Teletraffic Analysis of Spectrum Handover in Cognitive Radio Networks

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Abstract – **This paper presents a well-known and general model of a serving system with primary and secondary users. A novel approximate but simple and computationally efficient analytical approach for solving the state probabilities of the state-transition diagram of the system is developed. A new precise formula for evaluation of the call dropping probability of the secondary users is derived. Channel limitation as a method for reducing the call dropping probability is analyzed. The proposed analytical approach and formulae are validated through extensive simulation experiments. Numerical results are presented and conclusions are drawn.**

Keywords – **call blocking probability, call dropping probability, channel limitation, cognitive radio network, spectrum handover.**

I. INTRODUCTION

Cognitive radio (CR) is a type of radio in which communication systems are aware of their environment and internal state and can make decisions about their radio operating behavior based on that information and predefined objectives [1]. It is capable of adjusting automatically its behavior or operations to achieve desired objectives. A brief overview of CR is presented in [2].

One of the most prominent applications of CR is in dynamic spectrum access (DSA) networks. DSA is a new paradigm for spectrum regulation which is expected to alleviate the artificially created scarcity of spectrum resources caused by the traditional static command-and-control approach for spectrum regulation. Hierarchical spectrum overlay is a promising method for DSA. It allows secondary (unlicensed or cognitive) users (SUs) to temporarily utilize spectrum resources assigned to primary (licensed or incumbent) users (PUs) if these resources are not currently being used for PU transmission. SUs have to vacate the occupied resources as soon as PUs start using them for transmission, i.e. PUs have preemptive priority over SUs. The cognitive network utilizes opportunistically the available unoccupied spectrum of the primary network on a non-interference basis. Spectrum handover is an essential function of CR since it enables and assures link maintenance and service resilience and thus facilitates the quality of service (QoS) provisioning for SUs.

II. PERFORMANCE ANALYSIS

In our study we analyze a well-known general model ([3]-[5]) of a serving system with PUs and cognitive SUs in accordance with the hierarchical spectrum overlay approach for DSA. We assume that the primary network and the cognitive network provide multimedia services with different bandwidth demands and that the bandwidth of a PU call is k times greater than the bandwidth of a SU call. Let us define the term *channel* as the necessary mean bandwidth for a PU multimedia call to be served and the term *subchannel* as the necessary mean bandwidth for a SU multimedia call to be served. It is obvious that one channel comprises k subchannels. The total bandwidth of the serving system is assumed to comprise n channels and hence nk subchannels. We denote with i ($i = 1, \dots, n$) and j ($j = 1, \dots, nk$) the number of PU and SU calls in the system, respectively. The offered PU and SU traffic is modeled by two Poisson random processes with arrival rates λ_p and λ_s, respectively. The PU and SU call durations follow a negative exponential distribution with mean $1/\mu_p$ and $1/\mu_s$, respectively.

PUs have a preemptive priority over SUs. If a PU starts transmitting on a channel, all subchannels occupied by SUs within that channel have to be vacated immediately. If a channel is being used by a PU, the subchannels within that channel are unavailable to SUs. The service of PU calls is absolutely independent of the service of SU calls.

In the model perfect spectrum sensing and spectrum handover procedures are assumed. Under these conditions, SU call blocking occurs only if there is not an unoccupied subchannel in the system to serve a new SU call. Similarly, SU call dropping occurs only if there is not an unoccupied subchannel in the system to continue the service of a SU call during spectrum handover. Since one channel comprises k subchannels, up to k SU calls may be dropped simultaneously at the arrival of a new PU call.

The described model of the serving system can be presented by a 2-D continuous time Markov chain (Fig. 1). Let us denote with $P_{i,j}$ the probability that the system is in state (i,j), i.e. the steady state probability that there are i PU calls and j SU calls in the system. Based on the state-transition diagram in Fig. 1, we can derive the global balance equations:

$$\left(\lambda_p + \lambda_s\right) P_{0,0} = \mu_p P_{1,0} + \mu_s P_{0,1}; \qquad (1)$$

$$\left(\lambda_p + \lambda_s + i\mu_p\right) P_{i,0} = \lambda_p P_{i-1,0} + \left(i+1\right)\mu_p P_{i+1,0} + \mu_s P_{i,1}, \qquad (2)$$

where $0 < i < n$;

