M/M/k Queues Modelled by Using of Petri Net Simulator

Zoran Gacovski[1] and Emilija Kamceva[2]

Abstract - **In this paper we present a Petri net simulator that will be used for modeling of queues. The concept of queue is ubiquitous in the everyday life. Queues are present at the airports, banks, shops etc. Many factors are important in the studying of queues – some of them are: the average waiting time in the queue, the usage of the server, expected length of the queue etc. The paper is organized as follows: first we give a brief overview of the queuing theory. After that – we describe the Petri nets, as a tool for modeling of discrete event systems. We have implemented Petri net simulator that is capable for queue modeling. Finally we are presenting the simulation results for M/M/3 queue and its different properties. We've also made comparison between theoretically obtained results and our simulation results.**

Keywords – **Queuing theory, Petri net Simulator, M/M/k queue, Simulink.**

I. INTRODUCTION

The concept of queue line is one of the most adequate modeling techniques for the class of discrete-event system. Even though they evolved as an outcome of pure pragmatic reasons, in the early 1940 in Great Britain, their appliances developed and grow in a variety of aspects of the modern society, particularly concentrated on the Information and Communications Technologies. The exact analytic solutions provide the response for each relevant parameter toward system performance from the structural, dynamic and behavioral aspects. According to various researches, on the annual bases are spent more than 37 billion of hours on some of the queue strings types. Due to these reasons, the optimization of the attitude on this class of system has not only academic, but also enormously practical meaning. The length of this kind of waiting in everyday life is a stochastic process. For this kinds of "waitings" - information science has developed special branch of learning called the theory of queue lines.

The theory of line queues is a mathematical study of the length of the line. The theory of line queues represents and provides a mathematical analysis of several related processes, including arrival at the end of the line, waiting in line, and servicing the users by the repairers in order. The theory of queue lines allows distribution and calculation of several measures of performance, including the average time waiting in line or system, the expected number of customers waiting

to be served or the one who already have been served, and the probability to find the system in certain situations, such as empty, full, available service at the moment, or taking some time waiting to be serviced [1].

II. PETRI NETS IN MATLAB

Petri Nets offer profound mathematical background originating namely from linear algebra and graph theory. Various Petri Net tools offer convenient graphical environment and sometimes they provide complex simulation and analysis of various high level Petri Net classes. Petri Net (PN) is mathematical and graphical modeling tool well suited for describing and analyzing discrete events systems (DES). PNs allow to model and visualize systems, which contain concurrence, resource sharing or synchronization. These possibilities allow them to be used for various applications in areas including computer systems, communication protocols, flexible manufacturing systems and software verification [2,3].

Within the mentioned context, the initiative of developing instruments for simulation, analysis and design of PNs under MATLAB brought remarkable benefits for training and research, because Control Engineering people are familiar with the exploitation of *Graphical User Interfaces* (GUIs) [4] based on this popular software. Although a recent list of the programs developed for PNs includes many resources (Mortensen, 2003) running under different operating systems, our initiative was successful due to the large preferences shown for MATLAB.

It is worth separately mentioning that the overall design and implementation gives us the **PN Toolbox**, which is well integrated with MATLAB, and allows further developments in the modern direction of studying hybrid dynamics involving both DES and ODE models [5,6].

After ending a simulation experiment, several *Performance Indices* are available to globally characterize the simulated dynamics. Some of the indices recorded for the transitions of the net refer to: the total number of firings during the simulation (*Service Sum*), the mean frequency of firings (*Service Rate*), the mean time between two successive firings (*Service Distance*), the fraction of time when server is busy (*Utilization*). For the places of the net, the recorded indices refer to: the total number of arrived (*Arrival Sum*) and departed (*Throughput Sum*) tokens, the mean time between two successive instants when tokens arrive in (*Arrival Distance*) and depart from (*Throughput Distance*) the place, the mean time a token spends in a place (*Waiting Time*), the average number of tokens weighted by time (*Queue Length*) [7].

Only for timed or (generalized) stochastic PNs, the time evolution for both current and global values of a *Performance Index* may be displayed dynamically while in the *Step* and

[1]Zoran Gacovski is with the Faculty of Information Technology, FON University, Bul. Vojvodina, bb 1000 Skopje, Macedonia, E-mail: zoran.gacovski@fon.edu.mk.

[2]Emilija Kamceva is with the Faculty of Information Technology, FON University, Bul. Vojvodina, bb 1000 Skopje, Macedonia, E-mail: emilija.kamceva@fon.edu.mk.

Run Slow simulation modes by means of the *Scope* command. Another facility available only for timed or (generalized) stochastic PNs is *Design*, which can be used for the synthesis of the models. One or two *Design Parameters* varying within intervals defined by the user can be included in the model. For each test-point belonging to this (these) interval(s) a simulation experiment is performed in the *Run Fast* mode. The dependence of a *Design Index* on the *Design Parameter*(*s*) can be visualized as a graphical plot (2-D or 3-D, respectively) [8].

