Properties of Two Traffic Models with Changed Serving Intensities in Alternative Groups

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Abstract – **In this paper the two systems with overflow possibilities and changing serving intensities are investigated. A primary group with two Poisson traffics is considered whereupon the rejected calls from one of them are directed to the alternative groups and served with changed serving intensities. Generating function technique is used for the analytical solving model with secondary and ternary group and model which treated separately the channels with ordered selecting in secondary group. The analysis of the importance for using this models and comparison with the model with same serving intensities of overflow traffics is carried out.**

Keywords – **Changed serving intensity, Generating function, Ordered selecting, Overflow traffic, Traffic parameters**

I. Introduction

This paper deals with problems of traffic parameters analytical determination in the systems with overflow traffic which have the possibility of changing serving intensity. The models with overflow traffic became relevant in the second half of the last century by introducing the alternative routing in telecommunication networks. For the fundamental work in this field [1] can be considered and in [2] a substantial mathematical tool for the analytical solving of steady state equations with the generating function technique can be found.

Alternate path routing of overflow traffic has been widely used to improve the performance of hierarchical networks, including traditional circuit-switched networks [5], networks with dynamic routing [6], broadband networks, IP networks and hierarchical cellular networks [8]. Similar methods can also be used to improve traffic performance in heterogeneous wireless networks which support internetworking overflow. Traffic loss analysis in such networks is a challenging issue due to intensive computations involved, particularly in the cases where various services with multi-rate and multi-quality of service are supported [8]-[11].

The problem of the changed serving intensity is treated in several works as in [3] and [4]. Paper [7] seeks to provide the analysis of a model with overflow traffic components that originate from several Poisson traffics served by a common primary group. The overflow parts of the two of them are served in a secondary group but with changed and different serving intensities. The general solution of the system was obtained by generating function techniques and compared with the simpler model.

The interest for the analysis of the models with this characteristics results from the real needs. Besides the cases described in [3], [4] and [7], it should be noted that in some cases, this phenomenon can be reiterated, too. Characteristical examples are call centers, with operators using different average time for performing the same works, or the case of combined operator's and interactive voice response systems engagement. These models have special importance in the case of heterogeneous networks, when the alternating resources with different basic bandwidth unit, i.e. different serving intensity, are going to be chosen.

Call-level multi-rate teletraffic loss models aim at assessing the call-level QoS of IP based networks with resource reservation capabilities. This assessment is important for the bandwidth allocation among service classes (guaranteeing QoS), the avoidance of too costly over-dimensioning of the network and the prevention, through traffic engineering mechanisms, of excessive throughput degradation. Despite of its importance, the call-level performance modeling and QoS assessment is a challenge in the highly heterogeneous environment of next generation networks, due to the presence of elastic traffic, or complicated call arrival process. With the case of multi-rate traffic, the basic method for determining traffic characteristics is the application of the so-called Kaufman-Roberts formulas and their modifications [12].

This paper is organized as follows. We start with the analytical solution of the first model with secondary and ternary group where the intensity of serving is changed and the process of determining the unknown coefficient is proposed. Presence of the ternary group represents the extension of the model proposed in [7]. The second is the model where the intensity of serving is changed in each channel of the secondary group with ordered selecting. A general solution is also proposed and the procedure for determining the unknown coefficients is explained. Two Poisson traffics in primary group are observed in both models since that generalization does not complicate the solution. Both models are illustrated with simple unique numerical example and the influence of serving intensity change on losses is analyzed.

II. Model with Secondary and Ternary Group and Changed Serving Intensity

The serving system with two (or more) Poisson traffics, a_1 and a_2, with serving intensity $\mu_0 = 1$, offered to the primary group having c channels, whereupon the rejected calls of one of the traffics are offered to the secondary and ternary group,

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with s_1 and s_2 channels and with changed serving intensities, μ_1 and μ_2, is of interest for the analytical research (Fig. 1).