$$n\mu_p P_{n,0} = \lambda_p \sum_{j=0}^{k} P_{n-1,j}; \qquad (3)$$

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$$\left(\lambda_p+\lambda_s+j\mu_s\right)P_{0,j}=\lambda_s P_{0,j-1}+\mu_p P_{1,j}+\left(j+1\right)\mu_s P_{0,j+1}, \qquad (4)$$

where $0<j\le\left(n-1\right)k$;

$$\left(\lambda_p+\lambda_s+j\mu_s\right)P_{0,j}=\lambda_s P_{0,j-1}+\left(j+1\right)\mu_s P_{0,j+1}, \qquad (5)$$

where $\left(n-1\right)k<j<nk$;

$$\left(\lambda_p+nk\mu_s\right)P_{0,nk}=\lambda_s P_{0,nk-1}; \qquad (6)$$

$$\left(\lambda_p+i\mu_p+j\mu_s\right)P_{i,j}=\lambda_s P_{i,j-1}+\lambda_p\sum_{m=j}^{j+k}P_{i-1,m}, \qquad (7)$$

where $i>0;\ j>0;\ ik+j=nk$;

$$\begin{aligned}&\left(\lambda_p+\lambda_s+i\mu_p+j\mu_s\right)P_{i,j}=\lambda_p P_{i-1,j}+\lambda_s P_{i,j-1}+\\&+\left(j+1\right)\mu_s P_{i,j+1},\end{aligned} \qquad (8)$$

where $i>0;\ j>0;\ \left(n-1\right)k<ik+j<nk$;

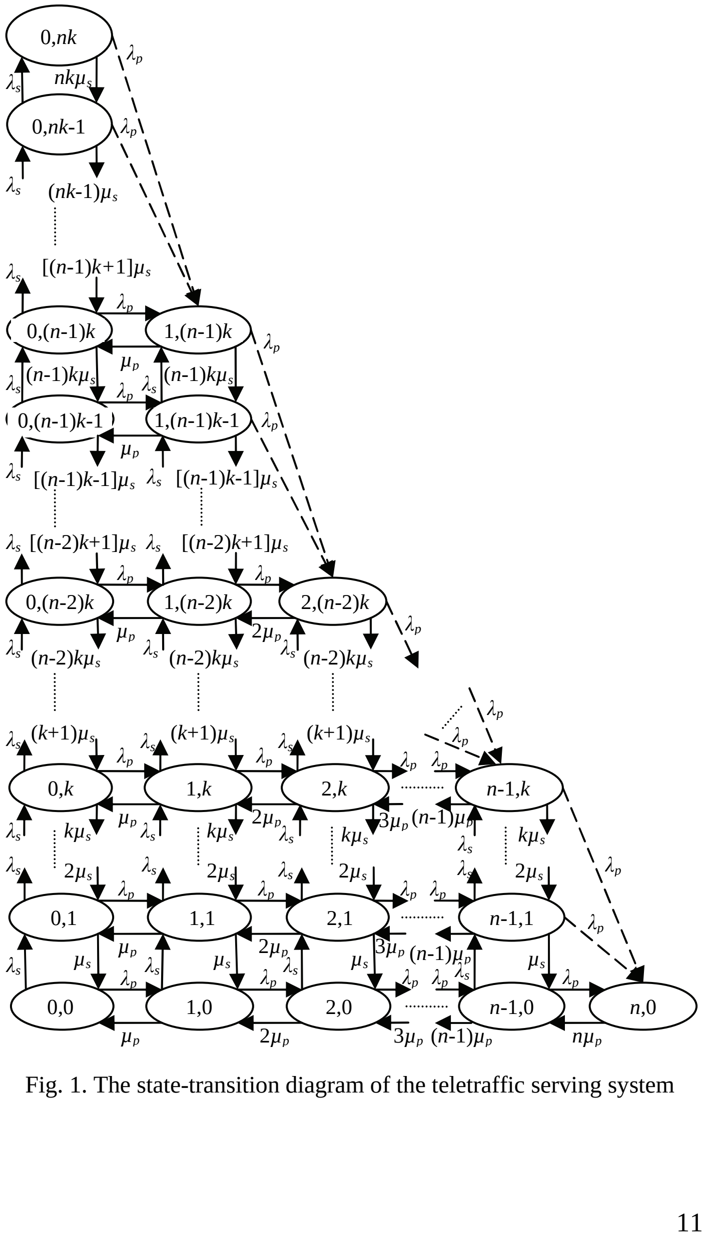

Fig. 1. The state-transition diagram of the teletraffic serving system

$$\begin{aligned}&\left(\lambda_p+\lambda_s+i\mu_p+j\mu_s\right)P_{i,j}=\lambda_p P_{i-1,j}+\lambda_s P_{i,j-1}+\\&+\left(i+1\right)\mu_p P_{i+1,j}+\left(j+1\right)\mu_s P_{i,j+1},\end{aligned} \qquad (9)$$