III. PETRI NET QUEUE SIMULATOR

The M/M/3 queue has been observed and toward it we have built a Petri network, shown on Figure 1. We have been carried out theoretical and simulated surveys for these queues. We have used the PN Toolbox for Matlab for our simulation. Typical for these types of queues is that theoretical results are calculated without considering the execution time for the system, while in simulation - the execution (service) time is also considered. From this point - obviously follows a diminutive difference between the obtained theoretical and simulation calculations.

During reviews and research of the queues, the most important properties that are required to be obtained are:

- λ - arrival rate of users
- μ - customer service rate
- ρ - occupancy rate of the server
- $E(X)$ - expected length of the line of customers in systems that have come and have been served
- $E(S)$ - average time in the system
- $E(W)$ - average waiting time in the line
- $E(R)$ - the flow of the users in the system

The M/M/3 queue model is shown on Figure1. It can be used to model discrete event systems, e.g., machines or operators, processing orders or communication equipment processing information.
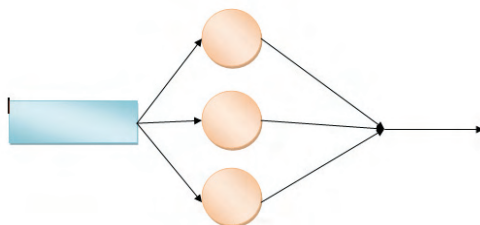

Fig. 1. Basic M/M/3 queue line

IV. SIMULATION RESULTS

Case 1 - The Figure 2 shows the model of M/M/3 queue after performed simulation for population of 10000. The value of the customer arrival rate is $\lambda = 0,1$ and the value of the service rate is $\mu = 6,667$. We are presenting both theoretically obtained and simulation results. The time required to perform this simulation is 428,323 seconds.
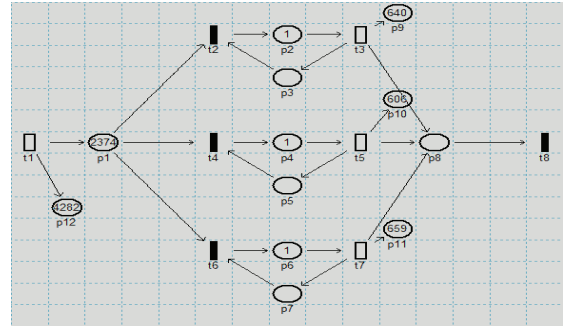

Fig. 2. Petri network for the M/M/3 queue - simulation for case 1

TABLE I
THEORETICAL AND SIMULATION RESULTS FOR THE CASE 1

M/M/3	lamda	mi	ρ	po	p1
Theoretical	0,100	0,667	0,050	0,868	0,130
Simulation	0,100	0,675	0,049	0,870	0,129
Error %	0,003	1,250	1,231		

	p2	p3	E(U)	E(X)	E(S)	E(W)
Theor.	0,010	0,000	0,150	0,150	1,500	0,000
Simul.	0,010	0,000	0,148	0,148	1,481	0,000

Case 2 - Figure 3 shows the M/M/3 queue after performed simulation for population of 10000 for $\lambda = 0,1$ и $\mu = 0,392$. In Table 2 are shown the results obtained theoretically and by simulation. The time required to perform this simulation is 2573,458 seconds.
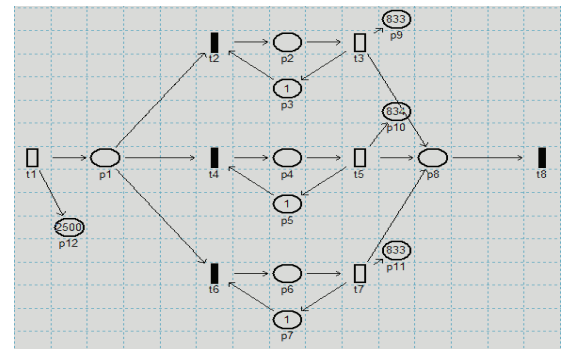

Fig. 3. Petri network for the M/M/3 queue - simulation for case 2

TABLE II
THEORETICAL AND SIMULATION RESULTS FOR THE CASE 2

M/M/3	lamda	Mi	ρ	po	p1
Theoretical	1,000	0,392	0,850	0,017	0,043
Simulation	0,971	0,324	1,000	0,000	0,000
Error %		2,858	17,428	14,646	

	p2	p3	E(U)	E(X)	E(S)	E(W)
Theor.	0,055	0,047	0,313	0,313	2,863	2,863
Simul.	0,000	0,000	0,333	0,343	3,431	3,333