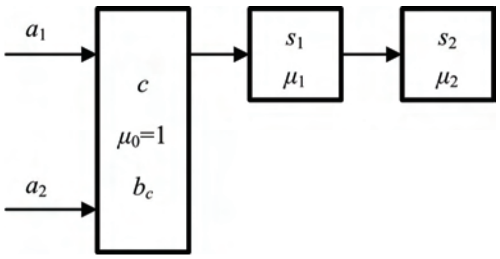

Fig. 1. Model with secondary and ternary group.

The model is described by $(c + 1)(s_1 + 1)(s_2 + 1)$ steady state equations. General solution for the state probabilities is reached by generating function technique, similar with [2], [7]

$$p(m,n_1,n_2) = (-1)^{n_1+n_2} \times \sum_{k_1=n_1}^{s_1} \binom{k_1}{n_1} \sum_{k_2=n_2}^{s_2} \binom{k_2}{n_2} R_{\mu_1 k_1+\mu_2 k_2}(m) C(k_1,k_2), \quad (1)$$

where (m, n_1, n_2) are states in groups with capacities (c, s_1, s_2). Here states represent the number of occupied channels. Also, relations

$$R_0(m) = \frac{(a_1+a_2)^m}{m!} \quad (2)$$

and

$$R_{(\mu n)}(m) = \sum_{i=0}^{m} \binom{(\mu n)+i-1}{i} \frac{(a_1+a_2)^{m-i}}{(m-i)!}, \quad n > 0. \quad (3)$$

are obtained. It should be noted that $(\mu n) = \sum \mu_i n_i$.

With further $R_{(\mu n)}(c) = R_{(\mu n)}$, from the boundary steady state equations $(m = c)$, using marginal generating functions technique, a recurrent formula for coefficients $C(n_1, n_2)$ is derived:

$$\begin{aligned} &-(\mu_1 n_1 + \mu_2 n_2) R_{\mu_1 n_1+\mu_2 n_2+1} C(n_1,n_2) \\ &= a_1 R_{\mu_1(n_1-1)+\mu_2 n_2} C(n_1-1,n_2) \\ &+ a_1 (-1)^{s_1+n_1} \binom{s_1}{n_1-1} R_{\mu_1 s_1+\mu_2 n_2} C(s_1,n_2) \\ &+ a_1 (-1)^{s_1+n_1} \binom{s_1}{n_1} R_{\mu_1 s_1+\mu_2 (n_2-1)} C(s_1,n_2-1) \\ &+ a_1 (-1)^{s_1+n_1+s_2+n_2} \binom{s_1}{n_1} \binom{s_2}{n_2-1} R_{\mu_1 s_1+\mu_2 s_2} C(s_1,s_2). \end{aligned} \quad (4)$$

With this procedure the problem is reduced to solving $(s_1 + 1)(s_2 + 1)$ equations for unknown $C(n_1, n_2)$. However, having in mind the fact that from the normalized condition for state probabilities $C(0, 0) = 1/R_1$ there is a possibility to find solution for $C(s_1, s_2)$ from the recurrent formula (4), then $C(s_1, n_2)$, and finally to express all the rest $C(n_1, n_2)$.

It is possible to define more parameters for this model and primary is the call loss of traffic a_1 that is in the form

$$b_{cs12} = p(c,s_1,s_2) = (-1)^{s_1+s_2} R_{\mu_1 s_1+\mu_2 s_2} C(s_1,s_2). \quad (5)$$

The call loss of traffic a_1 in the secondary group is

$$b_{s1} = p(c,s_1)/b_c = (-1)^{s_1} R_{\mu_1 s_1} C_{s1} R_1 / R_0, \quad (6)$$

where $b_c = B(a_1+a_2, c) = R_0 / R_1$ is Erlang loss in primary group, while in ternary we have $b_{s2} = p(c,s_1,s_2)/p(c,s_1)$. Coefficient C_{s1} can be obtained from explicit solution for system without ternary group [7]

$$C_{s1} = \frac{\dfrac{(-a_1/\mu_1)^{s_1}}{s_1! R_{\mu_1 s_1}} \displaystyle\prod_{i=0}^{s_1} \frac{R_{\mu_1 i}}{R_{\mu_1 i+1}}}{\displaystyle\sum_{n=0}^{s_1} \frac{(a_1/\mu_1)^{s_1-n}}{(s_1-n)!} \prod_{i=n+1}^{s_1} \frac{R_{\mu_1 i}}{R_{\mu_1 i+1}}}, \quad \prod_{i=s_1+1}^{s_1} \frac{R_{\mu_1 i}}{R_{\mu_1 i+1}} = 1. \quad (7)$$

