot model. The given solution in Bertrand game is only better for SP_1 in comparision with the one obtained in Stackelberg model with price leadership.

V. CONCLUSION

Analysis of recent research in the field of pricing telecommunications services has shown that research efforts are increasingly directed to the use of game theory mechanisms for defining the tariff system in telecommunication networks.

For the analysis of the tariff system and the relationship between competitive service providers, Stackelberg, Bertrand and Cournot, models can be used. In this paper we analyzed those game theoretic approaches for the same case of two competing SPs in NGN market. Bertrand model is suitable for solving the problems of determining service prices. Cournot model is convenient for modeling strategic choices of service providers that focus on the services quantity levels. Stackelberg model can be applicable to solve the problem of determining prices and/or service quantity level a provider offers to his users.

ACKNOWLEDGEMENT

This work is partially supported by the Ministry of the Science and technological development of the Republic of Serbia with the project TR32025.

REFERENCES

[1] B. M. Ninan, M. Devetsikiotis, "Game-Theoretic Resource Pricing For The Next Generation Internet", *Performance Evaluation and Planning Methods for the Next Generation Internet*, edited by A. Girard, B. Sanso and F. Vazquez Abad, Springer, 2005.

[2] Z. Ji, W. Yu, and K. J. R. Liu, "A Game Theoretical Framework for Dynamic Pricing-Based Routing in Self-Organized MANETs", *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1204-1217, 2008.

[3] P. Briest, M. Hoefer, and P. Krysta, "Stackelberg Network Pricing Games", *Proc. of the STACS 2008*, vol. 1, pp. 133-142, 2008.

[4] V. Radonjić, V. Aćimović-Raspopović, "Responsive Pricing Model with Fixed Bandwidth Usage for the Next Generation Internet", *Proc. of the ICEST 2008*, vol. 2, pp. 425-428, Nis, Serbia, 2008.

[5] M. J. Osborne, *An Introduction to Game Theory*, Oxford University Press, 2004.

[6] J. N. Webb, *Game Theory - Decisions, Interaction and Evolution*, Springer, 2007.

[7] A. Kelly, *Decision Making Using Game Theory - An Introduction for Managers*, Cambridge University press, New York, USA, 2003.

[8] P. Y. Nie, "Dynamic Stackelberg Games under Open-Loop Complete Information", *Journal of the Franklin Institute-Engineering and Applied Mathematics*, 342, pp. 737-748, 2005.

[9] S. V. Hoesel, "An Overview of Stackelberg Pricing in Networks", *European Journal of Operational Research*, vol. 189, no. 3, pp. 1393-1402, 2008.

[10] C. Courcoubetis, R. Weber, *Pricing Communication Networks*, John Wiley & Sons Ltd, 2003.

[11] T. Basar, R. Srikant, "A Stackelberg Network Game with a Large Number of Followers", *Journal of Optimization Theory and Applications*, vol.115, no.3, pp.479–490, 2002.