Case 3 - Figure 4 shows the M/M/3 queue after the performed simulation for population of 10000, for $\lambda = 1$ и $\mu = 0,337$, then in Table 3 are shown, the results obtained theoretically and by simulation. The time required to perform this simulation is 2524,93 seconds.
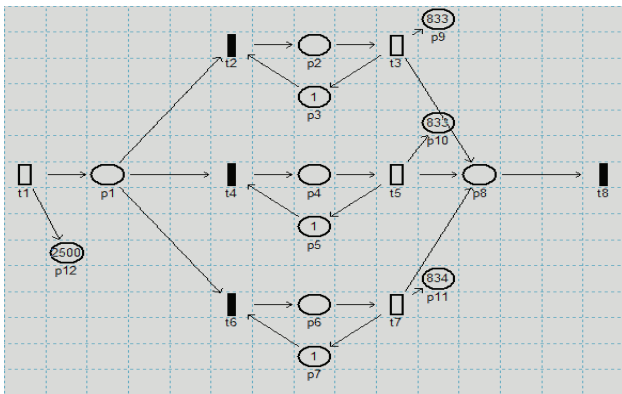

Fig. 4. Petri network for the M/M/3 queue - simulation for case 3

TABLE III
THEORETICAL AND SIMULATION RESULTS FOR THE CASE 3

M/M/3	lamda	mi	ρ	po
Theoretical	1,000	0,337	0,990	0,001
Simulation	0,990	0,330	1,000	0,000
Error %	0,987	1,990	1,023	

p1	p2	p3	E(U)	E(X)	E(S)
0,002	0,003	0,003	0,332	0,332	3,302
0,000	0,000	0,000	0,333	0,337	3,367

Case 4 - Figure 5 shows the M/M/3 queue after the performed simulation for population of 10000, for $\lambda = 1$ и $\mu = 0,337$, then in Table 4 are shown the results obtained theoretically and by simulation. The time required to perform this simulation is 2591,79 seconds.
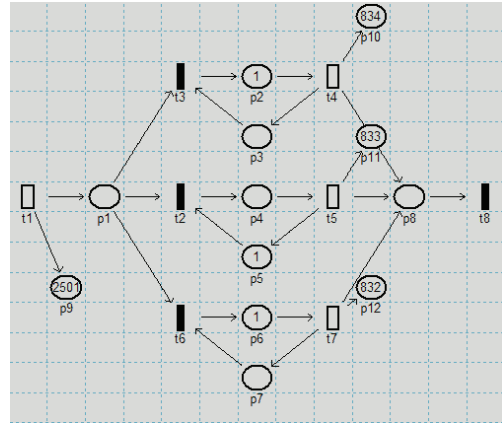

Fig. 5. Petri network for the M/M/3 queue - simulation for case 4

TABLE IV
THEORETICAL AND SIMULATION RESULTS FOR THE CASE 4

M/M/3	Lamda	Mi	ro	Po
Theoretical	1,000	0,303	1,100	-0,006
Simulation - 10000	0,965	0,321	1,001	0,000
Error %	3,503	6,061	9,017	

p1	p2	p3	E(U)	E(X)	E(S)
-0,019	-0,031	-0,034	0,342	0,342	3,642
0,000	0,000	0,000	0,334	0,346	3,457

After several simulations performed for the queue M/M/3 by using the concept of Petri nets, we have concluded that: by comparing the values of arrival rate and the service rate (theoretical and simulation research), we have obtained similar values with a small error, which expressed in percents - does not exceed 15%. This confirms the accuracy of our research of the M/M/3 queue.

If we compare the time of execution for different values of ρ, we can conclude that by increasing the value of ρ - the time of execution of the simulation is reduced (work of the system).

In the last case - for Petri network model of the M/M/3 queue, we've taken the value $\rho = 1.1111 > 1$. This contradicts the theoretical results, because of the definition $\rho < 1$. But for the same default values for the arrival and the

service rate, after performed simulation - we've received value $\rho < 1$, which confirms the accuracy of the simulator and the goal of our research. That shows a possible blockade in the system.
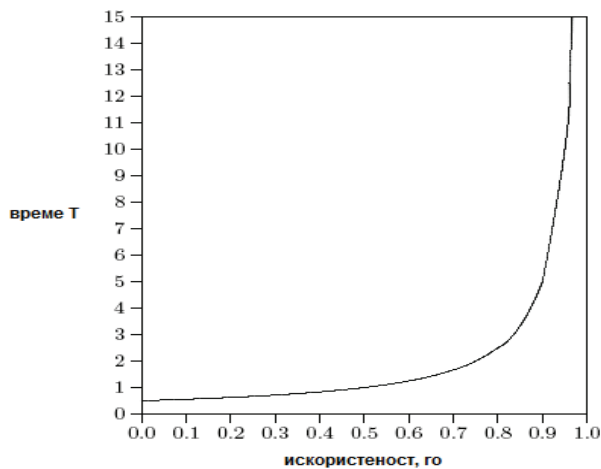

Fig. 6. The usage of the server.

V. CONCLUSION

In this paper we've presented a Petri net simulator that was used for modeling of queues. Many factors are important in the studying of queues – we have considered the following: the average waiting time in the queue, the usage of the server, expected length of the queue etc. First we gave a brief overview of the queuing theory. After that – we described the Petri nets, and a software tool for modeling of discrete event systems. We have implemented Petri net simulator (via PN Matlab toolbox) that is capable for queue modeling. Finally we have calculated the simulation results for M/M/3 queue and its different properties (and also its theoretical values). We've made comparison between our simulation results and theoretically obtained results and we have shown significant matching.

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