The following expressions can be used for determining the time loss in secondary

$$b_{ts1} = \sum_{m=0}^{c} \sum_{n_2=0}^{s_2} p(m,s_1,n_2) = (-1)^{s_1} R_{\mu_1 s_1+1} C_{s1} \quad (8)$$

and ternary group

$$b_{ts2} = \sum_{m=0}^{c} \sum_{n_1=0}^{s_1} p(m,n_1,s_2). \quad (9)$$

Traffic a_1 is offered to primary group, its calls formed traffic a_{1o} offered to the system

$$a_{1o} = a_1(1-b_c) + a_1 b_c (1-b_{s1})/\mu_1 + a_1 b_c b_{s1}/(\mu_1 \mu_2), \quad (10)$$

while the carried traffic is

$$a_{1c} = a_1(1-b_c) + a_1 b_c (1-b_{s1})/\mu_1 + a_1 b_c b_{s1} (1-b_{s2})/(\mu_1 \mu_2). \quad (11)$$

Now, the traffic loss in the system for a_{1o} can be obtained in the form

$$b_{a1o} = 1 - \frac{a_{1c}}{a_{1o}}. \quad (12)$$

It is obvious that the traffic offered to the system increases during the decreasing of serving intensity and vice versa.

In the bitrate and capacity domains, if arrival rate λ_i, $i = 1, 2$, is expressed in terms of calls per serving time $t_0 = 1/\mu_0$, equality $a_i = \lambda_i$ holds. The primary resources are of the capacity C, the mean value of information units per call r, while the basic bandwidth is b. Thus, the primary resource represents a group of $c = C/b$ channels, while serving intensity is $\mu_0 = b/r$.

In secondary and ternary resources with capacities S_i, $i = 1, 2$, basic bandwidths are b_i, while the corresponding groups has got $s_i = S_i/b_i$ channels and the relative serving intensities are $\mu_i = b_i/b$.

III. Model with Changed Serving Intensities in Secondary Group Channels

The previously used technique can also be applied for the case when the channels in secondary group are ordered selected and own serving intensity exists for each of them (Fig. 2).

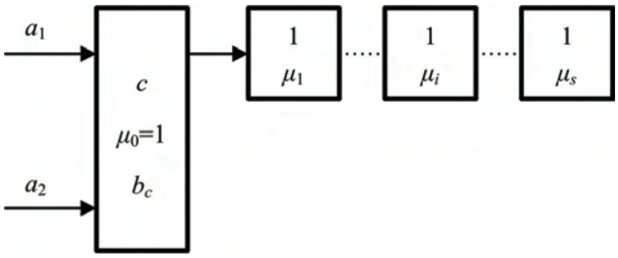

Fig. 2. Model with changed serving intensities in secondary group channels

Using generating function technique, we obtained the solution for state probabilities

$$p(m, n_1, ..., n_s) = (-1)^{n_{1s}} \sum_{k_1=n_1}^{1} ... \sum_{k_s=n_s}^{1} R_{(\mu k)_{1s}}(m) C(k_1, ..., k_s), \quad (13)$$

where $n_{ij} = n_i + \ldots + n_j$ and $(\mu n)_{ij} = \mu_i n_i + \ldots + \mu_j n_j$ (= (μn) for using (3)).