where $i>0;\ j>0;\ ik+j\le\left(n-1\right)k$;

$$\sum_{i=0}^{n}\sum_{j=0}^{(n-i)k}P_{i,j}=1. \qquad (10)$$

The system of Eqs. (1) - (10) contains $\frac{\left(n+1\right)\left(nk+2\right)}{2}$ unknown state probabilities and can be solved using an appropriate iterative method, such as the *Gauss-Seidel* method or the method of *successive over-relaxation* (SOR) [5],[6]. However, due to the high computational complexity of these iterative methods, their implementation in CR may be infeasible with respect to the real-time and power consumption design requirements on CR performance.

We develop and validate by simulation a novel approximate approach for solving the state probabilities of the diagram (Fig. 1) with relatively low computational complexity. Moreover, our approach is also applicable in case that channel limitation is used in the cognitive network for reducing the SU call dropping probability and thus guaranteeing the CR QoS provisioning. Channel limitation sets an upper bound on the maximum admissible number of calls in the system, i.e. it sets the call admission control (CAC) threshold. Let us denote with *l* the maximum allowable number of SU calls in the system. If channel limitation is applied, $0\le j\le l<nk$; otherwise: $0\le j\le l=nk$.

Because of the preemptive priority of PUs over SUs, our state-transition diagram (Fig. 1) differs from an ordinary multidimensional state-transition diagram and a trivial solution based on *state-based algorithms* or the *convolutional algorithm* [7] cannot be applied. Since the service of PUs is independent of the service of SUs and the PU and SU call arrival processes are i.i.d, we have:

$$P_{i,j}=P_iP_j^i=\frac{\dfrac{A_p^i}{i!}}{\displaystyle\sum_{m=0}^{n}\frac{A_p^m}{m!}}P_j^i,\ \ i<n; \qquad (11)$$

and

$$P_{n,0}=\frac{\dfrac{A_p^n}{n!}}{\displaystyle\sum_{m=0}^{n}\frac{A_p^m}{m!}}=B; \qquad (12)$$

where P_j^i is the conditional probability that there are *j* SU calls in the system provided that the number of PU calls is *i*; *B* is the PU call blocking probability.

Now we proceed to derive P_j^i by inspecting the columns of the diagram in Fig. 1 and considering limitation if applied. Let us introduce the notations:

$$t=\left(n-i-1\right)k,\ \ 0\le i<n; \qquad (13)$$

and

$$\rho=\frac{\mu_p}{\mu_s}. \qquad (14)$$

In states (i,j), where $i < n$ and $j > t$, SU call dropping occurs if a new PU call arrives, which is designated with the dashed transitions λ_p in Fig. 1. Since dropping decreases the number of SU calls in the system, the effective departure (service) rate of the SU calls in state (i,j) $\{i<n; j>t\}$ is assumed to be $j\mu_s + \dfrac{\lambda_p}{j-t}$. Based on this assumption and taking into account the optional use of limitation ($0<l\leq nk$), we solve the balance equations about column i of the state-transition diagram for SU traffic only. Thus we obtain:

$$P_j^i = \frac{\dfrac{A_s^j}{j!}}{\displaystyle\sum_{m=0}^{l} \frac{A_s^m}{m!}}, \quad t \geq l; j \leq l; i < n; \tag{15}$$

or

$$P_j^i = \frac{\dfrac{A_s^j}{j!}}{\displaystyle\sum_{m=0}^{t} \frac{A_s^m}{m!} + \frac{1}{t!} \sum_{x=t+1}^{\min[l,(n-i)k]} \frac{A_s^x}{\displaystyle\prod_{m=t+1}^{x} \left(m + \rho \frac{A_p}{m-t} \right)}}, \tag{16}$$

where $t < l; j \leq t; i < n;$

or

$$P_j^i = \frac{\dfrac{A_s^j}{t! \displaystyle\prod_{m=t+1}^{j} \left(m + \rho \frac{A_p}{m-t} \right)}}{\displaystyle\sum_{m=0}^{t} \frac{A_s^m}{m!} + \frac{1}{t!} \sum_{x=t+1}^{\min[l,(n-i)k]} \frac{A_s^x}{\displaystyle\prod_{m=t+1}^{x} \left(m + \rho \frac{A_p}{m-t} \right)}}, \tag{17}$$

where $t < l; t < j \leq \min\left[l,(n-i)k\right]; i < n.$

Substituting Eq. (15) or (16) or (17) into Eq. (11), P_{ij} is obtained.