The equation system for unknown coefficients $C(n_1, \ldots, n_s)$ obtained from the boundary equations with marginal generating function technique is

$$\begin{aligned} -(\mu n)_{1s} R_{(\mu n)_{1s}+1} C(n_1, ..., n_s) &= a_1 \sum_{i=1}^{s} ((-1)^{i-1+n_{1(i-1)}} \\ &\times R_{\mu_{1(i-1)}+\mu_i(n_i-1)+(\mu n)_{(i+1)s}} C(1, ..., 1, n_i - 1, n_{i+1}, ..., n_s) \\ &+ (-1)^{i+n_{1i}} \binom{1}{n_i - 1} R_{\mu_{1i}+(\mu n)_{(i+1)s}} C(1, ..., 1, n_{i+1}, ..., n_s)), \end{aligned} \quad (14)$$

where $n_{10} = 0$, $n_{(s+1)s} = 0$ and $C(0, \ldots, 0) = 1/R_1$. The number of equations now is 2^s - 1 instead of $(c + 1)2^s$ in the case of solving system steady state equation.

Based on the fact that

$$p(c, n_1, ..., n_{j-1}) = p(c, n_1, ..., n_{j-1}, 0) + p(c, n_1, ..., n_{j-1}, 1) \quad (15)$$

and using (13), it can be concluded that the equality $C(n_1, \ldots, n_{j-1}, 0) = C(n_1, \ldots, n_{j-1})$ holds. For $j = 2, 3, \ldots, s$ the system of 2^{j-1} equations

$$\begin{aligned} -(\mu n)_{1j} R_{(\mu n)_{1j}+1} C(n_1, ..., n_{j-1}, 1) &= a_1 \sum_{i=1}^{j} ((-1)^{i-1+n_{1(i-1)}} \\ &\times R_{\mu_{1(i-1)}+\mu_i(n_i-1)+(\mu n)_{(i+1)j}} C(1, ..., 1, n_i - 1, n_{i+1}, ..., n_j) \\ &+ (-1)^{i+n_{1i}} \binom{1}{n_i - 1} R_{\mu_{1i}+(\mu n)_{(i+1)j}} C(1, ..., 1, n_{i+1}, ..., n_j)), \end{aligned} \quad (16)$$

can be solved iteratively. This iterative procedure requires solving 2^{s-1} equations for unknown coefficients $C(n_1, \ldots, n_s)$ in the last step which, compared to the former case, makes only half of the equations. That is important for calculating time and convergence.

In the case of the same serving intensity in secondary group with j channel, $\mu_j = \mu$, $j = 1, \ldots, s$ expression for C_j from [7] can be used, so $C(1, \ldots, 1_{(j)}) = C_j$ and we must solve the system with $2^{j-1} - 1$ equation, i.e. $2^{s-1} - 1$ in the last step, if we wish to determine microstate probabilities. For $c = 1$, $a_2 = 0$ and $\mu = 1$ we could have a classical system with $(s + 1)$ channel and ordered selecting. If we use the steady state equations for determining the microstate probabilities, we need to solve 2^{s+1} equation which is around four times more than in the last step of developed procedure. The losses for these cases have explicit solutions [7].

IV. Numerical Example

As an example, simple but enough illustrative case is analyzed when $c = 10$, $s_1 = 1$, $s_2 = 1$ channel and that results from the first and second model. Then, (4) or (14) will be in the form

$$\begin{aligned} -(\mu_1 n_1 + \mu_2 n_2) R_{\mu_1 n_1 + \mu_2 n_2 + 1} C(n_1, n_2) &= a_1 (R_{\mu_1(n_1-1)+\mu_2 n_2} \\ &\times C(n_1 - 1, n_2) - (-1)^{n_1} \binom{1}{n_1 - 1} R_{\mu_1+\mu_2 n_2} C(1, n_2) - (-1)^{n_1} \\ &\times R_{\mu_1+\mu_2(n_2-1)} C(1, n_2 - 1) + (-1)^{n_1+n_2} \binom{1}{n_2 - 1} R_{\mu_1+\mu_2} C(1,1)) \end{aligned} \quad (17)$$

From the relation (17) for n_i the system of tree equations is formed, while $C(1, 0)$ is dependent only on coefficient $C(0, 0)$ and is equal with C_1 from (7).

Fig. 3 illustrates call losses in alternate groups with one channel for $a_1 = 6$ Erl., $a_2 = 2$ Erl. and different values of μ_1 and μ_2. The mutual relation between the losses curve b_{s2} for $\mu_2 = \mu_1$ and b_{s1} is characteristical and correspond to the simpler model from [7]. For different μ_2 there exist the values $\mu_1 > \mu_2$ when b_{s2} becomes greater then b_{s1}. For $\mu_1 = 0$ ternary group is going on secondary.

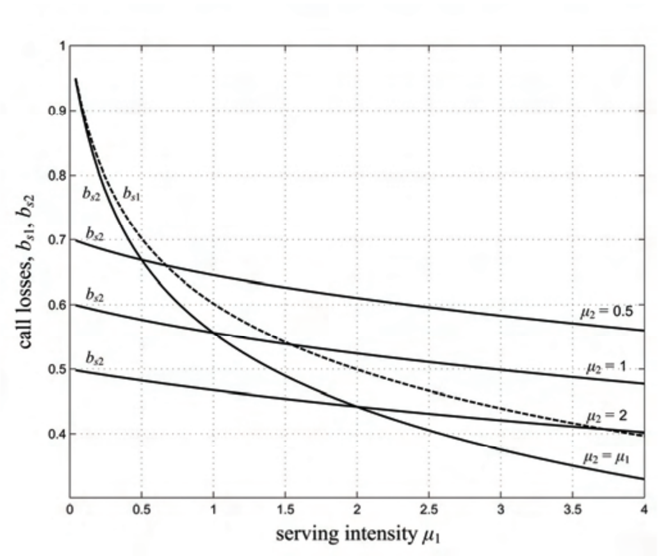

Fig. 3. Call losses in secondary and ternary group

Fig. 4 illustrates significant losses discrepancy concerning the difference of serving intensities. In this case, traffic loss in the system b_{a1o} is more sensitive to serving intensities changes than call losses b_{cs12}.

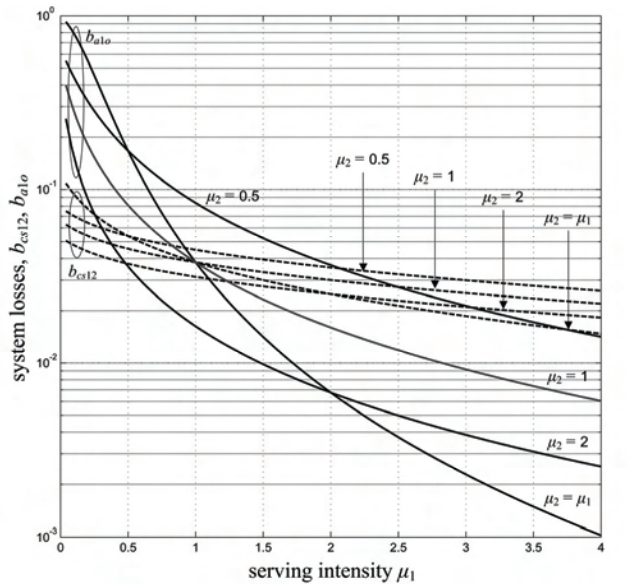

Fig. 4. Call and traffic losses in the system.

V. Conclusion

Models with overflow traffic have been relevant for over half a century and the need for their application exists in modern communication networks. In spite of the great number of papers concerning this topic, there are a few explicit solutions, when dealing with obtaining steady state equation system. Among the different models, the two are emphasized for which analytical solution is obtained. Analytical solution of the steady state equations presented in this work is a challenge which gives as a result fast calculation and possible system volume increment. For the simpler case representative enough for both models graphical representation and traffic parameters analysis is given.

Attained knowledge about parameters characteristics is of importance for traffic analysis of modern telecommunication networks in which, when using alternate routing, we can observe cases which these models are going to simulate. Results obtained using the analysis of traffic parameters behavior, justify the needs to dedicate special care to the case where phenomenon with changed serving intensity is remarked. The proposed solutions will be a motivation for further researches in accordance with the cases where various services with multi-rate and multi-quality of service are supported.

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