SU call blocking occurs if all subchannels are occupied:

$$P_b = \sum_{i=0}^{n} P_{i,\min[l,(n-i)k]}. \tag{18}$$

SU call dropping occurs only if $j > t$ and a new PU call arrives. We propose a new precise formula which evaluates the SU call dropping probability by the ratio of the mean number of dropped SU calls to the mean number of SU calls in the system (instead of just summing up the probabilities of the states in which SU call dropping occurs):

$$P_d = \frac{\displaystyle\sum_{i=0}^{n-1} \sum_{j=t+1}^{\min[l,(n-i)k]} (j-t) \left(1 - e^{-\frac{\lambda_p}{j\mu_s}} \right) P_{i,j} \Delta(l,t)}{\displaystyle\sum_{i=0}^{n-1} \sum_{j=1}^{\min[l,(n-i)k]} jP_{i,j}}, \tag{19}$$

where $\Delta(l,t)$ =1 if $l > t$ and $\Delta(l,t)$ = 0 otherwise.

The mean system service rate μ_{ss} for SU calls is:

$$\mu_{ss} = \sum_{i=0}^{n-1} \sum_{j=1}^{\min[l,(n-i)k]} j\mu_s P_{i,j}. \tag{20}$$

III. NUMERICAL RESULTS

A simulation model for performance evaluation of the serving system under consideration is developed to validate the proposed analytical approach. Various simulation experiments are performed and in all cases there is a good coincidence between analytical and simulation results, as shown in Figs. 2, 3, 4 and 5, which verifies and validates the developed new approximate approach and formulae.

We first analyze the effect of PU traffic load on the performance of the cognitive network. As A_p increases, P_b and P_d also increase, as shown in Figs. 2 and 3, which means that the cognitive traffic capacity decreases. Therefore, it is reasonable to deploy CR only in primary networks whose transmission resources are underutilized in order to provide significant cognitive capacity.

Next, we analyze the effect of applying channel limitation. Figs. 2 and 3 illustrate that due to limitation it is possible to reduce P_d at the price of increasing P_b, i.e. there is a trade-off relationship between these two parameters. In general, the use of limitation is desirable since the reduction in P_d improves and facilitates the QoS provisioning in the CR network. However, the effect of limitation is significant only if A_p is relatively small. If A_p is relatively large, the performance of the CR network is no longer determined by l but by A_p since in this case the number of PU calls (not l) limits the maximum possible number of SU calls that the system can serve. Consequently, in order to apply limitation efficiently and to guarantee the QoS provisioning in the CR network, A_p must be relatively small, i.e. we again conclude that the primary network has to be underutilized in order to provide service with QoS guarantee over CR.

Figs. 4 and 5 show that channel limitation decreases both the throughput and the capacity of the CR network since the maximum number of admissible SU calls is reduced.

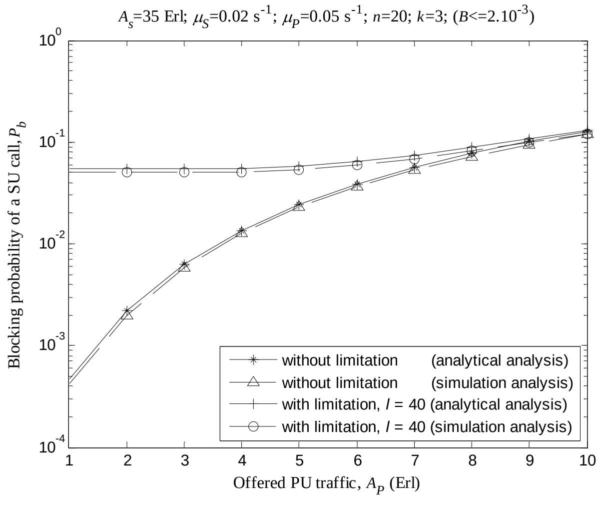

Fig. 2. SU call blocking probability versus the offered PU traffic

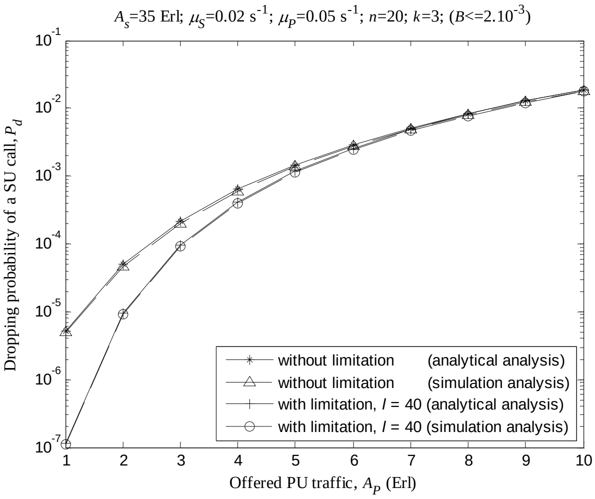

Fig. 3. SU call dropping probability versus the offered PU traffic

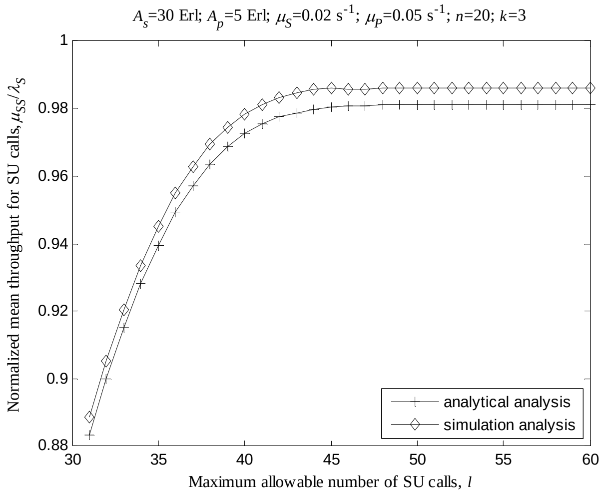

Fig. 4. Normalized mean service rate of SU calls versus the SU CAC threshold

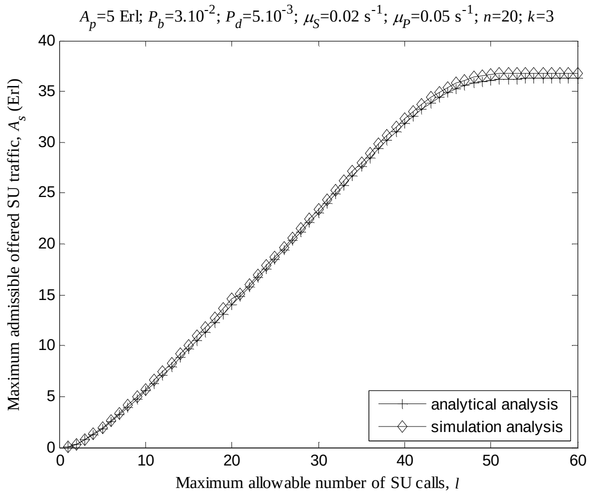

Fig. 5. Cognitive traffic capacity versus the SU CAC threshold

IV. CONCLUSION

In our paper we analyze a general model of a serving system with PUs and cognitive SUs in accordance with the hierarchical spectrum overlay approach for DSA, which makes the scope of our study generic and determines its wide applicability and theoretical significance.

A novel approximate but computationally efficient approach for solving the steady state probabilities of the state-transition diagram of the system and applicable in case of channel limitation is developed and validated by simulation. The main advantage of our approximate approach over the precise iterative methods for solving the state probabilities of the state-transition diagram of the system is its computational simplicity which facilitates its application and implementation in CR with respect to satisfying the real-time and power consumption design requirements on CR. A new precise formula for the SU call dropping probability is also proposed.

Our study corroborates that the effect of PU traffic on the capacity and performance of CR should always be considered. Channel limitation decreases the CR throughput and capacity but facilitates the SU QoS provisioning if prudently applied.

For future research work, we plan to use the analytical model presented in this paper for the design of a cognitive CAC algorithm capable of providing QoS for heterogeneous multimedia services over CR.

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